



JINNAH UNIVERSITY FOR WOMEN

Terminal Examination 2018* • Course Supervisor Prof. Dr. Syed Arif Kamal
Department Mathematics • Class BS-IV/MSc (Final) • Date December 12, 2018
Course Title Sport Mathematics • Course Code MAT 4222/MAT 621
Time Allowed 2 hours 30 minutes • Max Marks 60

Note: Attempt all questions. Marks of each question are shown in front of that question.

Question No.	Question Text	Marks Allocated
01	Write down the formula to determine focal length of a convex lens relating distances of object and image from the center of lens. If data are available for a set of 5 readings of these distances, how would you determine the focal length by fitting least-square lines (give at least 2 different methods).	10
02	Prove the following: (a) $\delta_{ij}A_j = A_i$ Using ϵ_{ijk} notation prove: (b) $\vec{A} \cdot \vec{B} \times \vec{C} = \vec{A} \times \vec{B} \cdot \vec{C}$	10 10
03	By drawing a diagram explain how scoliosis-like conditions are ruled out using differential-spinal-function testing.	11
04	(a) By setting up equation of simple pendulum (and drawing a labeled diagram) show that stable equilibrium occurs when the pendulum bob is closest to earth surface and unstable equilibrium occurs when it is farthest from the earth surface. (b) Briefly describe the role of energy-channelization in producing spastic gait. (c) Why is it mandatory that the child should be in dress code 0/0.5 for a proper evaluation of gait?	04 03 02
05	Write short note on any one of the following: (a) Fitness of an athlete (b) Degree of correction of spinal deformity	10

*Sample paper — actual paper should be different from this one



SOLUTION

Question 1

Write down the formula to determine focal length of a convex lens relating distances of object and image from the center of lens. If data are available for a set of 5 readings of these distances, how would you determine the focal length by fitting least-square lines (give at least 2 different methods).

Solution 1

if u represents distance of object from center of lens, v distance of image from center of lens and f the focal length, these distances are related by (Fig. 1)

$$(1) \quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow v = \frac{uf}{u - f}$$

Eq. (1) may be expressed in the following 2 different forms, both of which represent straight lines:

$$(2a) \quad \frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$$

$$(2b) \quad uv = f(u + v)$$

For (2a), plot $\frac{1}{u}$ and $\frac{1}{v}$. The data would apparently lie along a line with slope -1 and y intercept $\frac{1}{f}$. For (2b), plot $(u + v)$ and uv . The data would apparently lie along a line with slope f . For more precise results fit least square lines to determine the above-mentioned slope and y intercept.

Question 2a

Prove the following: $\delta_{ij} A_j = A_i$

Solution 2a

$$\text{LHS} = \delta_{ij} A_j = \delta_{i1} A_1 + \delta_{i2} A_2 + \delta_{i3} A_3$$

$$\text{For } i = 1, \text{ LHS} = \delta_{ij} A_j = \delta_{i1} A_1 + \delta_{i2} A_2 + \delta_{i3} A_3$$

$$= (1)A_1 + (0)A_2 + (0)A_3$$

$$= A_1$$

$$= \text{RHS}$$

Similarly, for $i = 2, 3$, LHS = RHS and the result is proved.

Question 2b

Using ϵ_{ijk} notation prove: $\vec{A} \cdot \vec{B} \times \vec{C} = \vec{A} \times \vec{B} \cdot \vec{C}$

Solution 2b

$$\text{LHS} = \vec{A} \cdot \vec{B} \times \vec{C} = A_i \epsilon_{ijk} B_j C_k = \epsilon_{ijk} A_i B_j C_k$$

$$\text{RHS} = \vec{A} \times \vec{B} \cdot \vec{C} = \epsilon_{ijk} A_j B_k C_i$$

Since i, j, k are dummy indices, j is replaced by i , k by j and i by k in the above equation, which now becomes

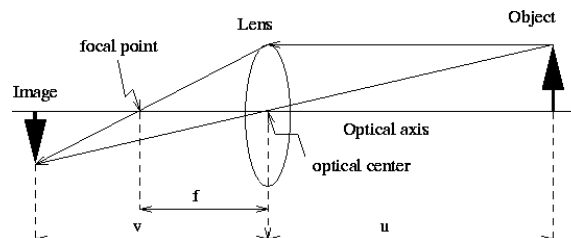


Fig. 1. Geometry of image formation



$$\text{RHS} = \epsilon_{kij} A_i B_j C_k$$

$= -\epsilon_{ikj} A_i B_j C_k$, because Levi-Civita symbol (a tensor density) picks up negative sign when adjacent indices are exchanged. Doing this again

$$\text{RHS} = -(-\epsilon_{ijk}) A_i B_j C_k$$

$$= \epsilon_{ijk} A_i B_j C_k$$

Hence, LHS = RHS and the result is proved.

