PROCEEDINGS of Second Workshop on TEACHING OF PHYSICS

December 27 - 28, 1986

APWA Govt. College For Women
KARACHI - PAKISTAN
HOW TO DEVELOP CREATIVE THINKING AND CRITICAL ANALYSIS?

SYED ARIF KAMAL* and KHURSHEED A. SIDIQUI
Department of Physics, University of Karachi
Karachi 75270, Pakistan.

Physics relates the abstract mathematical equations to real-life problems.
A few examples to develop creative thinking and critical analysis are presented.

INTRODUCTION

Creative thinking can be developed if the students are encouraged to develop alternate explanations of the topics discussed in the class and presented to check the validity of their assumptions.

The students can, critically, analyze a situation, if they have a thorough understanding of the principles involved. Critical analysis, also, requires an awareness of the validity and the limitations of the assumptions made for the solution of a problem.

In this session, a few problems and their solutions are discussed. Participants are requested to make similar problems for use in class discussions.

SAMPLE PROBLEMS

ONE: The relativistic mass of a particle is given by

\[ m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \] (1)

where \( m_0 \) is the rest (or proper) mass, \( v \) the velocity of particle and \( c \) the velocity of light in free space. For a photon \( v = c \) and, so, the denominator vanishes. In order to explain the finite mass (and energy) of a photon, its proper mass \( (m_0) \) is taken as zero.
**Problem:** When a light ray enters a medium having refractive index $n$, its velocity is decreased to $c/n$. If, now, we take $v = c/n$, the denominator is nonzero. With $m_0 = 0$, the mass (and the energy) of a photon vanishes. How could this be possible?

**TWO:** The Gauss’ theorem for a vector field $V$ may be written as

$$\oint_S V \cdot dS = \int_{\tau} \text{div} V \, d\tau$$

where LHS integral is over a surface $S$ and the RHS integral is over volume $\tau$, enclosed by the surface $S$. The Stokes’ theorem for a vector field $A$ may be expressed as

$$\oint_S A \cdot dl = \int_{\tau} \text{curl} A \cdot dS$$

where $\oint$ means line integral over a closed loop.

**Problem:** If we use Eq. (2) to convert RHS of Eq. (3) in the divergence form, we get

$$\oint_S \text{curl} A \cdot dS = \int_{\tau} \text{curl} A \cdot d\tau$$

However, $\text{div} \text{curl} A$ is, identically, zero. How could it be possible that we have for every vector $A$?

$$\oint A \cdot dl = 0$$

**THREE:** Suppose the energy-and-frequency relationship has other terms, in addition to the leading term, $h\nu$, i.e.,

$$E = h_1\nu + h_2\nu^{-2} + h_3\nu^{-5} + \ldots = \sum_{i=1}^{\infty} h_i \nu^{4-3i}$$

where $h_1$, $h_2$, $h_3$ have the same order of magnitude. However, the units of all these constants are different $h_{n+1}/h_n \sim 1 \text{ s}^3$. In the Planck spectrum, the effects of second- and higher-order terms are so small that they could not be detected.

**Problem:** Can the above relationship be a true law of nature?\

**FOUR:** Consider two experimenters having identical-perfectly-elastic spheres. Sphere“A” possessed by the experimenter at rest in a frame of reference $S$, and “B” poss-

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52 \hspace{1cm} CREATIVE THINKING AND CRITICAL ANALYSIS

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**Proceedings of 2nd Workshop on Teaching of Physics, Karachi, 1986**

