RELATIVISTICALLY STREAMING PLASMAS CONTAINING DYONS*

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Burman's generalized Ohm's law for relativistically streaming plasmas is obtained for particles endowed with both electric and magnetic charge. Binary plamas are also treated and the results are compared with Burman's results.

Burman et al. (1976) introduced a technique for treating energy dissipation and associated diffusion of magnetic field lines resulting from 'frictional' interactions between different species in nonrelativistic plasmas. Burman (1977) obtained generalized Ohm's law for relativistically streaming plasmas. Bacry and Kuber-Andre (1973) discussed the existance of particles endowed with both electric and magnetic charge (dyons). In this case the usual expression of Lorentz force is replaced by an expression in which both electric and magnetic charges are taken into account and Maxwell's equations are modified to involve monopoles. If the plasma contains such particles Burman's (1977) formulism has to be modified. The modified calculations show a symmetry between electric and magnetic forces which was not present in the Lorentz force expression.

Consider a plasma consisting of N species of any kind.

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For the rth component species $k_r = e_r/m_{or}$, $k_r' = g_r/m_{or}$ where e_r and g_r are the electric and magnetic charges (Bacry and Kuber-Andre 1973) and m_{or} is the proper mass (or rest mass), ρ_r is the mass density, ρ_{or} is the proper mass density, $\bar{v}_r$ the fluid or streaming 3-velocity with the corresponding Lorentz factor γ_r. We also neglect pressure gradients. The fluid velocities are unrestricted, but the thermal speeds are nonrelativistic implying that $\rho_r = \gamma_r^{\,2} \rho_{or}$. The force density that acts on the rth species through its 'frictional' interactions with the other species is denoted by $\rho_r \bar{F}_r$. As usual $\bar{E}$, $\bar{H}$, c and t denote the electric and magnetic fields, the free-space speed of light and the time coordinate. Bacry and Kuber-Andre (1973) gave the expression for the electromagnetic force acting on a particle of electric charge e and magnetic charge g and moving with velocity $\bar{v}$

$$(1) \qquad f = e(\bar{E} + \bar{v} \times \bar{B}) + g(\bar{B} - \bar{v} \times \bar{E})$$

in MKS units. Therefore Buram's (1977) equation of motion of the rth species is to be replaced by

$$(2) \quad \rho_r \bar{A}_r = k_r \rho_{or} \gamma_r (\bar{E} + c^{-1} \bar{v}_r x \bar{H}) + k_r' \rho_{or} \gamma_r (\bar{H} - c^{-1} \bar{v}_r x \bar{E}) + \rho_r \bar{F}_r$$

We consider the situation in which $\bar{B} = \bar{H}$ (approx.). The factor c^{-1} apprears because Gaussian system is used. Rindler (1966) (sections 57 and 58) defined $\bar{A}_r$ by

$$(3a,b) \quad \rho_r \bar{A}_r = \partial(\rho_r \bar{v}_r)/\partial t - \nabla.(\rho_r \bar{v}_r \bar{v}_r) = \rho_{or} \gamma_r (\partial/\partial t - \bar{v}_r.\nabla)(\gamma_r \bar{v}_r)$$

We define

$$(4a\text{-}c)\quad \rho = \sum \rho_r, \quad \rho\bar{v} = \sum \rho_r \bar{v}_r, \quad \hat{p} = \sum \rho_r(\bar{v}_r - \bar{v})(\bar{v}_r - \bar{v})$$

$$(4d\text{-}e)\quad \nu = \sum k_r \rho_{or} \gamma_r, \quad \nu' = \sum k_r' \rho_{or} \gamma_r$$

$$(4f\text{-}g)\quad \bar{j} = \sum k_r \rho_{or} \gamma_r \bar{v}_r, \quad \bar{j}' = \sum k_r' \rho_{or} \gamma_r \bar{v}_r$$

where the summation is over r from 1 to N. These relations define the mass density, barycentric velocity and pressure tensor of the plama as a whole (equations 4a-c), togrther with the net electric and magnetic charge densities and magnetic current densities (equations 4d-g).

Summing (3a) over all species, using (4a-c) and noting that $\sum \rho_r \bar{v}_r \bar{v}_r = \rho\bar{v}\bar{v} - \hat{p}$, we get

$$\sum \rho_r \bar{A}_r = \partial(\rho\bar{v})/\partial t + \nabla\cdot(\rho\bar{v}\bar{v}) + \nabla\cdot\hat{p}$$

Summing (2) over all species, using (4d-g) and taking ${}_r\bar{F}_r = 0$, we obtain the equation of motion of the plasma as a whole

$$(5)\quad \partial(\rho\bar{v})/\partial t + \nabla\cdot(\rho\bar{v}\bar{v}) + \nabla\cdot\hat{p} = \nu\bar{E} + c^{-1}\bar{j}\times\bar{H} + \nu'\bar{H} - c^{-1}\bar{j}'\times\bar{E}$$