Question 3

By drawing a diagram explain how scoliosis-like conditions are ruled out using differential-spinal-function testing.

Solution 3

A mandatory two-minute-stripped-scoliosis screening, which includes moiré examination of back, for school-going children in the age group 7-10 years and a follow-up of at-risk cases may prevent suffering of a lifetime. Fig. 2 illustrates decision matrix to detect possible existence of lateral curvatures and rotations of

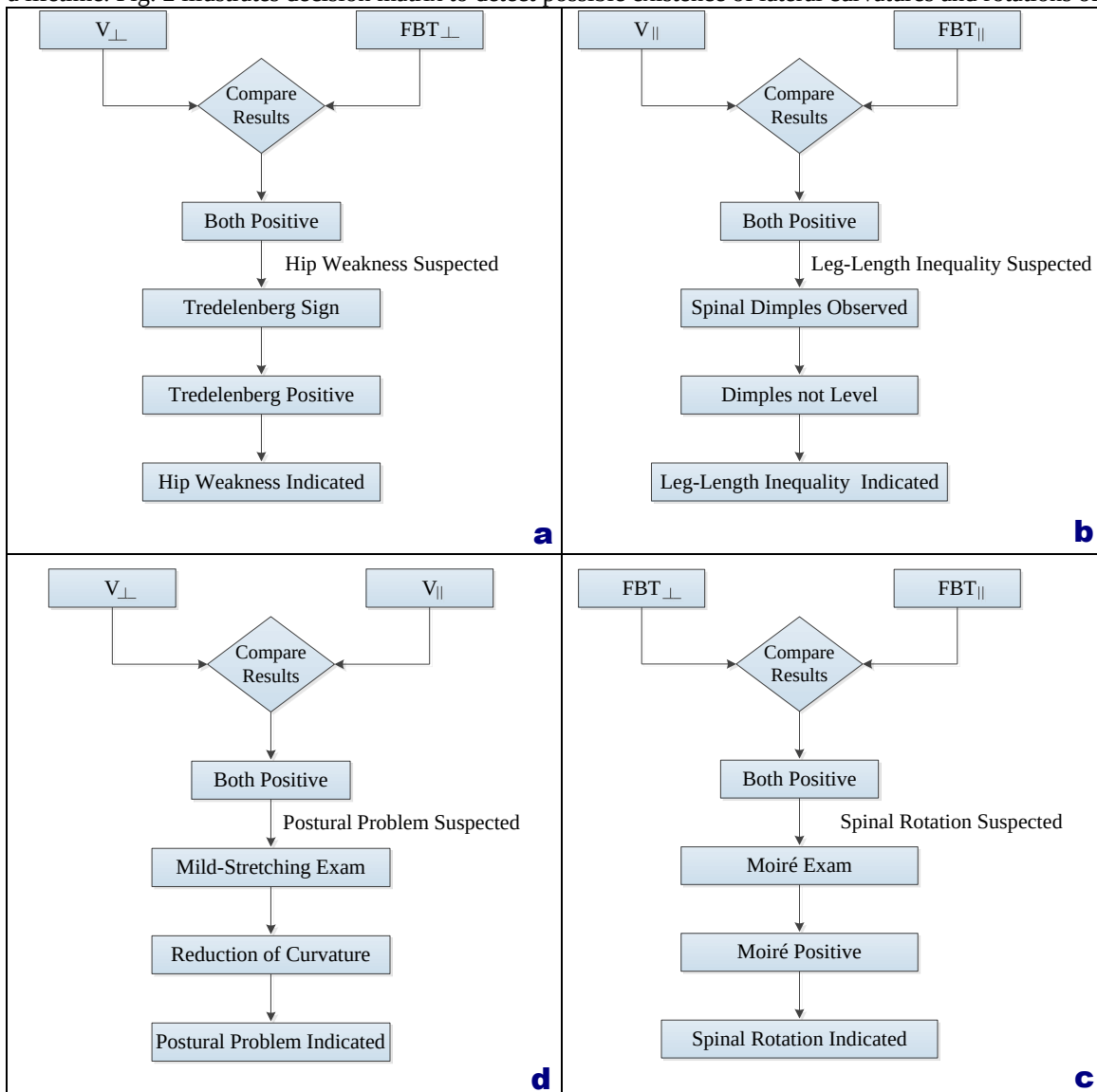


Fig. 2. Flowchart for decision matrix, which may be used to plan for efficient detection and effective treatment of scoliosis