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\*A better re-wording of the above question may be: How could one evaluate whether the above relationship might be a real law of nature — based on suggestion by J. Ryan Nielson
essed by the other at rest in $S'$, which has a speed $v$ relative to $S$ in the $x$ direction. As they pass, each of the experimenters projects own sphere with a speed $u$ in $y$ direction (as judged by self), so that a collision takes place. We shall suppose that $u$ is very small as compared to $v$. The velocity of sphere “B” as judged by $S$ is given by Eq. (6)

\begin{equation}
1 \beta - \frac{y'}{y''} = ct\frac{d}{d} \frac{\beta}{\sqrt{c^2}}
\end{equation}

where $\beta = v/c$ ($x'$ being constant for “B” as viewed from $S'$). If energy is not to be lost in the collision, the individual velocities of the sphere along their lines of centers are, simply, reversed. Thus, from the point of view of $S$, momentum can, only, be conserved if we put

\begin{equation}
m_0 u - mw = mw - m_0 u
\end{equation}

where $m_0$ is the mass of sphere “A” at rest relative to $S$ (except for an arbitrarily small velocity $u$), and $m$ is the apparent mass of an identical sphere “B”, which is passing with a velocity $v$. It, then, follows that

\begin{equation}
m(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}
\end{equation}

Problem: What is wrong with the above derivation?

**FIVE:** The length contraction and the time dilation (in special theory of relativity) are given by

\begin{equation}
dl' = dl\sqrt{1 - \beta^2}, \quad dt' = \frac{dt}{\sqrt{1 - \beta^2}}
\end{equation}

where $dl$ and $dt$ are length and time intervals in the laboratory frame ($S$), $dl'$ and $dt'$ are the values measured in a frame ($S'$) moving with velocity $c\beta$.

Problem: Dividing (8a) by (8b) and taking $dl/dt = c$, we get

\begin{equation}
\frac{dl'}{dt'} = \frac{dl}{dt} \sqrt{1 - \beta^2} = c\sqrt{1 - \beta^2} = c\sqrt{1 - \frac{v^2}{c^2}}
\end{equation}
How can we explain Eq. (8), when we know that velocity of light in free space is invariant in all frames of reference?

SOLUTIONS TO PROBLEMS

ONE: The photon, always travels with the velocity of light, $c$, in free space (vacuum). In a material medium, a photon is absorbed in an atom and excites an electron to a higher shell. This excited state is meta-stable. It de-excites by emission of a photon, which, then, travels again with velocity $c$ and, subsequently, absorbed by another atom. The process continues many times. Since, there is a time lag between the absorption and the emission of a photon, the effective velocity measured is $c/n$, not $c$.\footnote{This explanation has one drawback. It does not tell why photons of a wide range of frequencies could get absorbed in a material, when there are, only, certain discrete values of energy, which could be absorbed by the material depending on the energy states of the material. A better way to treat the problem may be to assert the dual nature of light. In a certain phenomenon, only one aspect of light is dominant, just like you can see only one side of a coin at a time. Refraction of light can be explained by wave nature of light. In fact, laws of reflection and refraction come out as consequences of Maxwell equations, when proper boundary conditions on electric-field vectors are applied. Particle aspect would not give a proper form of Snell’s law, because particles bend away from the normal in a denser medium. This approach should remove any confusion about the path and the velocity of a photon inside a material medium. It is interesting to note that the problem has intrigued many students of Physics. SAK, while an undergraduate student, contacted Nobel Laureates T. D. Lee and Abdus Salam (not, still, awarded Nobel Prize). Professor Fayazuddin answered on behalf of Professor Salam. Another attempt to resolve this conflict was made by Hafeez R. Hoorani. While a graduate student at Simon Fraser University, he asked Nobel Laureate Steven Weinberg, and later from Heins Rothe. The answers quoted in this paper are based on critical analyses of responses of all these torchbearers of physics.}

TWO: The Gauss’ theorem is applicable for integration over the entire surface, whereas the Stokes’ theorem deals with circulation around a patch and involves integration over a part of the surface. Therefore, we cannot substitute Eq. (3) in Eq. (2).

THREE: Calculate the shift in wavelength due to Compton scattering using Eq. (4). The answer comes out to

\[
\Delta \lambda = \frac{h}{mc^2}\delta E
\]
\[1 - \cos \theta = (\lambda' - \lambda) \frac{m_0 c}{h_1} + (\lambda'^2 - \lambda^2) \frac{m_0 c^4}{h_2} + (\lambda'^5 - \lambda^5) \frac{m_0 c^7}{h_3} + \ldots\]

where \(m_0\) is the electron rest mass.

