کراچی یونیورسٹی
University of Karachi
Department of Mathematics
MATH 686: Astronomy II

Time Allowed: 3 hours • *Maximum Marks*: 100 • *Date*: Wednesday, December 9, 2009 • *Paper Format*: A

*Student's Name*_____ (in CAPITAL LETTERS using a "**marker**")

Attempt ***Question*** **1** *and* **5** *other questions. Each part of* ***Question*** **1** *is of* 4 *marks & of* ***Questions*** **2-7** *is of* 8 *marks. Note down the time spent on solving each part of question and time spent on revision by making the following "Time Chart" on the answerbook front page* **(2 marks)**. *Time spent on revision must be at least 10% of the total time*

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Question No.	1a	1b	1c	1d	2a	2b	3a	3b	4a	4b	5a	5b	6a	6b	7a	7b	Revision
Time (minutes)																	

a) DO NOT TURN PAPER AROUND unless the invigilator says: *Start* now

b) This is a closed-book examination. Deposit all BOOKS, NOTES, MOBILE PHONES (switched off), DIGITAL DIARIES and LAPTOPS in the designated area. Remove everything from your desk, except markers, pens, pencils, stapler and calculator.

c) If you want to use a calculator, **it must bear a "*sticker*"** displaying your NAME and your SEAT NUMBER, large enough so that it is visible from a distance of 5 meters. **Absolutely, NO sharing of calculators.**

d) Use your own material. **Nothing can be borrowed from or given to** a friend.

e) The papers may be of different formats. Therefore, work on your own ***without consulting anyone*** (We have a record of your seating arrangement).

f) Write your "**NAME**" on all pages of ***your question paper*** (5 marks shall be deducted for failing to comply) [NOTHING ELSE SHOULD BE WRITTEN ON THE QUESTION PAPER] and "**PAPER FORMAT**" on the front page of ***your answerbook*** (the upper-right-hand corner) and ***the Yellow Sheet*** using a "**marker**". Start your work from **Page 2** of your answerbook. The only thing that could be written on the front page is the "*Time Chart*" (see above) and the "*Honor Statement*" (see below).

g) The following statement must be copied on the front page of your answerbook and signed **(2 marks)**: "*My signatures, below, testify that I am the person, whose name and photograph appear on the Admit Card. Upon my honor, I declare that the following work is my own, completed without giving or receiving unacknowledged help, without copying, or the use of any unfair means*." Signatures_____

h) This paper contains TWO PAGES (this page and the back page). On invigilator's signal (*Start now*) turn the paper around, check if you have the back page printed, correctly. Last line of the second page reads: <**END**>. Start working on the paper, immediately.

i) Put your pens down and your papers turned (so that this page is facing you) and the FRONT PAGE of your answerbook should be facing you as soon as you hear "**ALL PENS DOWN**". Failure to do as directed shall result in "deduction of 5 marks" from your score.

j) If you use extra copies, it is "**your responsibility**" to write YOUR NAME, COPY NUMBER and all OTHER INFORMATION on each copy used. All the extra copies must be stapled with the main copy, before turning in your paper (you may wish to bring in a stapler with you for this purpose).

k) If you have a question of "Fill in the blanks" in your paper you must write the complete sentence with the filled word underlined.

l) Nobody is allowed to leave the examination hall, **for whatsoever reason**, once the examination has started. Bring your own DRINKING WATER.

m) Students are not allowed to LEAVE THEIR SEATS or STAND UP during the examination. If you have a query, "raise your hand" and someone will help you.

n) All work, including rough work, must be on the official answerbook. No extra sheet may be used.

o) Students are *not* allowed to use RED anywhere. All work (except figures) must be in pen or ballpoint.

p) The result shall be displayed on my homepage on **Monday, December 14, 2009** at **1500h**. DO NOT contact the Course Supervisor. Students are not permitted to see the answerbooks.

q) Anyone found cheating in the examination should be facing disciplinary action, which may result in **EXPLUSION** or **SUSPENSION** for 2 or more years. **Absolutely, no conversation among students. DON'T TURN THE PAPER, YET.** *Wait for "signal" from the invigilator.*

Course Supervisor
Homepage: http://ngds-ku.org/kamal
e-mail: kamal(at the rate of)ngds-ku.org

*Student's Name*_____ (in CAPITAL LETTERS using a "**marker**")

1-*a)* State "Strong Noether Theorem". What benefits are achieved when the two-body problem of planetary motion is set up in the elliptic-astrodynamical-coördinate mesh?

