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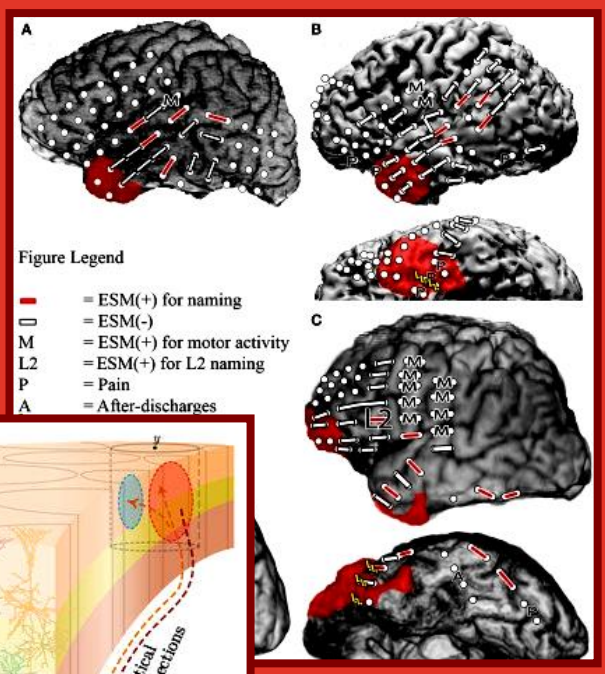
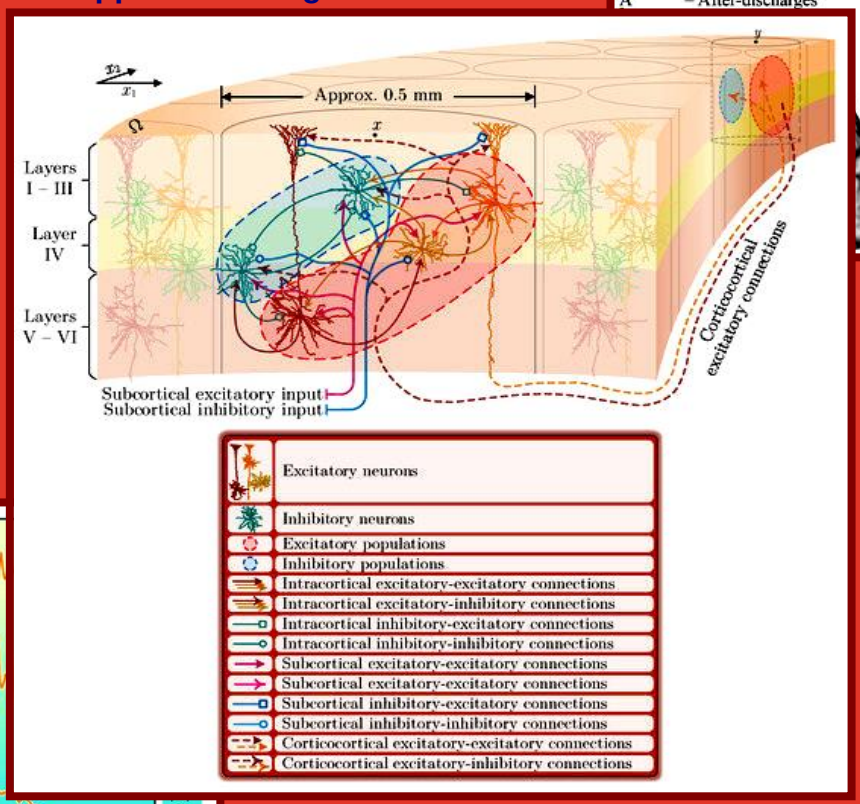
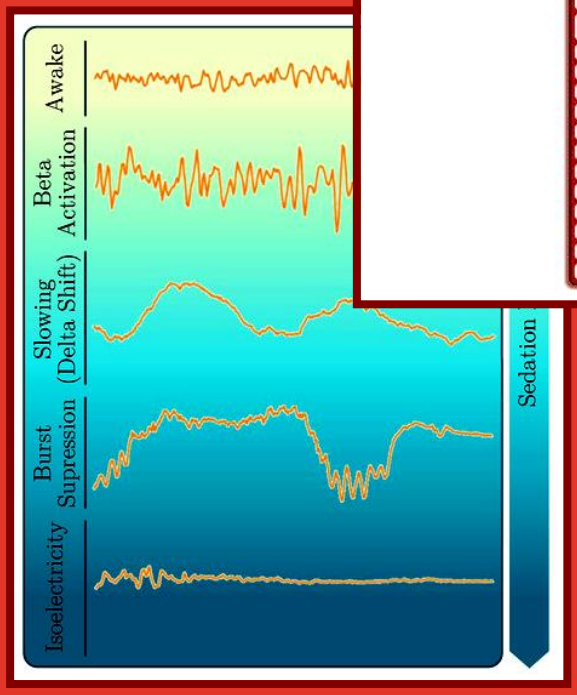
Effects of Weak Electromagnetic Fields on Global-Electrocortical Activity

