



## Down-Range and Cross-Range Errors for Short-Range-Ballistic Missiles (SRBMs)

Syed Arif Kamal<sup>\*, §</sup> 

PhD; MA, Johns Hopkins, United States (Astrophysics); MS, Indiana, Bloomington, United States (Gravitation Physics); MSc (Mathematical Physics with special courses in Space Physics)<sup>β</sup>; BSc (Honors); last 2 summa cum laude; Gold Medalist (4 Times); Throughout First-Class-First Position; Listed in “Marquis’ Who’s Who in the World” and “Dictionary of International Biography”

Referee, ‘Astrophysics & Space Science’ (Springer Nature), ‘Chemical Thermodynamics & Thermal Analysis’ (Elsevier) and ‘Optics & Laser Technology’ (Elsevier)  
Published in ‘Chinese Journal of Physics’ (Elsevier), member AIAA

Visiting Faculty, Department of Aeronautics and Astronautics, Institute of Space Technology; Consultant, GNC (Guidance, Navigation & Control) Laboratories and Ex-Senior Scientific Officer, Control-Systems Laboratories, SUPARCO (Plant), Pakistan Space & Upper Atmosphere Research Commission, Box 8402, PO Karachi 75270, Pakistan<sup>¶</sup>

Meritorious Professor, Ex-Acting Vice Chancellor, Ex-Dean, Faculties of Science & Engineering and Ex-Chairman, Departments of Applied Physics, Computer Science & Mathematics, University of Karachi, PO Box 8423, Karachi 75270, Pakistan

\*[profdrakamal@gmail.com](mailto:profdrakamal@gmail.com)

**Abstract** – Down-range error in the flight of a missile occurs, when the missile remains in the pre-assigned plane, but strikes the earth before or ahead of the intended target. Cross-range error occurs, when the missile does not remain in the preassigned plane. Control laws, the extended-cross-product steering and dot-product steering are expressed in the elliptical- as well as the astrodynamical-coördinate mesh. These laws expressed for coördinate perpendicular to the trajectory, but in the orbital plane (*e. g.*, radial coördinate,  $r$ , for the circular orbit, expressed in the plane-polar coördinate formulation) should eliminate down-range error, whereas, the same control expressed for coördinate normal to the orbital plane (*e. g.*, normal coördinate,  $z$ , which is normal to the circular orbit) should eliminate cross-range error. In this work, detailed calculations are given to compute these errors for SRBM (range 300-1000  $km$ ; acceleration due to gravity considered constant during the entire flight; curvature of earth and location of target at the end of flight time approximated as same as the one at the launch time). In the MRBM (Medium-Range-Ballistic Missile; range 1000-3000  $km$ ) modeling of acceleration due to gravity (altitude, rotation-of-earth, mass distribution of earth, difference between equatorial and polar radii) is performed, whereas in the IRBM (Intermediate-Range-Ballistic Missile; range 3000-5500  $km$ ) curvature of earth is taken in account. ICBM (Intercontinental-Ballistic Missile; range more than 5500  $km$ ) is most far-reaching missile, whose mission design is a fixed time-of-flight (transfer time), boundary-value problem, in which rotation-of-earth is taken into consideration to compute the location of target at the end of transfer time.

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**Keywords** – Elliptic-astrodynamical-coördinate mesh, Hyperbolic-astrodynamical-coördinate mesh, ICBM, IRBM, MRBM, Normal-component-cross-product steering, Normal-component-dot-product steering

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<sup>§</sup>Institutional e-mail: [sakamal@uok.edu.pk](mailto:sakamal@uok.edu.pk) • Current affiliation: Department of Physics (taught in courses in Institute of Space Science & Technology, 2021-2022; Institute of Space & Planetary Astrophysics, 2004-2006; Department of Computer Science, 1999-2006, Additional Charge as Chairman, Department of Computer Science, 2015-2016 and Faculty Member, Department of Mathematics, 1995-2016, Chairman, Department of Mathematics, 2003-2006, 2009-2012), University of Karachi; worked as Professor in Department of Mathematics, Jinnah University for Women, Karachi 74700, Pakistan, 2017-2019

<sup>β</sup>100% marks in all subjects during the final semester

<sup>¶</sup>Highest security clearance by virtue of author’s service in Pakistan Space and Upper Atmosphere Research Commission