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Cross-Lattice Formulation Applied to Liquid-Crystal Structure

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Reciprocal lattice is, widely, used in condensed-matter physics and materials science. The expressions of reciprocal-lattice vectors, having dimensions of inverse of length, are changed, https://www.ngds-ku.org/Presentations/Cross.pdf when different systems of units are used. In order to alleviate this problem, a modified lattice, termed as *cross lattice*, https://www.ngds-ku.org/Papers/J07.pdf was introduced. The cross-lattice vectors were constructed in such a way that they became independent of the coördinate system chosen. A coördinate-independent representation is a pre-requisite of representing generalized laws and forms the basis of principle of general covariance https://www.ngds-ku.org/Presentations/CIIT.pdf — this was the motivation to present the most general coördinate transformation. Let $\hat{\boldsymbol{a}}_1$, $\hat{\boldsymbol{a}}_2$, $\hat{\boldsymbol{a}}_3$, be the basis vectors for ordinarty lattice. Any vector $\boldsymbol{L}$, may be represented in terms of these linearly-independent bases $\boldsymbol{L} = L_k \hat{\boldsymbol{a}}_k$ (repeated indices denote summation according to Einstein convention; all latin indices take up the values 1, 2, 3). The recipriocal-latttice vectors for a non-rectangular lattice, may be constructed, using the basis vectors ($\in_{klm}$ is Levi-Civita symbol)

Fig. 1. Professor Dr. Syed Arif Kamal delivering his invited lecture during the Fourth Session, in which a new branch of mathematics was introduced, named as *condensed-matter mathematics*

$$\hat{\boldsymbol{g}}_k = \in_{klm} \frac{\hat{\boldsymbol{a}}_l \times \hat{\boldsymbol{a}}_m}{2\hat{\boldsymbol{a}}_1 \bullet \hat{\boldsymbol{a}}_2 \times \hat{\boldsymbol{a}}_3}$$

as $\boldsymbol{G} = G_k \hat{\boldsymbol{g}}_k = 2\pi n_k \hat{\boldsymbol{g}}_k$; the numbers n_k are integers, this condition is obtained by applying the periodicity condition $f(r+L) = f(r)$. The reciprocal lattice vectors satisfy the condition $\hat{\boldsymbol{a}}_i \bullet \hat{\boldsymbol{g}}_j = \delta_{ij}$; where δ_{ij} is Knoecker delta. A cross product of the direct- and the rciprocal-lattice vectors is dimensioneless, and named as *cross lattice*, $\hat{\boldsymbol{e}}_i = \hat{\boldsymbol{a}}_i \times \hat{\boldsymbol{g}}_i$ (no summation over i). The cross-lattice length can be written as $\boldsymbol{D} = D_k \hat{\boldsymbol{e}}_k = 2\pi n_k \hat{\boldsymbol{e}}_k$.The following results can, easily, be derived from the above deinition, $\boldsymbol{D} \bullet \boldsymbol{L} = \boldsymbol{D} \bullet \boldsymbol{G} = 0$; $\exp(i\boldsymbol{D} \bullet \boldsymbol{L}) = \exp(i\boldsymbol{D} \bullet \boldsymbol{G}) = 1$ ($i = \sqrt{-1}$). The eigenfunctions of cross lattice can be obtained by considring an anisotropic potential, which depends on angle, θ. The band must be well separated to make non-diagonal matrix elements vanish with respect to band index. Defining a modified Brillouin zone to a Wigner-Seitz cell in cross lattice, and letting $\boldsymbol{D}$ take all values inside this zone, all the coefficients vary and become functions of $\boldsymbol{D}$. However, because of the periodicity in the momentum space, they are all identical functions of $\boldsymbol{D}$, https://www.ngds-ku.org/Presentations/Physics3-1.pdf translated in cross-lattice space. Wannier functions are useful for systems, which are *isotroptic*, but *inhomogeneous*.The cross eigenfunctions, belonging to eigenstates of angular momentum in cross-lattice representation, are useful for systems, which are *homogeneous*, but *anisotropic*. Such systems are found in liquid crystals. We, now, have the necessary techniques availble to study these systems. Future work may involve mathematical derivation of eigenfunctions, which are both *inhomogeneous* and *anisotropic*. Application of mathematical techniques to problems in solid-state physics may bring out new avenues of research, forming the basis of ***Condensed-Matter Mathematics*** (Fig. 1).

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