Magnetic fields of the brain have been experimentally studied by many groups, especially for the localization of epileptic foci. It would be of interest to include the effects of of weak electromagnetic fields in our covariant as well as generalized coupling models. It is shown that a weak electromagnetic field modifies the frequencies. We have written the equation for the time variation of potential of a dendritic tree in the commoving frame of the signal. The commoving frame is fixed to the potential wavefront of the dendrite. When this equation was transformed into the laboratory frame, a magnetic vector potential appeared along with the electro-static potential.







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Syed Arif Kamal, Khursheed Athar Siddiqui, Shaikh Ansar Husain, Mohummed Naeem and Nasiruddin Khan

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Effects of Weak Electromagnetic Fields on Global Electrocortical Activity

SYED ARIF KAMAL[§], KHURSHEED A. SIDDIQUI, S. A. HUSAIN, MUHAMMAD NAEEM, and N. U. KHAN*
Department of Physics, University of Karachi, Karachi–75270, Pakistan

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Abstract. Effects of weak electromagnetic fields are considered on recently proposed covariant and generalized coupling models of global electrocortical activity. A method to calculate the ratio of components of signal velocities is given. First-order shift in frequencies is obtained in the presence of a weak, time-varying magnetic field.

Key words: Global electrocortical activity, magnetoencephalography, neuromagnetism.

1. Introduction

We have recently proposed a generalization (Kamal *et al.*, 1989) of a linear model of Wright and Kydd (1984) for global electrocortical activity. In this covariant model, we have written the equations of electrical potential in the dendritic trees in the comoving frame of the signal. Transformation to the laboratory frame has generated magnetic fields. (Recall that a stationary electron appears as current in the moving frame.)

A generalized coupling model of global electrocortical activity has also been proposed (Kamal, 1989). In this model, the electrical potentials of a synaptic gap not only depend on the neighboring potentials but also on their rate of change.

Magnetic fields of the brain have been experimentally studied by many groups, especially for the localization of epileptic foci (Romani, 1987; Ricci *et al.*, 1987; Narici *et al.*, 1987; Narici *et al.*, 1987). It would be of interest to include the effects of weak electromagnetic fields in our covariant as well as generalized coupling models. It is shown that a weak electromagnetic field modifies the frequencies.

2. The Covariant Model

We summarize the essential features of our covariant model (Kamal *et al.*, 1989). We have written the equation for the time variation of the potential of a dendritic tree in the comoving frame of the signal. The comoving frame is fixed to the

[§]PhD (Neuroscience); MA, Johns Hopkins, Baltimore, MD, United States; MS, Indiana, Bloomington, IN, United States • *paper mail*: Department of Physics, University of Karachi, PO Box 8423, Karachi 75270, Sindh, Pakistan • *e-mail*: profdrakamal@gmail.com • *telephone*: +92 21 9926 1300-15 ext. 2250 • *homepage* • *vita* 0000-0002-1711-4827

*Department of Mathematics, University of Karachi

potential wavefront of the dendrite. When this equation was transformed into the laboratory frame, a magnetic vector potential appeared along with the electrostatic potential.

