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GROUP STRUCTURE OF A COVARIANT MODEL OF GLOBAL ELECTROCORTICAL ACTIVITY

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ABSTRACT:

Group structure of a recently developed covariant model of global electrocortical activity suggests a possible link between the identity of the group and the phenomenon of brain death.

INTRODUCTION

We have recently developed a covariant model of brain dealing with the global electrocortical activity (Kamal. 1989; Kamal, Siddiqui and Husain, 1989; 1992; Kamal, Siddiqui, Husain, Naeem and Khan, 1992; Siddiqui and Kamal, 1992; Siddiqui, Kamal and Khan, 1990). This model is a generalization of Wright and Kydd's linear model (Wright and Kydd, 1984). In this model electrical potentials of different denritic segments are written as harmonic oscillator equations with damping coefficients, natural frequencies and coupling constants as open parameters. We have rewritten these equations in the comoving frame of the signal. Transformation to the laboratory frame has generated magnetic fields. In this paper we are considering the symmetries of the state transition matrix introduced in that model.

Essential theoretical features

Essential theoretical features of our model can be summarized as:

(a) Electrocortical recordings reflect the transformed spatial average of cortical potentials (Elul, 1972).
(b) The telencephalon is assumed to be a linear wave medium with regard to the gross wave potentials although the underlying microscopic interactions may be extremely non-linear.
(c) Closed and constant boundary conditions lead the linear waves to generate ac-

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tivity at a large number of resonant modes, each associated with a constant natural frequency.

(d) The values for the natural modes of the resonant frequencies are clustered about certain central values (Cramer's Central Limit Theorem).

(e) Ascending inhibitory systems act partly to damp resonant activity and partly as a source of noise like driving signals.

(f) An electrical potential in a comoving frame of the signal transforms as four-potential in the laboratory frame.

Mathematical model

A mass of unit sources coupled to each other are represented by (Kamal, Siddiqui and Husain, 1989).

$$\ddot{A}_l + \Delta_l(\tau)\,A_l + \eta_l{}^2(\tau)\,A_l = \sum_j \kappa_j{}^l(\tau)\,A_j \qquad (1)$$

were $\Delta_l(\tau)$, $\eta_l(\tau)$, and $\kappa_j{}^l(\tau)$ are 4 x 4 matrices generated by a similarity transformation with λ_l (Lorentz transformation) as the transformation matrix. Their eigenvalues $D_l(\tau)$, $N_l(\tau)$, and $K_j{}^l(\tau)$ are free parameters analogous to damping coefficients, natural frequencies and coupling constants respectively. A_j is in fact $A_l{}^\mu = [\phi/A]$; $\mu = 0,1,2,3$, which is the 4-potential. The state transition matrix is constructed by defining new variables $\Omega_k = f(A_l, A_l)$ $k = 1 \ldots m$, where $m = 2n$ (n is the number of dendritic trees considered in the model, usually of the order of 10^{15}). Let us define a dimensionless parameter $t = \tau/\varepsilon$ (ε is a scaling parameter which may be taken as the average time of travel of a signal between two neurons). The coordinates are defined as:

$\Omega_k = A_k$ if k is an odd number

$\Omega_k = dA_{k-1}/dt$ if k is an even number

In terms of Ω_k, eq. (1) can be written as

$$dZ/d\tau = AZ$$

where $\mathbf{Z} = [\omega_k]$ is a column vector and A is the state transition matrix.