Taking the scalar product of $\rho_r\bar{A}_r$ (eq. 2) with $\bar{v}_r$, summing the result over all species, using definition (4f,g) of $\bar{j}$ and $\bar{j}'$, defining $\bar{J}_r$ as $\rho_r(\bar{v}_r - \bar{v})$ and hence substituting for $\bar{v}_r$, and then using (5) leads to

$$(6)\quad \bar{j}\cdot\bar{E} + \bar{j}'\cdot\bar{H} = \bar{v}\cdot\left\{\partial(\rho\bar{v})/\partial t + \nabla\cdot(\rho\bar{v}\bar{v})\right\} + \bar{v}\cdot(\nabla\cdot\hat{p}) + \sum \bar{J}_r\cdot(\bar{A}_r - \bar{F}_r)$$

We analyze the last term which represents processes associated with the relative velocities of the species.

Since in ordinary space no more than three vectors can be linearly independent, the N relative momentum densities and the N 'frictional' forces can be expressed in terms of three basic nonorthogonal vector fields

(7a,b) $\qquad \bar{J}_r = \sum_{i=1}^{3} S_{ri}\bar{m}_i, \qquad \bar{F}_r = \sum_{i=1}^{3} A_{ri}\bar{m}_i$

The $\bar{m}_i$ can be chosen as generalized currents. By definitions (4a) and (4b) of and $\bar{v}$, the $\bar{J}_r$ sum to zero, so that

(8) $$\sum S_{ri} = 0$$

Using equations (7), (6) can be written as

(9) $$\bar{j}\cdot\bar{E} + \bar{j}'\cdot\bar{H} + \sum_{i=1}^{3}\bar{m}_i\cdot\bar{X}_i = \bar{v}\cdot\partial(\rho\bar{v})/\partial t + \nabla\cdot(\rho\,\bar{v}\bar{v}) + \bar{v}\cdot(\nabla\cdot\hat{p}) + Q$$

Here

(10a,b) $\qquad \bar{X}_i = -\sum S_{ri}\bar{A}_r, \qquad Q = \sum_{i=1}^{3}\sum_{j=1}^{3} K_{(ij)}\bar{m}_i\cdot\bar{m}_j$

with

(10c) $$K_{ij} = -\sum S_{ri}A_{rj}$$

the symmetric and skewsymmetric parts being denoted by $K_{(ij)}$ and $K_{/ij/}$ respectively.

Multiplying (2) by S_{ri}/ρ_r, summing over all the species and using (8) gives

(11) $$-\sum S_{ri}\bar{F}_r = \alpha_i\bar{E} + c^{-1}(\sum k_r S_{ri}\bar{v}_r/\gamma_r)\times\bar{H} + \alpha_i'\bar{H} - c^{-1}(\sum k_r' S_{ri}\bar{v}_r/\gamma_r)\times\bar{E} + \bar{X}_i$$

where

(12a,b) $\qquad \alpha_i = \sum k_r S_{ri}/\gamma_r, \qquad \alpha_i' = \sum k_r' S_{ri}/\gamma_r$

Suppose that α_1 and α_1' donot vanish, so that eq. (11) for i = 1 can be written as

$$(13)\quad -\sum S_{ri}\bar{F}_r = \alpha_1\bar{E} + c^{-1}\alpha_1\bar{u}_H \times \bar{H} + \alpha_1'\bar{H} - c^{-1}\alpha_1'\bar{u}_H' \times \bar{E} + \bar{X}_i$$

where $\bar{u}_H$ and $\bar{u}_H'$ are the velocity fields given by

$$(14a)\quad \bar{u}_H = \sum(k_r S_{r1}\bar{v}_r)(\gamma_r\alpha_1) = \bar{v} + \sum_{i=1}^{3}\sum(k_r S_{ri}\bar{m}_i S_{r1})/(\gamma_r\rho_r\alpha_1)$$

$$(14b)\quad \bar{u}_H' = \sum(k_r' S_{r1}\bar{v}_r)/(\gamma_r\alpha_1') = \bar{v} + \sum_{i=1}^{3}\sum(k_r' S_{ri}\bar{m}_i S_{r1})/(\gamma_r\rho_r\alpha_1')$$

