



UNIVERSITY OF KARACHI

Department of Mathematics

MATH 686 • Astronomy II

Course Supervisor: Professor Dr. Syed Arif Kamal

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Time Allowed: 3 hours • Maximum Marks: 100 • Date: Friday, November 15, 2002

Student's Name _____ (in CAPITAL LETTERS using a "marker") • Paper Format A

Attempt **Question 1** and 5 other questions. Each part of **Question 1** is of 4 marks & of **Questions 2-7** is of 8 marks. Note down the time spent on solving each part of question and time spent on revision by making the following "Time Chart" on the answerbook front page (2 marks). Time spent on revision must be at least 10% of the total time

Question No.	1a	1b	1c	1d	2a	2b	3a	3b	4a	4b	5a	5b	6a	6b	7a	7b	Revision
Time (minutes)																	

a) DO NOT TURN PAPER AROUND unless the invigilator says: *Start* now

b) This is a closed book examination. Deposit all BOOKS, NOTES, MOBILE PHONES (switched off), DIGITAL DIARIES and LAPTOPS in the designated area. Remove everything from your desk except markers, pens, pencils, stapler and calculator.

c) If you want to use a calculator, **it must bear a "sticker"** displaying your NAME and your SEAT NUMBER large enough so that it is visible from a distance of 5 meters. **Absolutely, NO sharing of calculators.**

d) Use your own material. **Nothing can be borrowed from or given to** a friend.

e) The papers may be of different formats. Therefore, work on your own **without consulting anyone** (We have a record of your seating arrangement).

f) Write your "NAME" on all pages of **your question paper** (5 marks shall be deducted for failing to comply) [NOTHING ELSE SHOULD BE WRITTEN ON THE QUESTION PAPER] and "PAPER FORMAT" on the front page of **your answerbook** (the upper right-hand corner) and **the Yellow Sheet** using a "marker". Start your work from **Page 2** of your answerbook. The only thing that could be written on the front page is the "Time Chart" (see above) and the "Honor Statement" (see below).

g) The following statement must be copied on the front page of your answerbook and signed (2 marks): "My signatures below testify that I am the person whose name and photograph appear on the Admit Card. Upon my honor, I declare that the following work is my own, completed without giving or receiving unacknowledged help, without copying, or the use of any unfair means." Signatures _____

h) This paper contains TWO PAGES (This page and the back page). On invigilator's signal (*Start now*) turn paper around, check if you have the back page printed correctly. Last line of second page is: <END>. Start working on the paper immediately.

i) Put your pens down and your papers turned (so that this page is facing you) and the FRONT PAGE of your answerbook should be facing you as soon as you hear "ALL PENS DOWN". Failure to do as directed shall result in "deduction of 5 marks" from your score.

j) If you use extra copies, it is "your responsibility" to write YOUR NAME, COPY NUMBER and all OTHER INFORMATION on each copy used. All the extra copies must be stapled with the main copy before turning in your paper (you may wish to bring in a stapler with you for this purpose).

k) If you have a question of "Fill in the blanks" in your paper you must write the complete sentence with the filled word underlined.

l) Nobody is allowed to leave the examination hall, **for whatsoever reason**, once the examination has started. Bring your own DRINKING WATER.

m) Students are not allowed to LEAVE THEIR SEATS or STAND UP during the examination. If you have a query, "raise your hand" and someone will help you.

n) All work, including rough work, must be on the official answerbook. No extra sheet may be used.

o) Students are **not** allowed to use RED anywhere. All work (except figures) must be in pen or ballpoint.

p) The result shall be displayed on the Departmental Notice Board on **Thursday, November 21, 2002**. DO NOT contact the Course Supervisor. Students are not permitted to see the answerbooks. **SOLUTION is available at the photocopy shop, when the examination ends.**

q) Anyone found cheating in the examination should be facing disciplinary action, which may result in **EXCLUSION** or **SUSPENSION** for 2 or more years. **Absolutely, no conversation among students. DON'T TURN THE PAPER, YET. Wait for "signal" from the invigilator.**

Student's Name _____ (in CAPITAL LETTERS using a "marker")

- 1-a) Why is the variable *TYPE* needed in the transfer-time equation? What should be the value of *TYPE* for a missile fired westward from Karachi?
- 1-b) How does one start from 12 degrees of freedom in a two-body problem and reduce them to 2 degrees in plane-polar coördinates?
- 1-c) What are the conventions to label *x*, *y*, *z* axes in an aircraft/rocket? What are the rotations about each of these axes called? Which one produces down-range error and which one cross-range error? Why does one introduce roll in the rocket?
- 1-d) What are the basic assumptions taken in the design of a short-range, an intermediate range (IRBM) and an intercontinental ballistic missile (ICBM)? Why is the word "ballistic" used in these names?
- 2-a) What are the links between dynamics and kinematics and vice versa in the two-body formulation?
- 2-b) Starting from the elliptical-astrodynamical lagrangian:

$$\mathcal{L} = \frac{1}{2} \{ m M a^2 (1 - e^2 \cos^2 E) / (m + M) \} (dE/dt)^2 + G m M / [a(1 - e \cos E)]$$

Obtain the following equation of motion:

$$(1 - e^2 \cos^2 E) (d^2 E / dt^2) + \frac{1}{2} e^2 \sin(2E) (dE/dt)^2 + [G(m + M) e \sin E] / [a^3 ((1 - e \cos E)^2)] = 0$$

- 3-a) By drawing a diagram illustrate how one defines the elliptic-astrodynamical-coördinate mesh (*x*, *E*) in terms of the plane-polar-coördinate mesh (*r*, *f*). What are the benefits achieved by using this formulation?
- 3-b) Obtain the relation: $\tan(\frac{1}{2} E) = ' \tan(\frac{1}{2} f)$, where $' = [(1 - e)/(1 + e)]^{1/2}$, and justify positive sign of the radical.
- 4-a) Why is the statement of cross-product steering, as given by RH Battin, incomplete? How does the "Extended-Cross-Product Steering" overcome this deficiency? Give its mathematical statement.
- 4-b) Give the mathematical statement of "Dot-Product Steering". Provide a general proof.
- 5-a) Why $x = h / ae$, and not $h = \ln [(1 + \hat{I}) / (1 - \hat{I})]$, $\hat{I} = (1 - e^2)^{1/2}$, is a good choice as an elliptic-astrodynamical coördinate? Complete the following table:

Energy	Eccentricity	Shape of Orbit	Type of Orbit	System (bound/free)	Number of Turning Points

- 5-b) Using equation of orbit $d^2(1/r)/dq^2 + (1/r) = -(m_R r^2 / \ell^2) F(r)$, find force law corresponding to the orbit:

$$r = p / (1 + e \cos f)$$

- 6-a) Consider a surface-to-surface missile thrown upward with a velocity, '*v*₀', at an angle, '*q*', with the horizontal. Consider motion under constant '*g*'. Compute the total transfer time (time-of-flight), range, flight-direction angle and velocity as a function of time, '*t*'.
- 6-b) In the following situations tell whether the use of reduced mass is (i) unnecessary, (ii) suggested, if accurate measurements need to be taken, (iii) required: A. missile dynamics, B. alpha-particle scattering, C. hydrogen atom, D. deuteron nucleus.
- 7-a) Obtain an expression of down-range error for surface-to-surface missile in Q. 6-a), if the fight-path angle changes from '*q*' to '*g* + *Dg*' after *t*₁ seconds elapsed from time-of-launch.
- 7-b) Justify that a projectile trajectory in constant gravity (parabola) is actually a limiting case of the trajectory computed from the Kepler's equation.

END