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ACCURACY AND PRECISION OF MOIRE FRINGE TOPOGRAPHY SYSTEM

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Abstract: Moiré fringe topography is a simple, noncontact, noninvasive, nondestructive, photogrammetric technique which provides three-dimensional map of a study subject/object (human body, precision instrument) without using X rays or any other ionizing radiation. The technique of moiré fringe topography consists of photographing the study object through a specially constructed screen. Dark fringes are produced because of the presence of screen. Accuracy and precision of shadow type, fishing line moiré fringe topography system are determined.

1. Introduction

The need to find a convenient three-dimensional measuring system led to the development of photogrammetry in the middle nineteenth century. Medical photogrammetry is the term used to cover all applications of photogrammetry in the broad field of medicine. These include stereo-photogrammetry, holography, integrated surface imaging system (ISIS), 3-D video laser scanning system, moiré fringe topography and rasterstereography.

The important advantages of photogrammetry over conventional methods for medical applications are (Lane 1983):

- Photogrammetry is a noninvasive, noncontact and nondisruptive technique. It avoids risks involved in hurting, infecting, or distorting the

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human subject being studied.

- Photogrammetry can make it easy to measure objects otherwise inaccessible or difficult to measure.
- Photogrammetry enables to keep permanent records in the form of photographic images on paper, videotape or diskette.
- Body structures, which could not be seen or illustrated before, may often be studied by photogrammetry.
- Any desired degree of accuracy ma be achieved by making a suitable choice of equipment and technique.
- Data obtained from photogrammetric measurements especially if put in coordinate form may be easily utilized by computer systems.

We are, therefore, in need of methods which are inexpensive, easy to implement, simple to be performed by moderately trained personnel and elegant enough to permit handling by various algorithms. Moiré fringe topography (Creath, Wyant 1992) and rasterstereography both have all these characteristics (Kamal 1998). The first one provides height maps and the second curvature maps. If we are able to record and process information from both these techniques we may obtain a complete profile of human trunk during every cycle of gait. Such information may be used in modeling of human gait and understanding of various neurological disorders.

2. Moire Fringe Topography System

Moiré fringe topography is a photogrammetric technique which gives a three-dimensional profile of the object under study. The technique is simple, noninvasive and noncontact in nature. As such the technique is most suitable to study objects which are difficult to access. Moiré system can be of shadow type, projection type or grating hologram type. In our laboratory we have developed shadow type moiré systems using fishing line and argenza (cloth) as well as a grating hologram type system. A rasterstereography system has also been developed. Construction of the moiré frame has been described elsewhere (Akram 1989; Ahmad, Hashmi 1990; Kamal, Bukhari, Akram 1990a,b). Moiré techniques are known to us since 70's (Takasaki 1970; 1973). These days, moiré fringe topography is being widely applied in the detection (Adair, van Wijk, Armstrong 1977;

Kamal, El-Sayyad 1981; Kamal, Naseeruddin, Waseem 1996; Kamal, Naseeruddin, Waseem, Firdous 1998; Kamal, Yosufzai 1979), documentation (Willner 1979), quantification (El-Sayyad, Kamal 1981; 1982; Kamal 1982a-c; 1983a,b; 1987; 1988; 1996a-c; Kamal, Akram, Bukhari 1988; Kamal, Akram, Siddiqui, Khan 1989; Kamal, Choudhry, Siddiqui 1996; Kamal, El-Sayyad 1981; Kamal, El-Sayyad 1981; Kamal, Lindseth 1980; Siddiqui, Choudhry 1990; Yosufzai, Kamal, Zubairi 1995; Zubairi 1994). Moiré exam must be an integral part of physical examination of every athlete (Akram, Kamal 1991).

Reproducibility of moiré technique has been investigated (Kamal 1990; Kamal, Benoni, Willner 1994). Some authors have performed error analysis and determined accuracy of their systems (Karras 1990; Klein, Rooze 1990; van Wijk 1980; 1981). There is a need to establish a protocol for checking the accuracy and the precision of our moiré systems.