In the comoving frame of the signal a mass of unit sources coupled to each other may be represented by (j runs from 1 to n subject to the condition that $i \neq j$)

$$\ddot{\phi}_i + \mathcal{D}_i(\tau)\,\dot{\phi}_i + \mathcal{N}_i^2(\tau)\,\phi_i = \sum_j \mathcal{K}_i^j(\tau)\,\phi_j\,, \tag{1}$$

where $\mathcal{D}_i(\tau)$, $\mathcal{N}_i(\tau)$, and $\mathcal{K}_i^j(\tau)$ are 4×4 matrices with eigenvalues $D_i(\tau)$, $N_i(\tau)$, and $K_i^j(\tau)$, respectively. These are free parameters analogous to damping coefficients, natural frequencies, and coupling constants. ϕ_i is a 4×1 column vector with the first entry as the only nonzero entry representing the electrical potential φ_i. The quantity τ is time in the comoving frame of the signal. Let us transform Equation (1) in the laboratory frame under the Lorentz transformation λ_i. Under this similarity transformation, the four vectors and matrices take the form

$$\phi_i \rightarrow A_i = \lambda_i \phi_i\,, \tag{2a}$$

$$\mathcal{D}_i(\tau) \rightarrow \Delta_i(\tau) = \lambda_i \mathcal{D}_i(\tau)\,\tilde{\lambda}_i\,, \tag{2b}$$

$$\mathcal{N}_i(\tau) \rightarrow \eta_i(\tau) = \lambda_i \mathcal{N}_i(\tau)\,\tilde{\lambda}_i\,, \tag{2c}$$

$$\mathcal{K}_i^j(\tau) \rightarrow \tilde{\mathcal{K}}_i^j(\tau) = \lambda_i \mathcal{K}_i^j(\tau)\,\tilde{\lambda}_i\,. \tag{2d}$$

$\tilde{\lambda} = g\lambda g$, where $g_{\mu\nu}$ is the metric tensor ($\mu, \nu = 0, 1, 2, 3$) with $g_{00} = -1, g_{rr} = 1$; $r = 1, 2, 3$, $g_{\mu\nu} = 0$, otherwise. Equation (1), therefore, becomes

$$\ddot{A}_i + \Delta_i(\tau)\,\dot{A}_i + \eta_i^2(\tau)\,A_i = \sum_j \tilde{\mathcal{K}}_i^j(\tau)\,A_j\,. \tag{3}$$

The state transition matrix was constructed by defining new variables $\Omega_k = f(A_k, \dot{A}_k)$, $k = 1, \ldots, m$, where $m = 2n$ (n is the number of dendritic trees considered in the model, usually of the order of 10^{15}). Let us define a dimensionless parameter $t = \tau/\varepsilon$ (ε is a scaling parameter which can be taken as the average time of travel of a signal between two neurons). The coordinates are defined as

$$\Omega_k = A_k\,, \qquad \text{if } k \text{ is an odd number,} \tag{4a}$$

$$\Omega_k = \mathrm{d}A_{k-1}/\mathrm{d}t\,, \quad \text{if } k \text{ is an even number.} \tag{4b}$$

In terms of Ω_k, Equation (3) can be written as

$$\mathrm{d}\mathbb{Z}/\mathrm{d}t = \mathbb{A}\mathbb{Z}\,, \tag{5}$$

where $\mathcal{Z} = [\Omega_k]$ is a column vector and $\mathcal{A}$ is the state transition matrix. The state transition matrix is a function of D's, N's, and K's and ε. The scaling parameter ε is introduced to make all the elements of the state transition matrix dimensionless. The state transition matrix is a linear transformation which connects the four potentials to their rate of change. Since we are transforming under a similarity transformation, the eigenvalues remain invariant. Hence,

$$D_{\text{lab}} = D_{\text{comoving}} , \tag{6a}$$

$$N_{\text{lab}} = N_{\text{comoving}} , \tag{6b}$$

$$K_{\text{lab}} = K_{\text{comoving}} . \tag{6c}$$

3. Predictions of Covariant Model (External Electromagnetic Fields)

Let us apply weak stationary electric and magnetic fields $\mathbf{E}_{\text{ext}}$ and $\mathbf{B}_{\text{ext}}$ which are assumed to be uniform throughout the region concerned. The electric and magnetic fields, are generated by scalar and vector potentials $\phi_{\text{ext}}(x, y, z)$ and $\mathbf{A}_{\text{ext}}(x, y, z)$ such that $\mathbf{E}_{\text{ext}} = -\nabla\varphi_{\text{ext}}$ and $\mathbf{B}_{\text{ext}} = \nabla \times \mathbf{A}_{\text{ext}}$. Let us write a four-potential as

$$A_{\text{ext}} = \begin{pmatrix} \varphi_{\text{ext}} \\ \mathbf{A}_{\text{ext}} \end{pmatrix} . \tag{7}$$