Symmetries of the state transition matrix

Let us consider the symmetries of state transition matrix. The state transition matrix

$$\begin{array}{ccccccc} 0 & 1 & 0 & 0 & \cdots\cdots & 0 & 0 \\ \eta_1^{2} & -\Delta_1 & \kappa_2^{1} & 0 & \cdots\cdots & k_m^{1} & 0 \\ 0 & 0 & 0 & 1 & \cdots\cdots & 0 & 0 \\ k_{12} & 0 & \eta_2^{2} & -\Delta_2 & \cdots\cdots & \kappa_m^{2} & 0 \\ 0 & 0 & 0 & 0 & \cdots\cdots & 0 & 0 \\ k_{13} & 0 & \kappa_2^{3} & 0 & \cdots\cdots & \kappa_m^{3} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots\cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots\cdots & 0 & 1 \\ k_{1n} & 0 & \kappa_2^{n} & 0 & \cdots\cdots & -\eta_n^{n} & -\Delta_n \end{array} \qquad (2)$$

is a linear transformation. It is in fact a set of matrices. Different matrices could be generated by assigning different values to D's, N's and K's. Note that $\eta = \varepsilon\eta$, $\Delta = \varepsilon\Delta$, $\kappa = \varepsilon^2\kappa$ are introduced to make all the elements of the state transition matrix dimensionless. Each entry in this state transition matrix a is itself 4 x 4 matrix. The matrix is neither symmetric nor hermitian. However, it is worthwhile to look if the matrix **A** forms a group the operation of matrix multiplication. To do so let us first transform the matrix by interchanging alternate columns, bringing the first in place of second ect. By block diagonalization we construct a non-singular matrix **A**. In the next section we shall look into the group structure formed by the set of matrices (**A**).

The determinant of **A** is just the negative of the determinant of Wright and Kydd's transition matrix A. For n = 2 the determinant of A may be written as:

$$\pi_{i=1}^{4} N_i^2 - \sum_{\substack{i<j \\ k<l}} |e_{ijkl}|\, N_i^2 N_j^2 K_l^k K_{kl} - N_1^2 \begin{bmatrix}342\\234\end{bmatrix} - N_1^2 \begin{bmatrix}492\\243\end{bmatrix}$$

$$-N_2^2 \begin{bmatrix}341\\134\end{bmatrix} - N_2^2 \begin{bmatrix}431\\143\end{bmatrix} - N_3^2 \begin{bmatrix}241\\124\end{bmatrix} - N_3^2 \begin{bmatrix}421\\142\end{bmatrix} - N_4^2 \begin{bmatrix}321\\132\end{bmatrix}$$

$$-N_4^2 \begin{bmatrix}231\\123\end{bmatrix} - \begin{bmatrix}2341\\1234\end{bmatrix} - \begin{bmatrix}3421\\1342\end{bmatrix} - \begin{bmatrix}4321\\1432\end{bmatrix} - \begin{bmatrix}4231\\1423\end{bmatrix} - \begin{bmatrix}3241\\1324\end{bmatrix}$$

$$\begin{bmatrix}2491\\1243\end{bmatrix} + \begin{bmatrix}21\\12\end{bmatrix}\begin{bmatrix}43\\34\end{bmatrix} + \begin{bmatrix}31\\13\end{bmatrix}\begin{bmatrix}42\\24\end{bmatrix} + \begin{bmatrix}41\\14\end{bmatrix}\begin{bmatrix}32\\23\end{bmatrix}$$

where $\begin{bmatrix}342\\234\end{bmatrix} = K_{23}K_{34}K_{42}$ etc.

The determinant looks complicated. However, we note that each term is of degree 2n having a degree k in powers of N_l^2 and a degree (2n - k) in powers of products of K_l^j where k ranges form O to 2n. A general determinant can be written as a polynomial in N's and K's with the coefficients K's determined by the irreducible representations of the classes of the permutation group S(2n). The class containing identity can be identified with the term $\prod N_\ell^2$ having a positive sign. The next comes with a negative sign. The sign alternates with the classes with the exception that the last class always comes with a negative sign. It is, therefore, concluded that:

(a) The determinatnt is independent of the damping coefficients D_ℓ's. Since the determinant is product of eigenvalues, the eigenvalues do not depend on the damping coefficients.
(b) We can always construct a nonsingular matrix out of the state transition matrix A.

