Proceedings of the Second Workshop on Teaching of Physics, edited by Hasnain AF, Karachi, Pakistan, December 27, 28, 1986, pp 51-56

HOW TO DEVELOP CREATIVE THINKING AND CRITICAL ANALYSIS?

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Physics relates the abstract mathematical equations to real-life problems. A few examples to develop creative thinking and critical analysis are presented.

INTRODUCTION

Creative thinking can be developed if the students are encouraged to develop alternate explanations of the topics discussed in the class and presented to check the validity of their assumptions.

The students can, critically, analyze a situation, if they have a thorough understanding of the principles involved. Critical analysis, also, requires an awareness of the validity and the limitations of the assumptions made for the solution of a problem.

In this session, a few problems and their solutions are discussed. Participants are requested to make similar problems for use in class discussions.

SAMPLE PROBLEMS

ONE: The relativistic mass of a particle is given by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \tag{1}$$

where m_0 is the rest (or proper) mass, v the velocity of particle and c the velocity of light in free space. For a photon $v = c$ and, so, the denominator vanishes. In order to explain the finite mass (and energy) of a photon, its proper mass (m_0) is taken as zero.

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Problem: When a light ray enters a medium having refractive index n, its velocity is decreased to c/n. If, now, we take $v = c/n$, the denominator is nonzero. With $m_0 = 0$, the mass (and the energy) of a photon vanishes. How could this be possible?

TWO: The Gauss' theorem for a vector field V may be written as

$$\iint_S \boldsymbol{V} \cdot d\boldsymbol{S} = \iiint_\tau div\boldsymbol{V}\ d\tau \tag{2}$$

where LHS integral is over a surface S and the RHS integral is over volume τ, enclosed by the surface S. The Stokes' theorem for a vector field A may be expressed as

$$\oint \boldsymbol{A} \cdot d\boldsymbol{l} = \iint_S curl\boldsymbol{A} \cdot d\boldsymbol{S} \tag{3}$$

where $\oint$ means line integral over a closed loop.

Problem: If we use Eq. (2) to convert RHS of Eq. (3) in the divergence form, we get

$$\iint_S curl\boldsymbol{A} \cdot d\boldsymbol{S} = \iiint_\tau div\ curl\boldsymbol{A}\ d\tau$$

However, $div\ curl\boldsymbol{A}$ is, identically, zero. How could it be possible that we have for every vector $\boldsymbol{A}$?

$$\oint \boldsymbol{A} \cdot d\boldsymbol{l} = 0$$

THREE: Suppose the energy-and-frequency relationship has other terms, in addition to the leading term, $h\nu$, *i. e.*,

$$E = h_1\nu + h_2\nu^{-2} + h_3\nu^{-5} + \ldots = \sum_{i=1}^{\infty} h_i \nu^{4-3i} \tag{4}$$

where h_1, h_2, h_3 have the same order of magnitude. However, the units of all these constants are different $h_{n+1}/h_n \sim 1\,s^3$. In the Planck spectrum, the effects of second- and higher-order terms are so small that they could not be detected.

Problem: Can the above relationship be a true law of nature?

FOUR: Consider two experimenters having identical-perfectly-elastic spheres. Sphere "A" possessed by the experimenter at rest in a frame of reference S, and "B" possessed by the other at rest in S', which has a speed v relative to S in the x direction. As they pass, each of the experimenters projects own sphere with a speed u in y direction (as judged by self), so that a collision takes place. We shall suppose that *u* is very small as compared to *v*. The velocity of sphere "B" as judged by *S* is given by Eq. (6)

(5) $$y = y',\ t = \frac{t' + vx'/c^2}{\sqrt{1-\beta^2}},\ dy = dy',\ dt = \frac{dt'}{\sqrt{1-\beta^2}}$$

(6) $$w = \frac{dy}{dt} = \frac{dy'}{dt'}\sqrt{1-\beta^2} = u\sqrt{1-\beta^2}$$

where $\beta = v/c$ (x' being constant for "B" as viewed from S'). If energy is not to be lost in the collision, the individual velocities of the sphere along their lines of centers are, simply, reversed. Thus, from the point of view of *S*, momentum can, only, be conserved if we put

(6) $$m_0 u - mw = mw - m_0 u$$

where m_0 is the mass of sphere "A" at rest relative to *S* (except for an arbitrarily small velocity *u*), and *m* is the apparent mass of an identical sphere "B", which is passing with a velocity *v*. It, then, follows that

(7) $$m(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Problem: What is wrong with the above derivation?

FIVE: The length contraction and the time dilation (in special theory of relativity) are given by

(8*a*, *b*) $$dl' = dl\sqrt{1-\beta^2},\ dt' = \frac{dt}{\sqrt{1-\beta^2}}$$

where *dl* and *dt* are length and time intervals in the laboratory frame (S), dl' and dt' are the values measured in a frame (S') moving with velocity $c\beta$.