The components of this four-potential are denoted by A^{μ}_{ext}; $\mu = 0, 1, 2, 3$. The presence of weak electromagnetic field will not affect the damping coefficients and coupling of the individual A's. In the presence of this weak magnetic field, the natural frequencies $N_i(\tau)$ will be modified to, say, $N'_i(\tau)$. In the lab frame Equation (3) now takes the form

$$\ddot{A}'_i + \Delta_i(\tau)\,\dot{A}'_i + \eta'^2_i(\tau)\,A'_i = \sum_j \tilde{\mathcal{K}}^j_i(\tau)\,A'_j , \tag{8}$$

where $A'_i = A_i + A_{\text{ext}}$. Since the signal velocities are very small as compared to the velocity of light, we take $t \simeq \tau$ (τ is time in the laboratory frame). The external field $A_{\text{ext}}(x, y, z)$ does not depend on time. We, therefore, have

$$\ddot{A}'_i = \ddot{A}_i , \quad \dot{A}'_i = \dot{A}_i .$$

Subtracting (3) from (8) and introducing $\eta'^2_i = \eta^2_i + \delta\eta^2_i$, ($\delta\eta_i$'s are 4×4 matrices having eigenvalues δ^{μ}_i; $\mu = 0, 1, 2, 3$ representing the shifts in frequencies) we have

$$\delta\eta^2_i A'_i = -\left(\eta^2_i - \sum_j \tilde{\mathcal{K}}^j_i\right) A_{\text{ext}} .$$

We assume that $\delta\eta_i$, $\mathcal{K}^j_i$, and η_i can all be simultaneously diagonalized. In other words, $[\delta\eta_i, \mathcal{K}^j_i] = 0$, $[\delta\eta_i, \eta_i] = 0$, etc. Applying the transformation $\delta\eta_i \to \tilde{\lambda}_i \delta\eta_i \lambda_i = \delta\mathcal{N}_i$, etc., to write the above equation in the comoving frame of the signal, we have

$$\delta\mathcal{N}_i^2\tilde{A}_i^l = -\left(\mathcal{N}_i^2 - \sum_j \mathcal{K}_i^j\right)(\tilde{A}_{\text{ext}})_i\,, \tag{9}$$

where

$$\tilde{A}_i^l = \phi_i + (\tilde{A}_{\text{ext}})_i\,;\quad (\tilde{A}_{\text{ext}})_i = \tilde{\lambda}_i A_{\text{ext}}\,.$$

The matrices $\delta\mathcal{N}_i$, $\mathcal{N}_i$ and $\mathcal{K}_i^j$ are already diagonalized. Equation (9) will yield four equations, each one sufficient to determine an eigenvalue δ_i^μ. The results are

$$(\delta_i^\mu)^2 = \left[\sum_j K_i^{j\mu} - (N_i^\mu)^2\right]\Theta_i^\mu\,, \tag{10}$$

where

$$\Theta_i^0 = (\gamma\varphi_{\text{ext}} - \gamma_i\mathbf{v}_i\cdot\mathbf{A}_{\text{ext}})\,/\,(\gamma_i\varphi_{\text{ext}} + \varphi_i - \gamma_i\mathbf{v}_i\cdot\mathbf{A}_{\text{ext}})\,. \tag{11a}$$

$$\Theta_i^r = 1\,;\quad r = 1,2,3\,. \tag{11b}$$

The following conclusions can be drawn:

(i) The shift in frequencies is a measure of the strength of coupling through the factor $\sum_j K_i^{j\mu} - (N_i^\mu)^2$.

