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BEHAVIOUR OF MASSIVE PARTICLES NEAR THE VELOCITY OF LIGHT

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ABSTRACT

Behaviour of massive particles near the velocity of light is discussed. It is shown that an accurate determination of velocity makes the Lorentz factor and energy indeterminate. The relativistic relation of mass is modified so that mass is non-infinite at $v = c$ in the light of uncertainty relations. An experiment is proposed to test the theory.

Introduction

The question whether one can legitimately assume special relativity to be valid in the quantum domain was discussed by Cohn (1968). It was demonstrated that, owing to the Heisenberg uncertainty relations, a very general type of detector is only able to resolve the spacetime to a limited degree. The question of validity of the uncertainty principle to the relativistic quantum theory was discussed by Landau and Peierls (1931). In recent years several investigators have devised a 'Stochastic Field Theory' whose basic premise is that spacetime is not sharp (Ingrahm, 1962, 1964; de Falco et al., 1982). In this paper behaviour of massive particles near the velocity of light is examined in the framework of quantum mechanics.

Uncertainty Considerations

Consider a free electron travelling in the y direction. Its motion is governed by the free Dirac Hamiltonian

$$H_D = c(\alpha_x p_x + \alpha_y p_y + \alpha_z p_z) + \beta m_o c^2 \quad (1)$$

where

$$\alpha_i = \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix}, \; i = x,y,z; \; \beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

m_o is the electron rest (proper) mass, p_i's are momentum operators, and c the velocity of light in free space. The matrices σ_i's are 2 x 2 Pauli spin matrices. Applying the uncertainty relation (Robertson, 1929)

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$$\Delta v_x \ \Delta E \geq 2c^2[(p_{x2}+p_{y2})(v_y/c)^4 + P_{y2}(vz/c)^4 + 2p_{y2}(v_y v_z/c^2)^2]^{1/2} \tag{8}$$

where we used $v^2 + v_{y2} + v_{z2} \sim c^2$, $P_y = mv_y$, $P_z = mv_z$. The last two substitutions are valid because we are considering free electron. The average value gives the motion of wave packet which rigorously obeys the laws of classical mechanics (Cohen-Tannoundji *et al.*, 1977). Using

$P_x = mv_x$ etc. and $p_{x2} + P_{y2} = (E/c)_2 - m_{o2}c_2 - P_{z2}$, I get

$$\Delta v_x \geq 2c(E/\Delta E)(v_y/c)^2 [1 + (v_z/c)^2 + (vvz\,2/cvy)_2 - (m_oc^2/E)^2]^{1/2} \tag{9}$$

Consider 200 GeV electrons produced at the Fermilab. Suppose that the focusing system is 3m away from the production area. $v_x \approx 3 \times 10^8$ m/s, $x = 3$m, $t \approx 10^{-8}$ s. The beam is focused to a space of 3 microns. Therefore $y = z = 3 \times 10^{-6}$ m, $v_y = y/t = z/t \approx 3 \times 10^2$ m/s. Substituting these values in (9), the uncertainity in the x-component of the velocity is given by

$$\Delta v_x \geq (0.12)/E \tag{10}$$

where ΔE is in GeV. To obtain minimum uncertainity in the velocity let us use maximam uncertainity in the energy (5% of the actual value). Therefore $\Delta E = 10$ GeV and so

$$\Delta v_x \geq (11.9 \times 10^{-3}) \text{ m/s}$$

For $E = 200$ GeV, $c - v_x = (9.75 \times 10^{-3})$ m/s obtained from the relation $m = m_o(1 - v^2/c^2)^{-1/2}$. Therefore the uncertainity in the velocity exceeds the difference between the velocity of light and the calculated velocity of the electron.

