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From the Linear Model of Wright and Kydd to the Covariant-Enhanced-Coupling Model of Global-Electrocortical-Activity: A Journey of 37 Years

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Brain being a very complex structure, modeling of functioning of brain is emerging as a most significant field of modern times. This topic has long been of interest for biologists, mathematicians and physicists. Brain is a very intricate system, which contains around the order of 10^{11} neurons with approximately 10^{15} synaptic connections. However, when the speaker started working on this problem in 1987, taking the linear model of Wright and Kydd as the baseline, and developing a covariant model by writing the state-transition matrix of the signal equations, transformed in the laboratory frame from the comoving frame, the complexity arose from the fact that this matrix was of the order of $10^{16} \times 10^{16}$ (for each synaptic connection, the generalized-electromagnetic potential had 4 components — one for the electrical potential and 3 for the magnetic-vector potential), hence 10^{32} parameters had to be handled. In 1987, the world's fastest supercomputer (Cray XL 3 in Los Alamos Laboratories) could not even write this matrix (these days, it is possible using memory sharing and cloud computing). The problem was handled in Department of Physics, University of Karachi employing mathematical (group-theoretical) techniques. The model of Wright and Kydd rests on many simplifications and overlooks issues of cell-to-cell coupling as well as details of anatomy. A mass of unit sources, which are coupled to one another may be described by a set of *n* equations (*n* is the number of synaptic connections), representing driven-harmonic oscillators, $\ddot{\varphi}_i + D_i(t)\dot{\varphi}_i + N_i^2(t)\varphi_i = K_i^j(t)\varphi_j$ ($\dot{\varphi}_i$ and $\ddot{\varphi}_i$ are the first and the second time derivatives of the electrical potential), written in Einstein convention, in which repeated (dummy) indices denote summation, appearing as superscript and subscript (no summation on *i*'s). Such a representation is suitable for advanced computational neuroscience for constructing models based on the tensorial representation (*e. g.*, the covariant-enhanced-coupling model), which should be invariant under the scaled-Poincaré transformations, the most general coördinate transformations put forward 14-*year* ago (Fig. 1). $D_i(t)$, $N_i(t)$, $K_i^j(t)$ are free parameters equivalent to damping coefficients, natural frequencies and coupling constants, respectively. These parameters were assigned physiological meaning based on the assumptions that these parameters have a finite variance about a mean and they are stochastically independent, hence justifying the applicability of the Central Limit Theorem of Cramer for large *n*. These free parameters were, then, replaced by their respective means. In the covariant model, the signal equations, in the segment of the dendritic tree, were written in the comoving frame of the signal. When transformed into the laboratory frame, a magnetic-vector potential appeared along with the electrostatic potential (origin of magnetoencephalogram). Effects of external weak magnetic fields as well as external weak electromagnetic fields were studied and group structure explored. In the generalized-coupling model the signal equations are modified as (the electrical potentials depended, not only, on the neighboring potentials, but also, on their first time derivatives — $M_i^j(t)$ being free parameters) $\ddot{\varphi}_i + D_i(t)\dot{\varphi}_i + N_i^2(t)\varphi_i = K_i^j(t)\varphi_j + M_i^j(t)\dot{\varphi}_j$. Subsequently, the covariant-generalized-coupling model was constructed and effects on EEG during weightlessness studied. In the enhanced-coupling model, signal equations were re-written as

$$\ddot{\varphi}_i + D_i(t)\dot{\varphi}_i + N_i^2(t)\varphi_i = K_i^j(t)\varphi_j + M_i^j(t)\dot{\varphi}_j + L_i^j(t)\ddot{\varphi}_j$$