- (a) If $\sum_j K_i^{j\mu} = (N_i^\mu)^2$, there will be no shift in frequencies.
- (b) If $\sum_j K_i^{j\mu} > (N_i^\mu)^2$, a slight increase in the frequencies will be observed.
- (c) If $\sum_j K_i^{j\mu} < (N_i^\mu)^2$, there will be a decaying exponential introduced in the EEG spectrum. Therefore, the frequencies will be modulated.

(ii) Upon examination of Equation (10), we note that $(\gamma_i\varphi_{\text{ext}} - \gamma_i\mathbf{A}_{\text{ext}}\cdot\mathbf{v}_i)$ is very small as compared to φ_i. Therefore, we expect that

$$\delta_i^0\ \alpha\ |\varphi_{\text{ext}} - \mathbf{v}_i\cdot\mathbf{A}_{\text{ext}}|^{1/2}\,. \tag{12}$$

Using Cramer's central limit theorem, we replace individual frequencies by their averages. Taking the averages, we expect that $\langle\delta^0\rangle\ \alpha\ |\varphi_{\text{ext}} - \mathbf{v}_i\cdot\mathbf{A}_{\text{ext}}|^{1/2}$. This conclusion may be checked experimentally.

In an electrically and magnetically shielded room, stationary electric and magnetic fields may be applied to the brain. In a steady-state condition an EEG is taken. The dominant frequency of the EEG is expected to vary in a manner similar to the expression (12). The other components δ_i^r; $r = 1,2,3$ are independent of the applied field.

(iii) If we apply magnetic field of same magnitude in the x, y, and z directions and observe δ_i^0 in each case, we can measure the ratios of velocities of signals in different directions

$$(\delta_i^0)_x : (\delta_i^0)_y : (\delta_i^0)_z = \sqrt{(v_i)_x} : \sqrt{(v_i)_y} : \sqrt{(v_i)_z} .$$

. If we apply the Central Limit Theorem of Cramer to replace each individual frequency and velocity by its average, the above equation may provide a way to evaluate the ratio of different components of signal velocity.

4. Generalized Coupling Model

We describe here the essential features of the generalized coupling model of global electrocortical activity presented elsewhere (Kamal, 1989).

In the linear model of Wright and Kydd (1984), the electrical potentials φ_i's are coupled to φ_j's through coupling parameters $K_i^j(t)$. However, we note that a change in potential φ_j induces a magnetic field because of the flow of current. A magnetic field will, in turn, exert Lorentz force on a charged particle and, hence, in general, φ_j will influence φ_i.

In our covariant model (Kamal *et al.*, 1989), the dependence of A_i's on $\dot{A}_j$'s was also suggested when we tried to multiply the two state transition matrices (Kamal, 1989). Upon multiplication of two-state transition matrices, we came up with nonzero coefficients for $\dot{A}_j$'s.

In the comoving frame of the signal passing through a segment of the dendritic tree, the electrical potential for mass of unit sources coupled to each other in the generalized coupling model may be represented by

$$\ddot{\phi}_i + \mathcal{D}_i(\tau)\,\dot{\phi}_i + \mathcal{N}_i^2(\tau)\,\phi_i = \sum_j [\mathcal{K}_i^j(\tau)\,\phi_j + \Re_i^j(\tau)\,\dot{\phi}_j] , \tag{13}$$

where $\Re_i^j(\tau)$'s are 4×4 matrices having eigenvalues $M_i^{j\mu}(\tau)$; $\mu = 0, 1, 2, 3$. The M_i^j's are free parameters given physiological meaning under the assumptions that they have a finite variance σ_M about $\langle M \rangle$, and they are stochastically independent. No particular type of distribution for $M_i^j(\tau)$ is assumed. All other symbols are as defined previously.