Let us consider the behaviour of an electron in a prescribed external electromagnetic field. The Hamiltonian of such an electron is written as

$$H = \vec{\alpha}.(\vec{p} - e\vec{A}) + \beta m_o + e\Phi \tag{11}$$

where Φ and $\vec{A}$ are the electric and the magnetic potentials related to the electric intensity and the magnetic induction vectors by

$$\vec{\psi} = -\Delta\Phi, \qquad \vec{\beta} = \Delta \times \vec{A}$$

In eqs. (11-16) we are taking $c = h = 1$. Applying the Foldy-Wouthysen transformation (Bjorken and Drell, 1964) three times we get

$$H''' = \beta x_1 + e\Phi - x_2(\beta, \vec{\xi}) \tag{12}$$

where

$$X_1 = m_o + (\vec{p} - e\vec{A})^2/2m_o - p^4/8m_{o3} \quad (13a)$$

$$\vec{X}_2(\beta,\chi) = (e/8m_{o2})^{(i}\vec{\xi} \text{ curl } \vec{\xi} + 2\vec{\xi}.\vec{\xi}xp + \text{div } \vec{\xi} + 4m_o\beta\xi.\beta) \quad (13b)$$

and

$$X_1 = \begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{bmatrix}; i = x,y,z \quad (13c)$$

The solution of

$$H^{w}\psi = E\psi \quad (14)$$

is used in (3) to obtain

$$\Delta\alpha_x \, \Delta\beta \geq (F/\Omega)\,[1/2 + (p - eA)^2/4m_o^2 - p^4/16\,m_o^4\,] \quad (15)$$

where

$$\Omega = [\,|a_+|^2 + |b|^2]^{1/2}[\,|a\_|^2 + |b|^2]^{1/2} \quad (16a)$$

$$a\pm = e\Phi - E \pm \chi_1 - \chi_2\,(1,z) \quad (16b)$$

$$b = i\chi_2\,(1, y) - x_2\,(1, x) \quad (16c)$$

$$F = eB_y - (e/4m_o)\,(\nabla \times \vec{\xi})_x + 1/2\,e(\vec{\xi} \times \vec{p})_y \quad (16d)$$

The right-hand side of (15) is obviously non-zero and so we agian get condition (6).

From this discussion it is clear that the spacetime relationships in quantum domain need to be modified to incorporate these uncertainties.

Relativistic Effective Mass

Consider a particle having any arbitrary velocity between O and c. The equation

$$E^2 = c^2p^2 + m_{o2}{}^c4 \quad (17)$$

may be written as

$$c(m^2 - m_o^2)^{1/2} = p = mv \quad (18)$$

A particle having mass E/c^2 ($=m$) with velocity c^2p/E ($=v$) may be considered as a particle having mass $(m^2 - m_o^2)^{1/2}$ moving with velocity c. Note that the particle is now expressed as an equivalent luxon (for properties of luxons see Recami and Mignani,

1974) of mass $(m^2-m_o^2)^{1/2}$ The quantity $(m^2-m_o^2)^{1/2}$ approaches zero as mass mo tends to m_o. This quantity is named as 'relativistic effective mass' denoted by mf. The equation of a particle may, therefore, be written in a form similar to the luxon equation

$$E_{f2} = {}^{c}2^{p}2$$

where $E_f = c^2 m_f$ is called 'relativistic effective energy'.

Hence any luxon equation may be changed to an equivalent bradyon (for properties of bradyons see Recami and Mignani, 1974) equation provided the mass of a luxon in the equation is replaced by the relativistic effective mass of a bradyon (Kamal, 1978; Kamal and Husain, 1979). Consider the luxon equation hk = mc. Replacing m by the relativistic effective mass $m_o (c^2/v^2-1)^{-1/2}$ and simplifying we get

$$hk = m_o v(1 - v^2/c^2)^{-1/2}$$

which is de Broglie relation for a massive particle (bradyon)

Introducing the concept of spin the 'relativistic effective mass operator' (REMO) for bradyons may be written as