Group Structure

The matrix A is neither symmetric nor hermitian. However, it is worthwhile to look if the matrix A forms a group under the operation of matrix multiplication.

(i) Closure Property:

Let us take two matrices A_1 and A_2. Upon examining the product A_1A_2 we note that the elements of the first row are of the form 0,1, 0,0,...., 0 as A_1 A_2. In the second row of A_1 and A_2, we have N's, D's, and ciphers. The elements of second row of A_1 A_2 have in some places nonzero entries in place of ciphers. Third row again contains 0,0,0,1,0,.., as in the original matrices. Since the matrix A is a linear transformation, the ciphers in the second row indicate that there is no dependence of θ_j's on θ_i in the particular situation considered. However, in general θ_i's may depend on θ_j's. This possibility is considered elsewhere (Kamal, 1989; Kamal, Siddiqui, Husain, Naeem and Khan, 1992). Since the form of A is retained under multiplication, the set of state transition matrices is closed under matrix multiplication. To do so we interchange alternate columns, bringing second in place of first etc.

(ii) Associativity:

Since A's are m x m matrices, they must satisfy the properties of matrix algebra, in particular associativity property of matrix multiplication.

(iii) Existence of Identity:

The identity is obtained by taking D_i = o, K_i^j = o, $N_\ell{}^2$ = -1 etc.

(iv) Existence of Inverse:

We have shown above that the matrix A is nonsingular. Therefore its inverse exists. It can be shown that the inverse is also a member of the set of nonsingular matrices constructed from the state transition matrices (A).

Therefore the set (A) consisting of nonsingular matrices constructed from the set of state transition matrices (A) forms a group.

Brain death as identity of the state transition matrix group

The most interesting conclusion comes from looking at the identity of the *state transition group*. Looking at (2) we find that the identity is obtained by taking $D_i = O$, $K_\iota^j = O$, $N_{i2} = -1$ etc. The first condition states that there is no damping present. The second condition means that there is no interaction present among the neighboring neurons i.e. the neurons are decoupled. The condition on $\eta_{\iota 2}$ gives the eigenvalues of natural frequency as $N_\iota = \pm\iota$. In the solution of (1), the expression $\exp(\iota N_\iota t)$ with the eigenvalues of η_ι as $-\iota$ does not represent a physiological situation. However, the eigen-values $+\iota$ represents a decaying exponential. On the electroencephalogram this would correspond to brain death (Doreland's, 1982) - a biological state manifested by absolute unresponsiveness to all stimuli, absence of all spontaneous muscle activity, and an iso-electric electroencephalogram for 30 minutes, all in the absence of hypothermia or intoxication by central nervous system depressants.

The physical picture

Physically we can visualize the identity of the state transition matrix group in the presence of a strong field. A strong magnetic field will decouple the neurons causing all the K_ι^j's to vanish. The neurons will, therefore, act independently and have no interaction with the neighboring neurons. For such independent oscillators Cramer's Central Limit Theorem could not be applied. There will be no resonance and the oscillations will die out quickly as suggested by the eigenvalues $+\iota$ in the expression $\exp(\iota N_\iota t)$. Damping could also be modellled by considering a single neuron in the temperature bath of other neurons. In the absence of any interaction with the neighboring neurons we expect no damping indicated by vanishing of the coefficients D_ι's.

CONCLUSION

This model, therefore, provides explanation of the phenomenon of brain death as well as a physical understanding of the nature of interaction of various neurons. A further step could be the diago-nalization of the state transition matrix. Since the eigenvalues do not depend on the damping coefficients we may be able to write the system of equations which are indendepent of D_ι's. Comparing write the equations with out original set of equations we may be able to estimate the values of D_ι's. Once the determinant is evaluated we may write a generalized inverse of the state transition matrix and develop a metric tensor formulation of our model on the lines suggested by Pellionisz and Llinas (1982).

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