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Avoiding Infinities from the Lorentz and the Poincaré Transformations

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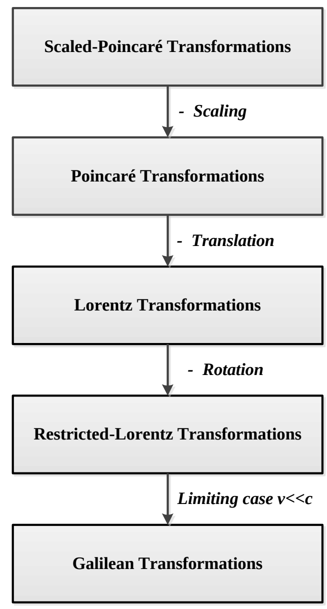

Fig. 1. Coördinate transformations

From the historical point-of-view, this paper gave history of conceptual development of relativity as well as mathematical formulation of Lorentz transformations. Contrary to the popular belief, giving credit of all contributions in special relativity to Albert Einstein, the paper put into perspective the fundamental conceptual contributions of Muslim scientists, *e. g.*, space and time, relative and absolute, long before, Einstein presented his theory. One year before the publication of Einstein's paper, Poincaré (1904) enunciated the principle of relativity. Also, complete mathematical framework was available through works of W. Voigt (1887), J. Larmore (1897) and H. A. Lorentz (1892, 1895, 1899, 1904). Long before that Ibn-é-Sina discussed space and time in his *Risala-é-Tabiyaat* (Journal of Physics). In his book, *Aghaz-o-Anjaam* (Beginning and End), Nasiruddin Tusi (1238 Iranian Year) says that time ascribes everything and, therefore, something is first and something is last relative to time. Time ordering of events (causality) occupies an important place in modern relativity theory. Tusi, further, observes that the entire universe is ascribed by space and in this connection, something is exposed and something is hidden relative to space. In addition, he comments that space and time are not complete in themselves. In theory of relativity space and time are not considered as separate things, but time is considered as a coördinate like space coördinates. Sadruddin Sheerazi, in *Asfar-é-Arbăa* (Four Journeys), says about the doubt of Fakruddin Raazi (*Ouoon-ul-Hikmat* — Springs of Knowledge) that Behmenyar mentioned this doubt and then, himself, refuted it. Behmenyar thinks that if the existence of motion is disproved, then it is that motion, which is called 'absolute motion'. He thinks that absolute motion is not, externally, present. But Sheerazi thinks that 'relative motion' is not, externally, present. He argues that absolute motion has not any such form in the external as that of things, which are stationary. One may notice that even Lorentz adhered to the notion of absolute rest and absolute motion, whereas Sheerazi and others introduced and discussed the concepts of relative motion. In his paper, *Zur Electrodynamik bewegter Korper* (on the Electro-dynamics of Moving Bodies), published in 1905, Albert Einstein combined these existing conceptual and mathematical formula-tions into an integrated and a unified approach, without giving reference to these contributions. Herman Minkowski formulated the relativity theory in terms of a four-dimensional-vector-field formulation. As early as 1911, it was shown that the assumption of existence of an invariant velocity was not necessary for the derivation of Lorentz transformations. Recami and Mignani (1974) generalized special relativity and set the relevant postulates in the form**:** (*i*) spacetime is homogeneous and space is isotropic, (*ii*) principle of relativity — physical laws of mechanics and electromagnetism are required to be covariant, when passing from one inertial frame to another frame in rectilinear, uniform relative motion and (*iii*) principle of retarded causality (equivalent to Dirac, Stuckelberg, Feynman and Sudershan reinterpretation principle) — negative-energy particles traveling forward in time do not exist. From only the postulates (*i*) and (*ii*) (without intervention of any assumption about the invariant character of the light speed) the remaining principles of special relativity may, actually, be deduced, such as the linearity of transformations and the existence of an invariant squared speed. Taking signature of the metric as $(-1,+1,+1,+1)$, the Poincaré transformations may be written as $X^{\alpha} = \Lambda^{\alpha}_{\delta} x^{\delta} + a^{\alpha}; \alpha, \delta = 0,1,2,3$ (differential form $dX^{\alpha} = \Lambda^{\alpha}_{\delta} dx^{\delta}$), where X^{α} and x^{δ} are spacetime-vector-field formulations in 4 dimensions, relating coördinates of the two frames in uniform, rectilinear motion. Λ is 4×4 transformation matrix, whose nonzero elements are $\Lambda^{1}_{1} = \Lambda^{4}_{4} = \gamma, \Lambda^{2}_{2} = \Lambda^{3}_{3} = 1, \Lambda^{1}_{4} = i\beta\gamma = -\Lambda^{4}_{1}$, $\beta = \dfrac{v}{c}, \gamma = 1\Big/\sqrt{1 - v^2/c^2}$ (v is relative velocity of the frames and c is velocity of light in free space). The condition $X = Y = Z = 0$ should give history of the space origin of the transformed frame with respect to the laboratory frame. A corollary of this condition is that $c\beta$ represents velocity of the frame α with respect to the frame δ. Poincaré transformations are undefined if frame velocity is taken equal to the velocity of light in free space, resulting in infinities appearing in field theories (one has to integrate field-equation terms from $-\infty$ to $+\infty$ to account for all possible values of energy and momentum). JP Hsu tried to avoid infinities by abandoning the idea of different times in different frames and introducing cosmological time, at the same time making (free space) velocity of light different in different frames. The author suggests to avoid infinities by (*a*) using non-inertial frame to work out Lorentz transformations, which may be generalized for Poincaré transformations; (*b*) realizing that the problem seems to exist because the factor γ approaches ∞ as $v \to c \Rightarrow \beta \to 1$, since in the relativistic quantum domain an accurate determination of v makes the factor $1/\gamma$ indeterminate[¶]; (*c*) appreciating that the singularity in Poincaré transformations at $v = c$ is a removable singularity. Defining a new coördinate mesh (scaled-Poincaré mesh), scaled[*] by powers of γ, the author is able to avoid infinities and preserve the form invariance of physical laws (Fig. 1). Introducing $dx^{\alpha}_{\mathrm{SC}} = dx^{\alpha}/\gamma$, scaled-Poincaré transformations may be expressed as $dX^{\alpha}_{\mathrm{SC}} = \Lambda^{\alpha}_{\delta} dx^{\delta}_{\mathrm{SC}}$. At $v = c, \beta = 1, \gamma = \infty$. Hence, the transformed coördinates become $dx^{0}_{\mathrm{SC}} = dx^{0} - dx^{1}$, $dx^{1}_{\mathrm{SC}} = dx^{1} - dx^{0}$, $dx^{2}_{\mathrm{SC}} = 0 = dx^{3}_{\mathrm{SC}}$, which are, clearly, finite. However, now the scaled coördinates depend on relative velocity of two frames[#]. The invariant ds^2 now takes the form $\gamma^2 ds^2_{\mathrm{SC}}$. Applying this scaling to momentum and current-density vector fields, one gets $p^{\alpha} \to \gamma p^{\alpha}_{\mathrm{SC}}$, $j^{\alpha} \to \gamma j^{\alpha}_{\mathrm{SC}}$. To preserve the form invariance of equations, the following scaling rules are proposed**:** (*i*) Corresponding to each super-index of a tensor field, divide the scaled coördinate by a factor of γ, in order to compensate for γ s appearing in the numerator for each of the involved transformation matrices; (*ii*) For each sub-index of a tensor field, multiply the scaled coördinate by a factor of γ. Using this recipe, invariance of equation of continuity $\nabla . \boldsymbol{j} + d\rho/dt = 0$ and the electrodynamic equations $F_{\alpha\beta,\gamma} + F_{\beta\gamma,\alpha} + F_{\gamma\alpha,\beta} = 0, F^{\alpha\beta}_{\ \ ,\beta} = 4\pi\, j^{\alpha}$ under scaled transformations was shown[§]. The scaling may be considered as mapping of a given spacetime-manifold-coördinate chart to the scaled-manifold-coördinate chart, in which velocity (numerical components) remains same, accelerations become undefined at $v = c$, momentum remains defined, suggesting that electrodynamics should be done using momentum approach instead of acceleration approach. This work circumvents the notorious difficulty of infinities without invoking any radical change in the physico-mathematical construct of relativity and, hence, the rest of physics. The formulation may be useful to astronomers, astrophysicists, field theorists, particle physicists as well as condensed-matter physicists, as the scheme has the potential to simplify formulations in quantum kinetic theory. Similar solutions need to be found out for infinities appearing in curvature-tensor fields useful in gravitation physics.

*Web address of this document***:** https://www.ngds-ku.org/Presentations/CIIT.pdf

[¶]The claim that a frame is traveling exactly at the velocity of light itself https://www.ngds-ku.org/Papers/J10.pdf implies that velocity has been determined exactly.

[*]Scaling, as coördinate transformation, is used in CAD/CAM and photocopying. Similar triangles in Euclidean geometry are related by scaling transformation.

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Quantum field theory • Relativity • Singularities

[#]This does not pose a problem as the transformed coördinates are, already, functions of frame velocity.

[§]The components of electromagnetic-tensor field are

$$F_{\alpha\beta} = -F_{\beta\alpha}, F_{0i} = -E_i, F_{ij} = \in_{ijk} B^k;\ F^{\alpha\beta} = -F^{\beta\alpha}, F^{0i} = E^i, F^{ij} = \eta^{ik}\eta^{jl} \in_{klm} B^m;\ i, j, k, l, m = 1,2,3$$