$$M = \delta_1 m + \delta_2 m_o \tag{20}$$

where δ_1 and δ_2 are anticommuting matrices such that $\delta_{12} = 1 = = - \delta_{12}$. If we write REMO as $M = (m^2 - m_o^2)^{1/2} = (H_2/c^4 - m_o^2)^{1/2}$ it is not of the form required by the general laws of the quantum theory (Dirac, 1974) on account of its being quadratic in H. The wave equation must be linear in the operator $\partial/\partial t$ or H, otherwise a conserved probability function cannot be defined (Bohm, 1951). For spin 1/2 particles, 2 x 2 matrices satisfying the properties are σ_x and $i\sigma_y$, σy and $i\sigma_z$, σ_z and $\iota\sigma_x$ and so on. Recall that σ's are Pauli spin matrices. The operator $M = \delta_1 m - \delta_2 m_o$ may also be taken as REMO with the same conditions imposed on δ_1 and δ_2. Since $p = cm_f$, the momentum operator $c(\delta_1 m + \delta_2 m_o)$ is defined in terms of quantities which are same in all coordinate systems (connected to one another by translation or rotation) which are at rest with respect to each other. The properties of REMO and its 4 x 4 representation are described elsewhere (Kamal, 1980; Al-Kurdi, 1978; Husain *et al.*, 1979). To solve $M\psi = M_f\psi$ where M_f is the eigenvalue of M, different combinations of Pauli spin matices were considered (Al-Kurdi, 1978; Husain *et al.*, 1979). The pair σ_y and $i\sigma_x$ was found most suitable for high speed particles. The operator $M = \sigma_y m + i\sigma_x m_o$ has eigenvalues $\pm(m^2 - m_o^2)^{1/2}$. Introducing

$$\psi_j(\vec{r}, t) = u_j \exp(i/h)\,(\vec{r}.\vec{p} - E.T);\; j = 1, 2 \tag{21}$$

we get

$$u_1\pm = iE_f\pm(m + m_o)^{-1}c^{-2}A\pm, u_2\pm = A\pm \quad (22)$$

which gives

$$|\psi|^2 = 2m(m + m_o)^{-1}|A\pm|^2 \quad (23)$$

There are both positive and negative energy states represented by

$$E_f\pm = \pm c^2(m^2 - m_{o2})^{1/2} \quad (24)$$

The probability is $|A\pm|^2$ when $v = 0$ and $2|A\pm|^2$ when $v=c$. Therefore we see that there is a finite probability of existance of a particle of non-zero rest mass travelling with an average speed equal to the speed of light in freee space. Such a particle is termed as a 'nooron' (or a photoparticle) $^+$.

In the next section we shall see how can we incorporate this finite probability of existance of particles having $v = c$ in the relativistic formula.

Modification in the Mass Formula

It was pointed out earlier that an accurate determination of velocity makes the factor $(1-v^2/c^2)^{1/2}$ indeterminate. To incorporate this uncertainity, a parameter $P=P(m_o,v)$ is introduced which is positive and gives the uncertainity in velocity. The relativistic mass formula, may therefore, be written as

$$m(v) = m_o(1-p^2v^2/c^2)^{-1/2} \quad (25)$$

For a non-zero rest mass, $P(m_o, c)$ is slightly less than unity. Therefore m is very large, but does not become infinite at $v = c$. If in the Lorentz transformations V (the velocity of moving frame) is replaced by PV, they leave the interval $(x_{k(1)} - x_{k(2)})^2$; $k=1, 2, 3, 4$ in the Minkowski space invariant and hence they form a Lorentz group (for properties of the Lorentz group see Roman, 1961).

The order of P may be estimated by writing the modified expression of momentum

$$p(v) = v\,m(v)\,P(v) = m_o vP(1 - p^2v^2/c^2)^{-1/2} \quad (26)$$

Let $p = p_c$ at $v = c$. Eq. (26), therefore, gives

$$p = \{1 + (m_o^2 c^4)/(c^2 P_c^2)\}^{-1/2} \sim 1 - (m_o^2 c^4)/(2c^2 P_c^2) \quad (27)$$

Using (Particle Data Group, 1982) $m_oc^2 = 0.9382796\ (27)$ GeV, $cp_c = 10^6$ GeV, we get $P \approx 1 - 0.44 \times 10^{-12}$. Therefore a test of this theory by a determination of P is almsot impossible.