For a shadow type moiré system (Terada 1974) the depth of a bright fringe may be expressed as

$$\Delta_n = nL[n - d/s]^{-1} \tag{1}$$

and a dark fringe as

$$\Delta_n = (n - ½)L[d/s - (n - ½)]^{-1} \tag{2a}$$

where 's' is the pitch of moiré grid, 'L' the distance from the moiré screen to the observer/ recording instrument lens center and d the distance from observer/recording instrument lens center to light source lens center. The light source used is a slide projector. Focus of the lens is so adjusted that a slide will project sharply on the moiré grid.

We have to use dark fringe formula in our calculations. The dark fringe formula may be rearranged as

$$(\Delta_n)^{-1} = (d/sL)(n - ½)^{-1} - L^{-1} \tag{2b}$$

A graph between $(n - ½)^{-1}$ and $(\Delta_n)^{-1}$ shall, therefore, be a straight line with slope d/sL. Therefore, pitch of the moiré grid may be computed from

$$s = d/L(\text{slope}) \tag{3}$$

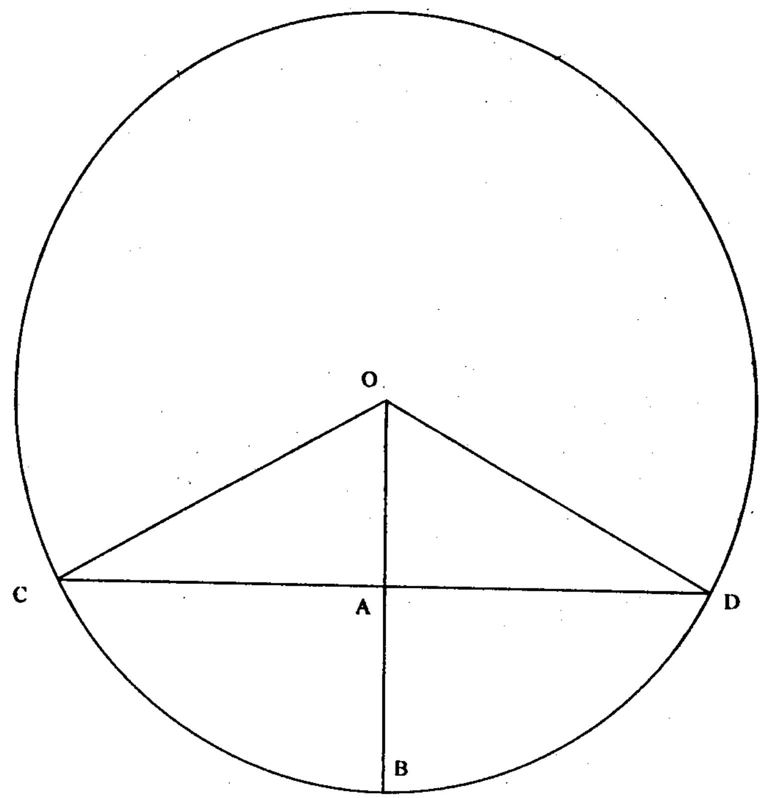

Fig.1. Geometry of the reference object (right-circular cylinder)

Depth of the n^{th}-order fringe may be computed considering Fig. 1 which shows the cross section of a right circular cylinder (our standard measuring object). The zeroeth-order fringe falls at B. Let the n^{th}-order fringe be observed at D on the left and C on the right. The depth $\square_n$ = AB may be computed from geometrical considerations. It can be easily verified that

$$\Delta_n = r[1 - (1 - \delta_n^2/4r^2)^{1/2}] \qquad (4)$$

where 'r' is the radius of cross section of the cylinder, δ_n = DC, length of the line segment joining n^{th}-order fringe on the left to the corresponding n^{th}-order fringe on the right.