Problem: Dividing (8*a*) by (8*b*) and taking $dl/dt = c$, we get

(9) $$\frac{dl'}{dt'} = \frac{dl}{dt}\sqrt{1-\beta^2} = c\sqrt{1-\beta^2} = c\sqrt{1 - \frac{v^2}{c^2}}$$

How can we explain Eq. (8), when we know that velocity of light in free space is invariant in all frames of reference?

SOLUTIONS TO PROBLEMS

ONE: The photon, always travels with the velocity of light, *c*, in free space (vacuum). In a material medium, a photon is absorbed in an atom and excites an electron to a higher shell. This excited state is meta-stable. It de-excites by emission of a photon, which, then, travels again with velocity *c* and, subsequently, absorbed by another atom. The process continues many times. Since, there is a time lag between the absorption and the emission of a photon, the effective velocity measured is c/n, not *c*[¶].

TWO: The Gauss' theorem is applicable for integration over the entire surface, whereas the Stokes' theorem deals with circulation around a patch and involves integration over a part of the surface. Therefore, we cannot substitute Eq. (3) in Eq. (2).

THREE: Calculate the shift in wavelength due to Compton scattering using Eq. (4). The answer comes out to

$$1-\cos\theta=(\lambda'-\lambda)\frac{m_0c}{h_1}+(\lambda'^{-2}-\lambda^{-2})\frac{m_0c^4}{h_2}+(\lambda'^{-5}-\lambda^{-5})\frac{m_0c^7}{h_3}+\ldots.$$

[¶]This explanation has one drawback. It does not tell why photons of a wide range of frequencies could get absorbed in a material, when there are, only, certain discrete values of energy, which could be absorbed by the material depending on the energy states of the material. A better way to treat the problem may be to assert the dual nature of light. In a certain phenomenon, only one aspect of light is dominant, just like you can see only one side of a coin at a time. Refraction of light can be explained by wave nature of light. In fact, laws of reflection and refraction come out as consequences of Maxwell equations, when proper boundary conditions on electric-field vectors are applied. Particle aspect would not give a proper form of Snell's law, because particles bend away from the normal in a denser medium. This approach should remove any confusion about the path and the velocity of a photon inside a material medium. It is interesting to note that the problem has intrigued many students of Physics. SAK, while an undergraduate student, contacted Nobel Laureates TD Lee and Abdus Salam (not, still, awarded Nobel Prize). Professor Fayazuddin answered on behalf of Professor Salam. Another attempt to resolve this conflict was made by Professor Hafeez R Hoorani of National Center for Physics, Islamabad. While a graduate student at Simon Fraser University, he asked Nobel Laureate Steven Weinberg, and later from Heins Rothe. The answers quoted in this paper are based on critical analyses of responses of all these torchbearers of physics.

where m_0 is the electron rest mass.

$$\text{Ratio of successive terms} = \frac{h_{n+1}}{h_n}\left(\frac{c}{\lambda}\right)^3 \sim 10^{39}\,(c \sim 10^8\,\text{cm s}^{-1},\ \lambda \sim 10^{-5}\,\text{cm})$$

Therefore, the higher-order terms dominate making the series divergent. Since, the experimental results do not indicate contributions from any other term than the first, Eq. (4) could not represent a physical situation.

FOUR: Eq.. (6) is not correct. In elastic collision, velocities are reversed, only, if masses are equal. If a ball strikes, elastically, with a railroad wagon, the velocities are not reversed.

FIVE: For an analytical treatment of the length-contraction problem, consider two points of a body at the coördinates x_1, x_2 in the frame *S*.

$$x_1 = \frac{x_1' + vt_1'}{\sqrt{1-\beta^2}},\ x_2 = \frac{x_2' + vt_2'}{\sqrt{1-\beta^2}}$$

The apparent distance between these points as measured in S' is obtained by setting $t_1' = t_2'$. Then

$$x_2 - x_1 = \frac{x_2' - x_1'}{\sqrt{1-\beta^2}}$$

or

$$dl' = dl\sqrt{1-\beta^2}$$

Therefore, Eq. (8*a*) is derived by considering $dt' = 0$. To obtain the time-dilation formula, consider the time-coördinate transformations

$$t_1' = \frac{t_1 - v x_1/c^2}{\sqrt{1-\beta^2}},\ t_2' = \frac{t_2 - v x_2/c^2}{\sqrt{1-\beta^2}}$$

For a clock at rest in frame *S*, $x_1 = x_2$ and so

$$t_2' - t_1' = \frac{t_2 - t_1}{\sqrt{1-\beta^2}}$$

or

$$dt' = \frac{dt}{\sqrt{1-\beta^2}}$$

Therefore, Eq. (8*b*) is derived by considering, $dx = 0$. Since, Equations (8*a*) and (8*b*) are derived using different assumptions; they cannot be divided to interpret dl/dt as the velocity in the laboratory frame.

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