1-*b)* Compute the scale factors h_x, h_E, h_z for the elliptic-astrodynamical-coördinate mesh.

1-*c)* Why an on-board computing system is preferred to telemetric control from ground? Describe one measure taken by the mathematician and one measure taken by the programmer to reduce the computing time of the data-processing system of INS (Inertial Navigation System).

1-*d)* In the following equation name the first, the second, the third and the fourth term:
$$\vec{F}_{eff} = m\vec{a}_f - m\vec{w}\times(\vec{w}\times\vec{r}_R) - 2m\vec{w}\times\vec{v}_R - m\frac{d\vec{w}}{dt}\times\vec{r}_R$$

2-*a)* Starting from two-body lagrangian derive the equation of orbit:
$$\frac{d^2}{dq^2}\left(\frac{1}{r}\right)+\frac{1}{r}=\frac{m_R r^2}{l^2}F(r)$$

2-*b)* Using equation of orbit, find force law corresponding to the orbit:
$$r=\frac{p}{1-e\sin q}$$

3-*a)* How does one start from 12 degrees of freedom in a two-body problem and reduce them to 2 degrees in plane-polar coördinates?

3-*b)* Use the law of conservation of angular momentum to (*i*) show that planetary orbits must lie in a plane; (*ii*) prove Kepler's second law of planetary motion

4-*a)* Starting from the elliptic-astrodynamical lagrangian:
$$L=\frac{mMa^2(1-e^2\cos^2 E)}{2(m+M)}\dot{E}^2+\frac{GmM}{a(1-e\cos E)}$$
obtain the following equation of motion:
$$(1-e^2\cos^2 E)\ddot{E}+\tfrac{1}{2}(e^2\sin 2E)\dot{E}^2+\frac{me\sin E}{a^3(1-e\cos E)^2}=0$$

4-*b)* Show that Kepler's equation, $\sqrt{m}(t-t)=a^{3/2}(E-e\sin E)$, is a particular solution of the equation of motion.

5-*a)* By drawing a distance-energy diagram define and explain stable, unstable and neutral equilibria.

5-*b)* Starting from the force law, $F=-\frac{K}{r^n}$, show that stable orbits can exist only if $n < 3$.

6-*a)* By drawing appropriate diagram show that the velocities, Δv_1 (which must be imparted to satellite at the lower-circular orbit of radius a_1) and Δv_2 (which must be imparted to satellite at the higher-circular orbit of radius a_2), may be expressed as [Hohmann Orbital Transfer]**:**
$$\Delta v_1=\sqrt{\frac{2m}{a_1}-\frac{2m}{a_1+a_2}}-\sqrt{\frac{m}{a_1}};\Delta v_2=\sqrt{\frac{m}{a_2}}-\sqrt{\frac{2m}{a_2}-\frac{2m}{a_1+a_2}}$$

6-*b)* Obtain an expression of down-range error for surface-to-surface missile, if the fight-path angle changes from g to $g + \Delta g$ after t_1 seconds elapsed from time-of-launch.

7-*a)* Let the bearing be j at the time of launch. At time, $t = t_1$, a disturbance is introduced which makes the bearing $j + \Delta j$. This would introduce a cross-range error ΔR. Compute this error assuming that 'g' remains constant during the flight.

7-*b)* What is the major difference between the orbits of satellites and missiles? What is a polar satellite and a geostationary satellite? Give 2 applications of polar satellites.

Web address of this document: http://www.ngds-ku.org/M685-6/Exams685-6/686-09_Paper.pdf

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