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Two-Body Problem in the Elliptic-Astrodynamical-Coördinate Mesh

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Fig. 1. Prof. Dr. Syed Arif Kamal delivering his invited lecture during the Plenary Session, in which a new branch of mathematics was introduced, named as *astromathematics*

The plane-polar coördinates are not the natural choice for setting up two-body problem. The elliptic-astrodynamical coördinate mesh (ξ, E, z) — the first one is ellipse-shape coordinate, $\frac{1}{2ae}\ln\frac{1+\in}{1-\in}$ (a is semi-major axis of elliptical orbit, e is eccentricity, $\in = \sqrt{1-e^2}$), the second one is elliptic-eccentric anomaly, the third one is representing direction of angular momentum of the orbit — have been custom-designed to handle orbital problems based on central-force motion of two bodies https://www.ngds-ku.org/pub/confabst.htm#C60**:** which exhibit 12 degrees-of-freedom (3 translational and 3 rotational degrees-of-freedom for each of the two bodies). Mathematical framework available at present can not generate exact solutions for more than 2 degrees-of-freedom. The 12 degrees are reduced to 6 by considering each of the 2 bodies as point masses and neglecting structures. Further, setting up the problem in the center-of-mass (CM) frame-of-reference, separates the problem into 2 terms, the first one represents motion of CM and the second motion about CM. Realizing that CM is either at rest or moving with a uniform velocity in the absence of external forces, the first term is constant and, hence, plays no role in the lagrangian equations, which, only, involve derivatives. Hence, the problem is reduced to 3 degrees-of-freedom. In the absence of external torques, angular momentum (a vector quantity having both magnitude and direction) is conserved. A fixed direction of angular momentum in space forces two-body orbits to lie in a plane, making the problem 2 dimensional. The orbital equation of motion is formulated using the elliptic-astrodynamical-coördinate mesh, evolved from the elliptic-cylindrical coördinates. This, further, reduces the degrees-of-freedom and the problem becomes a true one-parameter problem. Using a special substitution, the first integral of the dynamical equation (a nonlinear one) is obtained. Kepler's equation is shown to a particular solution https://www.ngds-ku.org/Papers/C56.pdf of the resulting dynamical equation. In addition, scale factors of the elliptic-astrodynamical-coördinate mesh are obtained as**:** $h_\xi = \frac{A}{B}$; $h_E = a\sqrt{1-e^2\cos^2 E}$; $h_z = 1$ ($A = 1-e\cos E$, $B = \frac{\xi}{a}$). This formulation brings out 3 constants of motion, instead of the customary 2. Control laws used in maneuvering targeted spacecrafts and satellites, the normal-component-dot-product steering https://www.ngds-ku.org/Papers/C55.pdf and the ellipse-orientation steering https://www.ngds-ku.org/pub/confabst.htm#C64**:** are expressed using this formulation, which could be used to check and correct deviations from the guidance path using the cross-range-corrected-[$] and the multi-stage- https://www.ngds-ku.org/Papers/C72.pdf Lambert schemes as well as the multi-stage-Q system[¥]. Technological implications of this formulation include *Air-Spacecraft of the Third Millennium*, a thematic aircraft proposed after taking into account green-engineering principles[£]. Mathematical structure of this formulation forms the basis of ***Astromathematics***, which focuses on geometry to study orbits from a kinematical perspective (Fig. 1). In contrast to *astrodynamics*, the force expressions do not, explicitly, appear in the formulation of astromathamatics. Even if there appears a need to study force interactions, these are expressed as space-time-curvature equivalents. This formulation seems to be, generically, more suitable for accelerated frames governed by *geometrodynamics*.

*Keywords***:** Astromathematics • Equation of motion • Keplarian motion

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