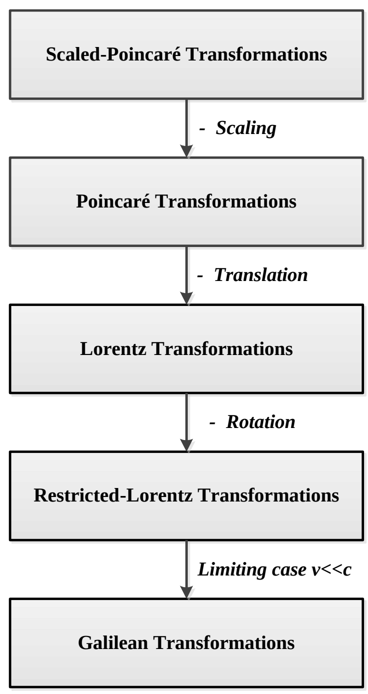

Fig. 1. From the Galilean transformations (1632) to the scaled-Poincaré transformations (2009) — a journey of 377 *years*

$$\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
-N_1^2 & -D_1 & -1 & K_1^2 & M_1^2 & L_1^2 & \cdots & K_1^n & M_1^n & L_1^n \\
0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\
K_2^1 & M_2^1 & L_2^1 & -N_2^2 & -D_2 & -1 & \cdots & K_2^n & M_2^n & L_2^n \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
K_3^1 & M_3^1 & L_3^1 & K_3^2 & M_3^2 & L_3^2 & \cdots & K_3^n & M_3^n & L_3^n \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\cdots & & & & & & & & & \\
\cdots & & & & & & & & & \\
K_n^1 & M_n^1 & L_n^1 & K_n^2 & M_n^2 & L_n^2 & \cdots & -N_n^2 & -D_n & -1
\end{bmatrix}$$

Fig. 2. State-transition matrix for the covariant-enhanced-coupling model of global-electrocortical activity

to include enhanced coupling, which depended on φ_i's, $\dot{\varphi}_i$'s and $\ddot{\varphi}_i$'s ($L_i^j(t)$ being free parameters having finite variance about a mean). From this model, the covariant-enhanced-coupling model was constructed and group structure explored. The covariant-enhanced-coupling-state-transition matrix was a linear transformation (Fig. 2). The set of these matrices formed a group under the binary operation of matrix multiplication. The group identity corresponded to the physiological state of *brain death*. Effects of external weak magnetic fields were studied. This lecture is dedicated to the loving memory of **Professor Samuel J. Williamson** (November 6, 1939, West Reading, Pennsylvania - April 25, 2005, New York). He obtained his BS and ScD (equivalent to PhD) in Physics in 1961 and 1965, respectively, from MIT (Massachusetts Institute of Technology, Cambridge, Massachusetts). He started his professional career at MIT's Francis Bitter National Magnet Laboratory (which the speaker had an opportunity to visit in 1990) and remained there until 1971, when he joined Department of Physics at the New York University. In 1977 he was promoted to professorship and in 1987 got additional appointment as Professor of Neural Science. In 1989 he become a University Professor and served there till his retirement in 2000 (Fig. 3). The speaker had a chance to meet Professor Williamson, when the former presented a paper in *the Seventh International Conference on Biomagnetism* held during August 14-18, 1989, and then again in 1991, when the legendary professor gave him a tour of his research labs.

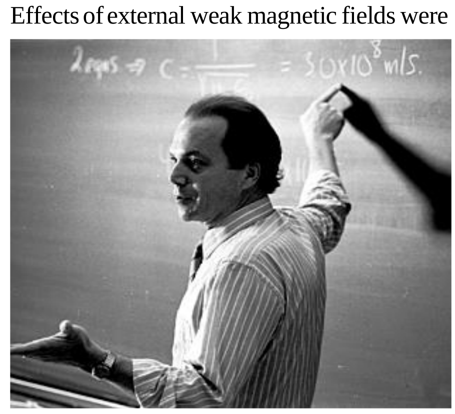

Fig. 3. Samuel J. Williamson

*Keywords***:** Covariant model • Generalized-coupling model • Magnetoencephalography • Mathematical definition of brain death • Neuromagnetic response

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