Experimential Test

An experimental test of this theory is provided by considering a theory of faster-than-light particles (Kamal, 1981 b; 1982a) presented by one of the authors (SAK). In this theory energy is a periodic function of velocity

$$c^2m(v) = E(v) = E(v + 2c) = c^2m(v + 2c);\quad -\infty < v < \infty \tag{28}$$

We choose $P(v - 2nc) = P(-v) = P(v) = P(v + 2nc)$; $n = 1, 2,...$ to have the theory maximum symmetry. Note that

$$c^2m(c-\varepsilon) = E(c-\varepsilon) = E(c + \varepsilon) = c^2m(c + \varepsilon)$$

and $p(c-\varepsilon) = p(c+\varepsilon)$. Assuming that energy and momentum are continuous and smooth functions in the vicinity of $v = c$, we have

$$\lim_{v\to c} dE/d(m_o cv) = (m_o c)^{-1} \{\lim_{\varepsilon\to 0} E(c + \varepsilon) - E(c - \varepsilon)\}/2e = 0 \text{ (29a)} \tag{29a}$$

$$\lim_{v\to c} d(cp)/d(m_o cv) = m_o^{-1} \lim_{\varepsilon\to 0} G(\varepsilon)/2\varepsilon$$

where

$$G(\varepsilon) = (c + \varepsilon)\, m(c + \varepsilon)\, P(c + \varepsilon) - (c - \varepsilon)\, m(c - \varepsilon)\, P(c - \varepsilon)$$

This gives

$$\lim_{v\to c} d(cP)/d(m_o cv) = m_o^{-1}\, m(c)\, P(c) \neq 0 \tag{29b}$$

Therefore (for details see Kamal, 1982b)

$$\lim_{v\to c} dE/d(cp) = 0 \tag{30}$$

The energy-momentum equation in Einsteinian relativity (17) gives $dE/d(cp) = v/c$. Therefore

$$\lim_{v\to c} dE/d(cp) = 1 \tag{31}$$

Note that the energy-momentum equation (17) may not be applied in our case because it is true for v < c. For v > c, we have (Kamal, 1918b; 1982a)

$$E^2 = (cp - 2E)^2 + m_o^2c^4, c < v < 3c \quad (32)$$

Einsteinian relativity predicts that the energy-momentum curve will asymptotically approach the line E = cp. This theory predicts a point of inflection near the velocity of light. Fig. I shows E-v curve in the extended relativity with finite energy at v = (2n - l)c. Fig. 2 shows the proposed E-p curve in the extended relativity.