A similarity transformation under λ_i transforms the various four-vectors and matrices as

$$\phi_i \to A_i = \lambda_i \phi_i , \tag{14a}$$

$$\mathcal{D}_i(\tau) \to \Delta_i(\tau) = \lambda_i \mathcal{D}_i(\tau)\,\tilde{\lambda}_i , \tag{14b}$$

$$\mathcal{N}_i(\tau) \to \eta_i(\tau) = \lambda_i \mathcal{N}_i(\tau)\,\tilde{\lambda}_i , \tag{14c}$$

$$\mathcal{K}_i^j(\tau) \to \tilde{\mathcal{K}}_i^j(\tau) = \lambda_i \mathcal{K}_i^j(\tau)\,\tilde{\lambda}_i , \tag{14d}$$

$$\mathfrak{K}_i^j(\tau) \to \mathcal{M}_i^j(\tau) = \lambda_i \mathfrak{K}_i^j(\tau)\, \tilde{\lambda}_i \,. \tag{14e}$$

Equation (13), therefore, becomes

$$\ddot{A}_i + \Delta_i(\tau)\, \dot{A}_i + \eta_i^2(\tau)\, A_i = \sum_j [\tilde{\mathcal{K}}_i^j(\tau)\, A_j + \mathcal{M}_i^j(\tau)\, \dot{A}_j] \,. \tag{15}$$

To write down the state transition matrix, we introduce a new set of coordinates $\Psi_k = f(A_i, \theta \dot{A}_i)$; $k = 1, 2, \ldots, m$ where $m = 2n$ (n is the number of dendritic trees considered in the model usually of the order of 10^{14}, where θ is a scaling parameter which can be taken as the average time of travel of the signal in the laboratory frame between two neurons. In particular,

$$\Psi_1 = A_1 \,, \quad \Psi_2 = \theta \dot{A}_1 \,, \quad \Psi_3 = A_2 \,, \quad \Psi_4 = \theta \dot{\Psi}_3 = \theta \dot{A}_2 \,,$$

etc. All the generalized coordinates Ψ_k have the same dimensions. To write the state transition matrix in such a way that all its elements are dimensionless, we introduce $\hat{\tau} = \tau/\theta$. The rate of change of $\hat{\mathbf{Z}} = [\Psi_k]$ is related to $\hat{\mathbf{Z}}$ by

$$\mathrm{d}\hat{\mathbf{Z}}/\mathrm{d}\hat{\tau} = \hat{\mathbf{A}}\hat{\mathbf{Z}} \,. \tag{16}$$

The state transition matrix $\hat{\mathbf{A}}$ now takes the form

$$\begin{bmatrix}
0 & 1 & 0 & 0 & \cdots & 0 & 0 \\
-\hat{\eta}_1^2 & -\hat{\Delta}_1 & \hat{\mathcal{K}}_1^2 & \hat{\mathcal{M}}_1^2 & \cdots & \hat{\mathcal{K}}_1^m & \hat{\mathcal{M}}_1^m \\
0 & 0 & 0 & 1 & \cdots & 0 & 0 \\
\hat{\mathcal{K}}_2^1 & \hat{\mathcal{M}}_2^1 & -\hat{\eta}_2^2 & -\hat{\Delta}_2 & \cdots & \hat{\mathcal{K}}_2^m & \hat{\mathcal{M}}_2^m \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\hat{\mathcal{K}}_3^1 & \hat{\mathcal{M}}_3^1 & \hat{\mathcal{K}}_3^2 & \hat{\mathcal{M}}_3^2 & \cdots & \hat{\mathcal{K}}_3^m & \hat{\mathcal{M}}_3^m \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & \cdots & 0 & 1 \\
\hat{\mathcal{K}}_n^1 & \hat{\mathcal{M}}_n^1 & \hat{\mathcal{K}}_n^2 & \hat{\mathcal{M}}_n^2 & \cdots & -\hat{\eta}_n^2 & -\hat{\Delta}_n
\end{bmatrix} \tag{17}$$

where

$$\hat{\eta}_i(\hat{\tau}) = \theta \eta_i(\tau), \quad \hat{\Delta}_i(\hat{\tau}) = \theta \Delta_i(\tau), \quad \hat{\mathcal{K}}_i^j(\hat{\tau}) = \theta^2 \tilde{\mathcal{K}}_i^j(\tau), \quad \text{and} \quad \hat{\mathcal{M}}_i^j(\hat{\tau}) = \theta \mathcal{M}_i^j(\tau) \,.$$