If we write

$$(15)\qquad \bar{X}_1 = \alpha_1\bar{E}' + \alpha_1'\bar{H}'$$

the generalized Ohm's law can be written as

$$(16)\quad -\sum S_{ri}\bar{F}_r = (\alpha_1\bar{E} + \alpha_1'\bar{H}) + c^{-1}(\alpha_1\bar{u}_H \times \bar{H} - \alpha_1'\bar{u}_H' \times \bar{E}) + (\alpha_1\bar{E}' + \alpha_1'\bar{H}')$$

where $\bar{E}'$ and $\bar{H}'$ are the equivalent electric and magnetic fields arising from the inertial or acceleration term in the equation of motion (2). Camparing (16) with Burman's (1977) result for the generalized Ohm's law

$$(17)\quad -\sum S_{ri}\bar{F}_r = (\alpha_1\bar{E}) + c^{-1}(\alpha_1\bar{u}_H \times \bar{H}) + (\alpha_1\bar{E}')$$

we note that $\alpha_1\bar{E}$ is replaced by $(\alpha_1\bar{E} + \alpha_1'\bar{H})$, $(\alpha_1\bar{u}_H \times \bar{H})$ is replaced by $(\alpha_1\bar{u}_H \times \bar{H} - \alpha_1'\bar{u}_H' \times \bar{E})$ and $(\alpha_1\bar{E}')$ by $(\alpha_1\bar{E}' + \alpha_1'\bar{H}')$. Eq. (16) is symmetric between electric and magnetic fields whereas eq. (17) is not so.

Now we discuss binary plasmas. Ardavan (1976) obtained a generalized Ohm's law for relativistic non-neutral binary plasmas with inertial effects included. He restricted the plasmas to those in which the particles are nonrelativistic in the proper frame of the plasma. Burman (1977) removed this restriction by obtaining a generalized Ohm's law for binary plasmas using eq. (17). Here we obtain generalized Ohm's law for binary plasmas containing dyons. Let n_r and n_{or} denote the number density and proper number density of particles of the rth species, Ardavan's (1976) symbol ρ denotes (in the present notation) $\sum m_r n_r$ that is, $\sum \rho_r / \gamma_r$ reducing to ρ/γ on putting $\gamma_r = \gamma$ (approx.). But with two species present, only the first of the $\overline{m}_i$ is needed in (7a). Since $\overline{J}_1 + \overline{J}_2 = 0$, $S_{11} + S_{21} = 0$ and so from (12)

$$\alpha_1 = k_1 S_{11}/\gamma_1 + k_2 S_{21}/\gamma_2, \qquad \alpha_1' = k_1' S_{11}/\gamma_1 - k_2' S_{21}/\gamma_2$$

Therefore

$$(18a) \qquad S_{11}/\alpha_1 = (k_1/\gamma_1 - k_2/\gamma_2)^{-1} = -S_{21}/\alpha_1$$

$$(18b) \qquad S_{11}/\alpha_1' = (k_1'/\gamma_1 - k_2'/\gamma_2)^{-1} = -S_{21}/\alpha_1'$$

With eq. (18) the generalized Ohm's law (16) becomes

$$(19) \quad \overline{F}_2 - \overline{F}_1 = (k_1/\gamma_1 - k_2/\gamma_2)(\overline{E} + c^{-1}\overline{u}_H \times \overline{H} + \overline{E}') + (k_1'/\gamma_1 - k_2'/\gamma_2)(\overline{H} - c^{-1}\overline{u}_H' \times \overline{E} + \overline{H}')$$

with

$$(20a) \quad \overline{u}_H = (k_1\overline{v}_1/\gamma_1 - k_2\overline{v}_2/\gamma_2)/(k_1/\gamma_1 - k_2/\gamma_2)$$

$$(20b) \qquad = \overline{v} + \overline{i}(k_1/\gamma_1^3\rho_{o1} + k_2/\gamma_2^3\rho_{o2})/(k_1/\gamma_1 - k_2/\gamma_2)^2$$

$$(20c) \quad \overline{u}_H' = (k_1'\overline{v}_1/\gamma_1 - k_2'\overline{v}_2/\gamma_2)/(k_1'/\gamma_1 - k_2'/\gamma_2)$$

$$(20d) \qquad = \overline{v} - \overline{i}'(k_1'/\gamma_1^3\rho_{o1} - k_2'/\gamma_2^3\rho_{o2})/(k_1'/\gamma_1 - k_2'/\gamma_2)^2$$

where

$$(20e,f) \qquad \bar{i} = \alpha_1 \bar{m}_1, \qquad \bar{i}' = \alpha_1' \bar{m}_1$$

$$(20g,h) \qquad \bar{E}' = (\bar{A}_2 - \bar{A}_1)/(k_1/\gamma_1 - k_2/\gamma_2)$$

$$\bar{H}' = (\bar{A}_2 - \bar{A}_1)/(k_1'/\gamma_1 - k_2'/\gamma_2)$$

The unspecified parameters α_1 and α_1' have cancelled from the generalized Ohm's law (19) and from the expressions for $\bar{u}_H$, $\bar{u}_H'$, $\bar{E}'$ and $\bar{H}'$. No restriction has been placed on $\bar{F}_r$. If we look and Burman's (1977) result

$$(21) \qquad \bar{F}_2 - \bar{F}_1 = (k_1/\gamma_1 - k_2/\gamma_2)(\bar{E} + c^{-1}\bar{u}_H \times \bar{H} + \bar{E}')$$

we note that $\bar{F}_2 - \bar{F}_1$ depends on two terms in our calculations but there was only one term in Burman's calculations.

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