3. Accuracy and Precision

Accuracy is a measure of how close the observations are to the accepted value. Precision is a measure of how close the observations are to the arithmetic mean. The pitch of shadow type, fishing line system may be computed using fringe formula and by direct measurement of the width and spacing of the grid (fishing lines in our case). Direct measurement of the pitch, s_0, is considered as the reference value which is to be used to determine accuracy. The other values computed from the depth of the 1^{st}-, 2^{nd}- and 3^{rd}-order fringes on a standard object (right circular cylinder) may be $s_1, s_2, s_3, \ldots, s_N$ for different sets of observations. Accuracy and precision are, therefore, given by

$$\text{Accuracy} = 100\,[1 - \sigma(s_0)/s_0]; \quad \text{Precision} = 100\,[1 - \sigma(<s>)/<s>] \qquad (5a,b)$$

where $<s> = (\Sigma\, s_i)/N, \sigma(x) = [(1/N)\Sigma(s_i - x)^2]^{1/2}$; the summation runs over $i = 1, 2, \ldots, N$.

Table 1: Fringe Depths for a Standard Object

Observation set no. N	Fringe no. n	Length of line segment CD δ_n (cm)	Depth of n^{th} fringe Δ_n (cm)	$(n - ½)^{-1}$	Δ_n $(cm)^{-1}$
	1	1.9	0.180	2.00	5.55
1	2	2.85	0.427	0.67	2.34
	3	3.5	0.680	0.40	1.47
	1	1.9	0.180	2.00	5.55
2	2	3.0	0.478	0.67	2.09
	3	3.9	0.884	0.40	1.13
	1	1.9	0.180	2.00	5.55
3	2	3.2	0.553	0.67	1.81
	1	2.05	0.211	2.00	4.73
4	1	2.05	0.211	2.00	4.73
	2	3.25	0.573	0.67	1.75

4. Experimental

Experiment was constructed in the Optics Laboratory. A paper was pasted on the top and the bottom surfaces of the cylinder and positions of the zeroeth-order, the first-order, the second- order and the third-order fringes were marked using a fine marker. The experiment was repeated for different positions of the cylinder and a total of 4 sets of observations were taken Fringes beyond the third-order were not visible. In two of the cases fringes beyond the second order were not visible. For this experiment 'L' was taken as 170 cm and 'd' as 70 cm, diameter of cylinder was 5.185 cm. Data collected were classified and least square lines were fitted to compute the slope. From the slope pitch of the moiré grid was estimated. Accuracy and precision of these estimates were obtained taking s = 0.15 cm as the reference value (Akram 1989). Tables 1 and 2 show the data analysis and the results. Accuracy of our fishing-line moiré system came out to be 99.61% and precision 91.21%. These values were reasonable under the present experimental conditions.

Table 2: Fitting of Least Square Line and Computation of Pitch

Equation of the least square line: $(\Delta_n)^{-1} = (d/sL)(n - ½)^{-1} - L^{-1}$
Reference value of the pitch, s_0 = 0.15 cm
Distance from the moiré grid to the observer, L = 170 cm
Distance from the observer to the slide projector, d = 70 cm

Observation set N	Slope of the least square line (cm)	Pitch of the moiré system s_N = d/L(slope) (cm)	Accuracy (%)	Precision (%)
1	2.50	0.164		
2	2.71	0.152	99.61	91.21
3	2.90	0.142		
4	2.29	0.179		

5. Discussion and Conclusion

Accuracy and precision of shadow type moiré system have been determined. A value of accuracy of 99.61% suggests that moiré technique can be used for fine measurements. However, a precision of 91.21% indicates that we need to improve our marking and observing techniques. This may be improved if still photographs are taken and viewed under the microscope to improve measurements. We also need to determine the accuracy and the precision of argenza (cloth) shadow type moiré system as well as grating hologram type moiré system. We are also in the process of developing an inexpensive, lightweight, portable projection-type moiré system which could be used for mass screening in schools. Projection-type systems developed in Japan are heavy and expensive. Photogrammetric techniques like moiré fringe topography, stereophotography, holography and rasterstereography shall play an increasing role in industry, health care and education in the third millennium we are going to enter within the next two years.

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