The state transition matrix $\hat{\mathbf{A}}$ is a linear transformation. To consider the group structure, let us transform the matrix $\hat{\mathbf{A}}$ by interchanging alternate columns, bringing the first column in place of the second, etc. This matrix forms a group under matrix multiplication. The identity is obtained by taking $\hat{\Delta}_i = 0$, $\hat{\eta}_i^2 = -1$. The first condition states that there is no damping present. The neighboring neurons, therefore, do not exert any forces on the neuronal activity. In other words, the neurons are decoupled. The condition on $\hat{\eta}_i^2$ gives the eigenvalues of natural frequency as $N_i = \pm i$. In the solution of (15), the expression $\exp(iN_i\tau)$ with the eigenvalue $N_i = -i$ does not represent a physiological situation. However, the eigenvalue $+i$ represents a decaying exponential, which would correspond to *brain death* on the electroencephalogram.

5. Predictions of Generalized Coupling Model (External Electromagnetic Fields)

We shall now see if there is any change in our results (10, 11a,b) if we include generalized coupling parameter. We already know that only the natural frequencies will be modified in the presence of external, stationary weak fields. In the lab frame, Equation (15) now takes the form

$$\ddot{A}_i + \Delta_i(\tau)\,\dot{A}_i + \eta_i^2(\tau)\,A_i' = \sum_j [\tilde{\mathcal{K}}_i^j(\tau)\,A_j' + \mathcal{M}_i^j(\tau)\,\dot{A}_j'] \,. \tag{18}$$

Subtracting (15) from (18) and introducing $\eta_i'^2 = \eta_i^2 + \delta\eta_i^2$, we have

$$\delta\eta_i^2 A_i' = -\left(\eta_i^2 - \sum_j \tilde{\mathcal{K}}_i^j\right) A_{\text{ext}} \,. \tag{19}$$

This is the same result as the one we obtained in the covariant model above. We note that all the factors related to generalized coupling with A have cancelled out. Therefore, we conclude that a weak, uniform, stationary electromagnetic field will give the same first-order shift in frequencies in the presence of generalized coupling.

Let us introduce a four-potential $A_{\text{ext}}(t)$ which is generated by electric and magnetic fields $\mathbf{E}_{\text{ext}}(t)$ and $\mathbf{B}_{\text{ext}}(t)$ varying linearly with time. Since $A_i' = A_i + A_{\text{ext}'}$ and $A_{\text{ext}}(x, y, z, t)$ has a linear time dependence, we have

$$\ddot{A}_i' = \ddot{A}_i \,, \quad \dot{A}_i' \neq \dot{A}_i \,.$$

Subtracting (15) from (18) and introducing $\delta\eta_i$ as the first-order correction to η_i, we have

$$\delta\eta_i^2 A_i' = -\left(\eta_i^2 - \sum_j \tilde{\mathcal{K}}_i^j\right) A_{\text{ext}} - \left(\Delta_i - \sum_j \mathcal{M}_i^j\right) \dot{A}_{\text{ext}} \,. \tag{20}$$

If all the matrices can be simultaneously diagonalized, (20) can be written in the comoving frame of the signal as

$$\delta\mathcal{N}_i^2 \tilde{A}_i' = -\left(\mathcal{N}_i^2 - \sum_j \mathcal{K}_i^j\right) (\tilde{A}_{\text{ext}})_i - \left(\mathcal{D}_i - \sum_j \mathfrak{R}_i^j\right) (\dot{\tilde{A}}_{\text{ext}})_i \,, \tag{21}$$

where $(\tilde{A}_{\text{ext}})_i = \tilde{\lambda}_i A_{\text{ext}}$. The matrices $\delta\mathcal{N}_i$, $\mathcal{N}_i$, $\mathcal{K}_i^j$, and $\mathfrak{R}_i^j$ are already diagonalized. Equation (21) will yield four equations which can be solved to determine eigenvalues δ_i^μ. The results are

$$(\delta_i^\mu)^2 = \left[\sum_j K_i^{j\mu} - (N_i^\mu)^2\right] \Theta_i^\mu + \left[\sum_j M_i^{j\mu} - D_i^\mu\right] \Xi_i^\mu \,, \tag{22}$$

