

Gauss' Law: Choice of the Gaussian Surface and Form of the Electric-Field Vector

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Fig. 1. Receiving shield from Professor Mohammed Idrees Khan (center) after delivering lecture on Gauss' law (Mohammed Khalid Rajput is seen behind the rostrum)

Gauss' law is one of the fundamental laws of electromagnetism, which is, primarily, used to determine the electric-field vector, if the system possesses certain symmetries. This is possible, provided a proper Gaussian surface is selected, according to the following criteria:

- Field point (where the electric-field vector needs to be determined) must lie on the Gaussian surface.
- It should be a closed surface (well-defined interior and exterior) — *most common types of Gaussian surfaces are boxes, cylinders and spheres.*
- It could be an imaginary or a real (coinciding with a physical boundary) surface — *for example, spherical surface, with center at origin, could lie inside a spherical-charge distribution of uniform density (an imaginary surface) or the boundary (a real surface).*
- It should be chosen considering symmetries of the system, as suggested by strong-Noether's theorem <https://www.ngds-ku.org/Presentations/Noether.pdf> — *for example, boxes, cylinders and spheres are chosen for surface-, line- or spherical-charge distributions, respectively.*
- Direction of the electric-field vector should either be normal or tangential to any chosen section of surface — *for a uniform-line-charge distribution, a cylindrical surface is selected as a Gaussian surface (the electric-field vector is normal everywhere on the curved surface and tangential on both of the plane surfaces)*
- Magnitude of the electric-field vector should be constant throughout each section of surface, allowing one to take it outside the integral — *for example, spherical surface with center at origin is, generally, chosen as Gaussian surface for a spherical-charge distribution of uniform density; magnitude of electric field, being a function, only, of radial coordinate (distance from center of distribution), is constant over this surface*

Examples were worked out for computing electric fields generated by (i) an infinite plane charge sheet, with constant surface-charge density and (ii) an infinite line charge, with constant line-charge density, with special emphasis to determine the form of electric-field vector. Similarities and differences of electricity and magnetism as well as electricity and gravitation were highlighted. An expression of Gauss' law for gravitation was presented and illustrated with computation of gravitational field inside and outside the earth. In the context of Ampère-circuital law, line integral of magnetic-flux density was computed along a closed curve. Criteria for choosing this curve, to most efficiently determine the magnetic field, were, also, mentioned. The speaker had the profound privilege to receive shield from the legendary physics teacher, Professor Mohammed Idrees Khan (Fig. 1).

Keywords: Ampère -circuital law • Similarities/Differences of electricity and gravitation • Similarities/Differences of electricity and magnetism • Space-time symmetries • Strong-Noether's theorem

Web address of this document: <https://www.ngds-ku.org/Presentations/Gauss.pdf>

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