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The Control Laws (the Extended-Cross-Product Steering and the Dot-Product Steering) Expressed in the Hyperbolic-Astrodynamical-Coördinate Mesh

Syed Arif Kamal*,§

Meritorious Professor and Ex-Acting Vice Chancellor and Ex-Dean, Faculties of Science & Engineering, University of Karachi, PO Box 8423, Karachi 75270, Pakistan

Visiting Faculty, Department of Aeronautics and Astronautics, Institute of Space Technology; Consultant, GNC (Guidance, Navigation and Control) Laboratories and Ex-Senior Scientific Officer, Control-Systems Laboratories, SUPARCO (Plant), Pakistan Space and Upper Atmosphere Research Commission, PO Box 8402, Karachi 75270, Pakistan

*profdrakamal@gmail.com

Abstract – The plane-polar coördinates not being the optimal choice for setting up the two-body problem, the two-body problem was formulated in the elliptic-astrodynamical-coördinate and the hyperbolic-astrodynamical meshes. Lagrangian, Hamiltonian and the equation of motion have been written for these coördinate meshes. Earlier, control laws, the extended-cross-product steering (to drive a vector to zero, it is sufficient to anti-align its time rate-of-change with the vector itself) and the dot-product steering (to drive a vector to zero, the dot product of this vector with its time rate-of-change added to the product of respective magnitudes must approach zero) have been formulated in the elliptic-astrodynamical-coördinate mesh. Here, these control laws are expressed in the hyperbolic-astrodynamical-coördinate mesh. The coördinate triad in the elliptic-astrodynamical-coördinate mesh, (ξ, E, z) [ξ the elliptical-shape coödinate, E the elliptic-eccentric anomaly and z the z coödinate in the catesian-coördinate mesh, representing direction of the relative angular momentum of orbit] is replaced by (κ, H, z) [κ the hyperbolic-shape coördinate and H the hyperbolic-eccentric anomaly] the hyperbolic-astrodynamical-coördinate mesh. For eliminating the down-range error, the velocity vector, perpendicular to desired orbit, but lying in the orbital plane, must be brought to zero, whereas for eliminating the cross-range error the velocity vector, normal to the orbital plane, must be brought to zero. For keeping a check on the down-range error, the extended-cross-product steering takes the form, $\vec{\boldsymbol{v}}_{\mathrm{K}} \times \dfrac{d\vec{\boldsymbol{v}}_{\mathrm{K}}}{dt} \to 0, \dfrac{d\left|\vec{\boldsymbol{v}}_{\mathrm{K}}\right|}{dt} < 0,$ and the dot product steering, $\vec{\boldsymbol{v}}_{\mathrm{K}} \cdot \dfrac{d\vec{\boldsymbol{v}}_{\mathrm{K}}}{dt} + \left|\vec{\boldsymbol{v}}_{\mathrm{K}}\right|\left|\dfrac{d\vec{\boldsymbol{v}}_{\mathrm{K}}}{dt}\right| \to 0.$ For vanishing of the cross-range error, the expessions employ the z coördinate — the extended-cross-product steering, $\vec{\boldsymbol{v}}_{\mathrm{Z}} \times \dfrac{d\vec{\boldsymbol{v}}_{\mathrm{Z}}}{dt} \to 0, \dfrac{d\left|\vec{\boldsymbol{v}}_{\mathrm{Z}}\right|}{dt} < 0,$ and the dot-product steering $\vec{\boldsymbol{v}}_{\mathrm{Z}} \cdot \dfrac{d\vec{\boldsymbol{v}}_{\mathrm{Z}}}{dt} + \left|\vec{\boldsymbol{v}}_{\mathrm{Z}}\right|\left|\dfrac{d\vec{\boldsymbol{v}}_{\mathrm{Z}}}{dt}\right| \to 0.$

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Keywords – Orbit theory, Hyperbolic-eccentric anomaly, Hyperbolic-shape coördinate, Satellite-launch vehicle (SLV), Two-body problem

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§Current affiliation: Department of Physics, University of Karachi