where

$$\Xi_i^0 = (\gamma_i \varphi_{\text{ext}} - \gamma_i \dot{\mathbf{A}}_{\text{ext}} \cdot \mathbf{v}_i) / (\gamma_i \varphi_{\text{ext}} + \varphi_i - \gamma_i \mathbf{A}_{\text{ext}} \cdot \mathbf{v}_i) , \tag{23a}$$

$$\Xi_i^r = \Im(\dot{A}_{\text{ext}}^r) / \Im(A_{\text{ext}}^r) , \quad r = 1,2,3 , \tag{23b–d}$$

where

$$\Im(A_{\text{ext}}^r) = -\gamma_i \varphi_{\text{ext}} + A_{\text{ext}}^r + (\mathbf{v}_i \cdot \mathbf{A}_{\text{ext}}) v^r (\gamma_i - 1) / |\mathbf{v}_i|^2 .$$

Note that the term $(\gamma_i \varphi_{\text{ext}} - \gamma_i \mathbf{A}_{\text{ext}} \cdot \mathbf{v}_i)$ in Equations (22) and (23a) is very small as compared to φ_i. Consider time-varying external fields which are applied in the x-direction. If their magnitudes are changed from $-\varepsilon$ to $+\varepsilon$ in a period of time Δt, i.e. on the average $|\mathbf{A}_{\text{ext}}| = 0$ $\varphi_{\text{ext}} = 0$, we expect that $\delta_i^0 \ \alpha \, |\dot{\varphi}_{\text{ext}} - \mathbf{v}_i \cdot \dot{\mathbf{A}}_{\text{ext}}|^{1/2}$. Using Cramer's Central Limit Theorem to replace the individual frequencies by their average, we get

$$\langle \delta_i^0 \rangle \ \alpha \, |\dot{\varphi}_{\text{ext}} - \mathbf{v}_i \cdot \dot{\mathbf{A}}_{\text{ext}}|^{1/2} . \tag{24}$$

This prediction may be checked experimentally.

Because of the presence of the terms φ_{ext} and $\mathbf{A}_{\text{ext}}$ in (23b–d), the frequencies will become unbounded when $\varphi_{\text{ext}} = 0$ and $|\mathbf{A}_{\text{ext}}| = 0$. This is not physically possible. We, therefore, conclude that

$$\Sigma \, M_i^{jr} = D_i^r , \quad r = 1,2,3 . \tag{25}$$

This may be considered as a condition of *impedance matching*.

6. Discussion and Conclusion

One may wonder what is meant by shifts in EEG frequencies when one knows that there are no well-defined frequencies in EEG. Although there is no single well-defined frequency in EEG, every rhythm has a finite range and variance of frequencies. Therefore, according to Cramer's Central Limit Theorem (Wright and Kydd, 1984; Kamal, 1989), the frequencies may be assumed to be clustered around their average values. Hence, the shift in frequencies must be taken as a shift in average frequencies for the purpose of testing this model.

Another question may be raised whether the external magnetic fields do have an effect on EEG large enough to be measured experimentally. Klitzing (1989) suggests that static magnetic fields increase the power intensity of EEG of man. However, no studies have been conducted to study the shifts as predicted in this model.

Once the generalized coupling is written depending on both A_j's and $\dot{A}_j$'s, we may be able to quantize the system and field-theoretic techniques may be applied to study global electrocortical activity. The model, however, treats the neural activity in one dimension. In reality, the neural activity takes place in three dimensions.

The next step could be towards the construction of a tensorial model of global electrocortical activity in two dimensions. Generalized coupling coefficients may be introduced which couple the products $\varphi_i\varphi_j$'s, $\varphi_i\dot{\varphi}_j$'s, as well as $\dot{\varphi}_i\dot{\varphi}_j$'s to individual φ_k's. Once this is done, lattice gauge techniques may be applied to our system.

There is a need to estimate the numbers D_i's, N_i's, K_i^j's, and M_i^j's to obtain quantitative predictions from this model. Simulation studies involving coupled harmonic oscillators are planned to be compared with the digitized electroencephalograms provided by local neurologists (Siddiqui *et al.*, 1990).

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