Ratio of successive terms = \(\frac{h_{n+1}}{h_n} \left( \frac{c}{\lambda} \right)^3 \sim 10^{39} (c \sim 10^8 \text{ cm s}^{-1}, \lambda \sim 10^{-5} \text{ cm})\)

Therefore, the higher-order terms dominate making the series divergent. Since, the experimental results do not indicate contributions from any other term than the first, Eq. (4) could not represent a physical situation\(^3\).

**FOUR:** Eq. (6) is not correct. In elastic collision, velocities are reversed, only, if masses are equal. If a ball strikes, elastically, with a railroad wagon, the velocities are not reversed.

**FIVE:** For an analytical treatment of the length-contraction problem, consider two points of a body at the coordinates \(x_1, x_2\) in the frame \(S\).

\[x_1 = \frac{x'_1 + vt'_1}{\sqrt{1 - \beta^2}}, \quad x_2 = \frac{x'_2 + vt'_2}{\sqrt{1 - \beta^2}}\]

The apparent distance between these points as measured in \(S'\) is obtained by setting \(t'_1 = t'_2\). Then

\(^3\)Another related problem, which I worked out with sophomore students taking physics major was pair production, when a photon was tried to be stopped. Assuming that it produced an electron-positron pair, I set up the problem to compute the angle between the tracks of electron and positron by applying energy-linear-momentum conservation. The problem was analyzed by taking 2 MeV as energy of photon, equally shared by electron and positron. The analysis showed that cosine of half-angle between electron-positron track was greater than unity. I’ll manage to start the problem in such a way that class timings would end at this stage. I’ll then dismiss the class and ask the students to think through the problem. The next day, I’ll ask the class for the reason of the anomaly. In rare cases, some students would mention that the pair production must occur near a heavy nucleus to conserve energy and momentum. Otherwise, I’ll mention this small sentence given in pre-university physics textbooks, which none of the students have paid attention to. With this activity, the students do not forget this fine point.
\[ x_2 - x_1 = \frac{x'_2 - x'_1}{\sqrt{1 - \beta^2}} \]

or
\[ dl' = dl \sqrt{1 - \beta^2} \]

Therefore, Eq. (8a) is derived by considering \( dt' = 0 \). To obtain the time-dilation formula, consider the time-coordinate transformations
\[
\begin{align*}
t'_1 &= \frac{t_1 - v x_1/c^2}{\sqrt{1 - \beta^2}}, \\
t'_2 &= \frac{t_2 - v x_2/c^2}{\sqrt{1 - \beta^2}}
\end{align*}
\]

For a clock at rest in frame \( S \), \( x_1 = x_2 \) and so
\[ t'_2 - t'_1 = \frac{t_2 - t_1}{\sqrt{1 - \beta^2}} \]

or
\[ dt' = \frac{dt}{\sqrt{1 - \beta^2}} \]

Therefore, Eq. (8b) is derived by considering, \( dx = 0 \). Since, Equations (8a) and (8b) are derived using different assumptions; they cannot be divided to interpret \( dl/dt \) as the velocity in the laboratory frame.

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Proceedings of Second Workshop on Teaching of Physics

December 27, 28, 1986

CONTENTS

Interface between Science and Technology 01
Begum Ra’ana Liaquat Ali Khan Lecture
Salim Muhmud

Thin-Film Studies in Department of Physics,
University of Karachi 25
Riaz Ahmed Hashmi

Physics Makes the Deaf and the Dumb Equations of Mathematics to Speak 40
Khursheed Athar Siddiqui and Syed Arif Kamal

How to Develop Creative Thinking and Critical Analysis? 51
Syed Arif Kamal and Khursheed Athar Siddiqui

Fourier-Transform Spectroscopy and Interferometry 57
Syed Khadim Husain

Teaching of Modern Physics 67
Aquila Islam

Development of Research in Physics 70
with the Help of Local Resources
Nasiruddin Bukhari