

Planetary-Orbit Modeling based on Astrodynamical Coördinates

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The planetary orbits are, in general, the elliptical orbits with sun at one of the foci (Fig. 1). The problem is, traditionally, formulated using the plane-polar coördinates, not a natural choice for such type of trajectories. The two-body problem has 12 degrees-of-freedom (3 translational, 3 rotational for each of the two bodies), which were reduced to 6 by neglecting structures. Further, setting up the problem in the center-of-mass (CM) frame-of-reference, separated the lagrangian into 2 terms, the first one representing motion of CM and the second motion about CM. Realizing that CM was either at rest or moving with a uniform velocity in the absence of external forces, the first term was constant and, hence, played no role in the lagrangian equations, which, only, involved derivatives. Hence, the problem was reduced to 3 degrees-of-freedom. In the absence of external torques, angular momentum (a vector quantity having both magnitude and direction) was conserved. A fixed direction of angular momentum in space forced two-body orbits to lie in a plane, making the problem 2 dimensional. The orbital equation of motion was formulated using the elliptic-astrodynamical-coördinate mesh, evolved from the elliptic-cylindrical-coördinate mesh. This further reduced the degrees-of-freedom and the problem became a true one-parameter problem. Kepler's equation was shown to be a particular solution of this equation. This formulation generated 3 (instead of the customary 2) constants of motion*.

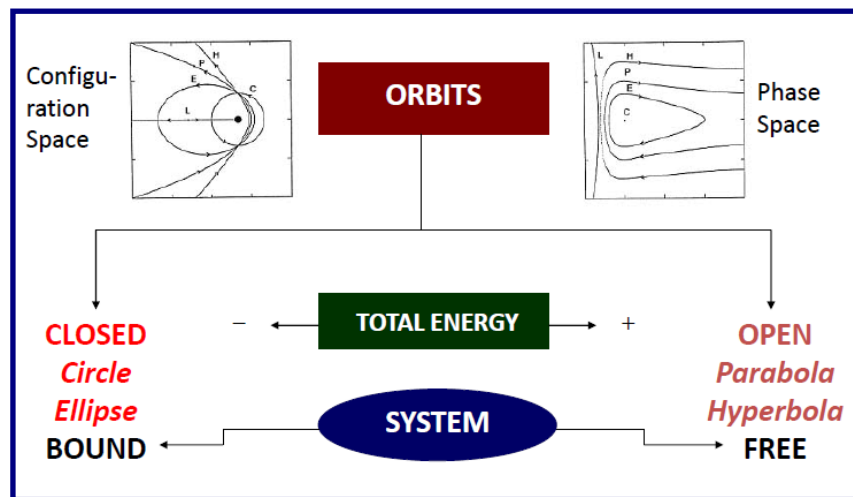


Fig. 1. Types of orbits

Keywords: Two-body problem • Central force • Constants of motion • Center-of-mass frame • Kepler's equation

Web address of this document: <https://www.ngds-ku.org/Presentations/Planetary.pdf>

*5-year later these results have been published in conference proceedings — Kamal, S. A. (2003). Incompleteness of cross-product steering and a mathematical formulation of extended-cross-product steering. *Proceedings of the Fourth International Bhurban Conference on Applied Sciences and Technologies (IBCAST 2002)*, June 10-15, 2002, Volume 1, Advanced Materials, Computational Fluid Dynamics and Control Engineering, edited by H. R. Hoorani, A. Munir, R. Samar and S. Zahir, National Center for Physics, Islamabad, Pakistan, pp. 167-177, full text: <https://www.ngds-ku.org/Papers/C56.pdf>