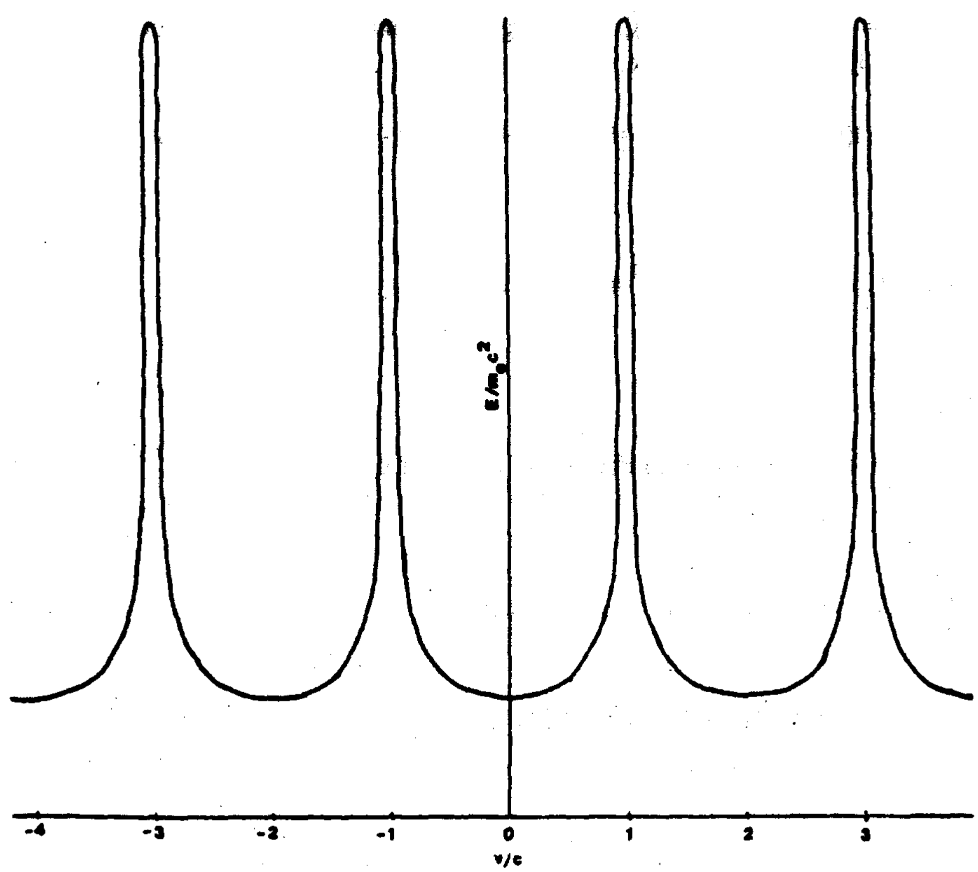

Fig. 1: E/m_oc^2 as a function of v/c in the extended relativeity. Note that E/m_oc^2 is finite at v = (2n - 1)c.

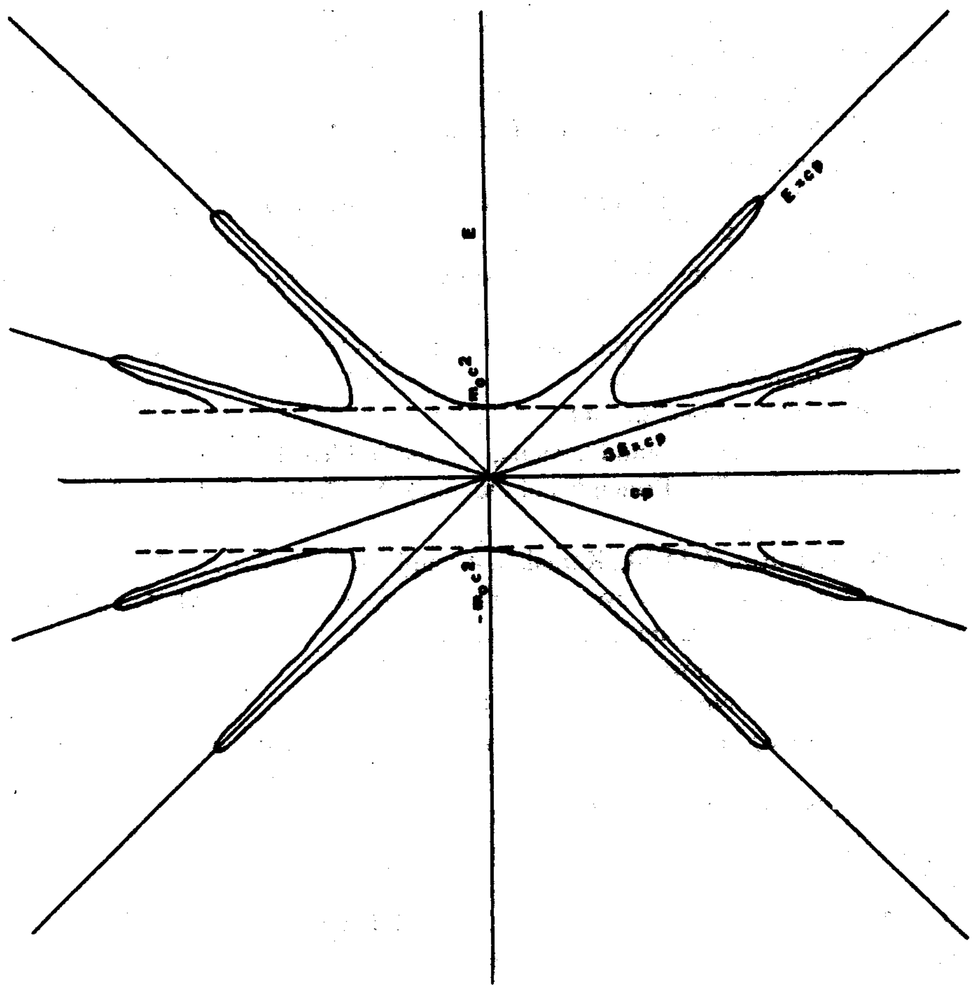

Fig.2: E as a function of cp in the extended relativity. Note the continuity of E-cp curve.

Conclusion

It appears that there have been no serious efforts to check the energy-momentum relationship (17) near the velocity of light. Even the calculated numbers (Particle Data Group, 1982) donot fit very well and there appear to be some deviations. However, since the data are not comprehensive, no conclusions can be drawn.

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