UNIVERSITY OF KARACHI

Department of Mathematics

MATH 686 • Astronomy II

Course Supervisor: Professor Dr. Syed Arif Kamal

Homepage: http://ngds-ku.org/kamal • *e-mail***:** kamal(at the rate of)ngds-ku.org

*Time Allowed***:** 3 hours • *Maximum Marks***:** 100 • *Date***:** Wednesday, December 3, 2003

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*Student's Name*_____ (in CAPITAL LETTERS using a "**marker**") • *Paper Format* A

*Attempt **Question 1** and 5 other questions. Each part of **Question 1** is of 4 marks & of **Questions 2-7** is of 8 marks. Note down the time spent on solving each part of question and time spent on revision by making the following "Time Chart" on the answerbook front page* **(2 marks)**. *Time spent on revision must be at least 10% of the total time*

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Question No.	1a	1b	1c	1d	2a	2b	3a	3b	4a	4b	5a	5b	6a	6b	7a	7b	Revision
Time (minutes)																	

a) DO NOT TURN PAPER AROUND unless the invigilator says: *Start* now

b) This is a closed book examination. Deposit all BOOKS, NOTES, MOBILE PHONES (switched off), DIGITAL DIARIES and LAPTOPS in the designated area. Remove everything from your desk except markers, pens, pencils, stapler and calculator.

c) If you want to use a calculator, **it must bear a "*sticker*"** displaying your NAME and your SEAT NUMBER large enough so that it is visible from a distance of 5 meters. **Absolutely, NO sharing of calculators.**

d) Use your own material. **Nothing can be borrowed from or given to** a friend.

e) The papers may be of different formats. Therefore, work on your own ***without consulting anyone*** (We have a record of your seating arrangement).

f) Write your "**NAME**" on all pages of ***your question paper*** (5 marks shall be deducted for failing to comply) [NOTHING ELSE SHOULD BE WRITTEN ON THE QUESTION PAPER] and "**PAPER FORMAT**" on the front page of ***your answerbook*** (the upper right-hand corner) and ***the Yellow Sheet*** using a "**marker**". Start your work from **Page 2** of your answerbook. The only thing that could be written on the front page is the "*Time Chart*" (see above) and the "*Honor Statement*" (see below).

g) The following statement must be copied on the front page of your answerbook and signed **(2 marks)**: "*My signatures below testify that I am the person whose name and photograph appear on the Admit Card. Upon my honor, I declare that the following work is my own, completed without giving or receiving unacknowledged help, without copying, or the use of any unfair means*." Signatures_____

h) This paper contains TWO PAGES (This page and the back page). On invigilator's signal (*Start now*) turn paper around, check if you have the back page printed correctly. Last line of the second page is: <**END**>. Start working on the paper immediately.

i) Put your pens down and your papers turned (so that this page is facing you) and the FRONT PAGE of your answerbook should be facing you as soon as you hear "**ALL PENS DOWN**". Failure to do as directed shall result in "deduction of 5 marks" from your score.

j) If you use extra copies, it is "your responsibility" to write YOUR NAME, COPY NUMBER and all OTHER INFORMATION on each copy used. All the extra copies must be stapled with the main copy before turning in your paper (you may wish to bring in a stapler with you for this purpose).

k) If you have a question of "Fill in the blanks" in your paper you must write the complete sentence with the filled word underlined.

l) Nobody is allowed to leave the examination hall, **for whatsoever reason**, once the examination has started. Bring your own DRINKING WATER.

m) Students are not allowed to LEAVE THEIR SEATS or STAND UP during the examination. If you have a query, "raise your hand" and someone will help you.

n) All work, including rough work, must be on the official answerbook. No extra sheet may be used.

o) Students are *not* allowed to use RED anywhere. All work (except figures) must be in pen or ballpoint.

p) The result shall be displayed on my homepage on **Wednesday, December 10, 2003** at **0900h**. DO NOT contact the Course Supervisor. Students are not permitted to see the answerbooks.

q) Anyone found cheating in the examination should be facing disciplinary action, which may result in **EXPLUSION** or **SUSPENSION** for 2 or more years. **Absolutely, no conversation among students. DON'T TURN THE PAPER, YET.** *Wait for "signal" from the invigilator.*

*Student's Name*_____ (in CAPITAL LETTERS using a "**marker**")

1-a) State "Strong Noether Theorem". What benefits are achieved when the two-body problem of planetary motion is set up in the elliptic-astrodynamical-coördinate mesh?

1-b) Why is the variable ***TYPE*** needed in the transfer-time equation? What should be the value of ***TYPE*** for a missile fired westward from Karachi?

1-c) How does one start from 12 degrees of freedom in a two-body problem and reduce them to 2 degrees in plane-polar coördinates?

1-d) Why were (r, θ) used in the classical treatment instead of (x, y)?

2-a) For a potential energy curve of the form: $U(q) = mgl(1-\cos q)$, find the points where the body will be in equilibrium? Which of these points correspond to stable equilibrium and which to unstable equilibrium?

2-b) Which system does the above curve correspond to? When does one use the stable equilibrium condition and when the unstable equilibrium condition?

3-a) What are the links between dynamics and kinematics and vice versa in the two-body formulation?

3-b) Starting from the elliptical-astrodynamical lagrangian:

$$L = \frac{mMa^2(1-e^2\cos^2 E)}{2(m+M)}\dot{E}^2 + \frac{GmM}{a(1-e\cos E)}$$

Obtain the following equation of motion:

$$(1-e^2\cos^2 E)\ddot{E} + \tfrac{1}{2}(e^2 \sin 2E)\dot{E}^2 + \frac{me\sin E}{a^3(1-e\cos E)^2} = 0$$

4-a) Show that the Kepler's equation, $\sqrt{m}(t-t) = a^{3/2}(E - e\sin E)$, is a particular solution of the equation of motion

4-b) Show that $(\hat{e}_x, \hat{e}_{E,,}\hat{e}_z)$ spans a three-dimensional space.

5-a) Why is the statement of Cross-Product Steering, as given by RH Battin, incomplete? How does the "Extended-Cross-Product Steering" overcome this deficiency? Give its mathematical statement.

5-b) Give the mathematical statement of "Dot-Product Steering". Provide a general proof.

6-a) Compute the scale factors h_x, h_E, h_z for the elliptic-astrodynamical-coördinate mesh.

6-b) Show that equation of ellipse in cartesian coördinates with center at (–*ae*, 0) is equivalent to:

$$r = \frac{p}{1+e\cos f}; \quad 0 \le f \le 180^0$$

7-a) Justify that a projectile trajectory in constant gravity (parabola) is, actually, a limiting case of the trajectory computed from the Kepler equation (ellipse).

7-b) By drawing appropriate diagram show that the velocities, Δv_1 (which must be imparted to satellite at the lower-circular orbit of radius a_1) and Δv_2 (which must be imparted to satellite at the higher-circular orbit of radius a_2), may be expressed as [Hohmann Orbital Transfer]:

$$\Delta v_1 = \sqrt{\frac{2m}{a_1} - \frac{2m}{a_1+a_2}} - \sqrt{\frac{m}{a_1}}; \Delta v_2 = \sqrt{\frac{m}{a_2}} - \sqrt{\frac{2m}{a_2} - \frac{2m}{a_1+a_2}}$$

END