of the spinal column in the context of differential-spinal-function testing. The decision is made in two levels. In the first level 2 tests were conducted and the results compared to suspect a possible condition. Then a third test was administered to indicate that condition. 4 tests were selected for this purpose: visual (standing), visual (sitting), forward bending (standing) and forward bending (sitting) — postural problem suspected through positive visual examinations (standing and sitting), indicated through positive mild-stretching test (if the deformity was not corrected after mild stretching, it was indicative of lateral curvatures); leg-length inequality suspected through positive visual and forward-bending tests (both standing), indicated through uneven spinal dimples; hip weakness suspected through positive visual and forward-bending tests (both sitting), indicated through positive Tredelenburg sign; spinal rotation suspected through positive forward-bending tests (standing and sitting), indicated through either positive moiré or positive forward-bending tests (back and front views) on opposite sides.

Question 4a

By setting up equation of simple pendulum (and drawing a labeled diagram) show that stable equilibrium occurs when the pendulum bob is closest to earth surface and unstable equilibrium occurs when it is farthest from the earth surface.

Solution 4a

Consider a simple pendulum, which is hung from a fixed support. Let ℓ be the length of pendulum, which consists of a bob of mass m — the length ℓ includes length of string, length of hook used to attach bob to the string and radius of the bob. When this pendulum bob is raised to an angle, θ (measured from the vertical), the potential energy, $U(\theta)$, may be expressed as (Fig. 3):

$$(1) \quad U(\theta) = mgh = mg\ell(1 - \cos\theta)$$

where g is the acceleration due to gravity. To obtain extrema, we differentiate the above and equate to zero

$$(2) \quad \left. \frac{dU(\theta)}{d\theta} \right|_{\theta=\theta_0} = mg\ell \sin\theta \Big|_{\theta=\theta_0} = mg\ell \sin\theta_0 = 0$$

Solving for θ_0 , we get two values, i. e., 0° and 180° . In order to decide, which one corresponds to minimum or maximum energy, we check sign of the second derivative of potential energy at $\theta_0 = 0^\circ, 180^\circ$ corresponding to normal as well as inverted pendulum:

$$(3) \quad \left. \frac{d^2U(\theta)}{d\theta^2} \right|_{\theta=\theta_0} = \frac{d}{d\theta} \left(\frac{dU(\theta)}{d\theta} \right) \Big|_{\theta=\theta_0} = \frac{d}{d\theta} (mg\ell \sin\theta) \Big|_{\theta=\theta_0} = mg\ell \cos\theta \Big|_{\theta=\theta_0} = mg\ell \cos\theta_0$$

At $\theta_0 = 0^\circ$, value of the second derivative is $mg\ell$. This value is positive. Hence, the potential energy is minimum at this point, corresponding to ‘stable equilibrium’. At $\theta_0 = 180^\circ$, value of the second derivative is $-mg\ell$. This value is negative. Hence, the potential energy is maximum at this point, corresponding to ‘unstable equilibrium’.

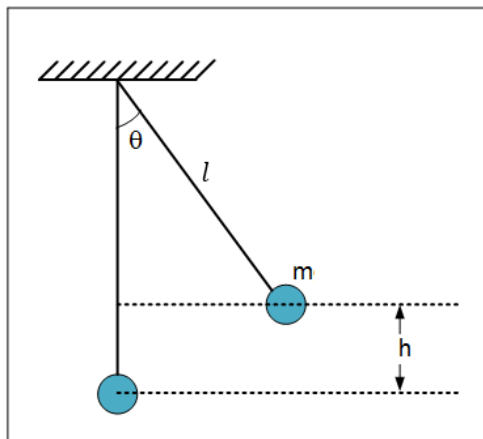


Fig. 3. Oscillation of pendulum: Calculation of potential energy



Question 4b

Briefly describe the role of energy-channelization in producing spastic gait.

Solution 4b

Energy channelization was introduced in 2014. For stunted and wasted children, UN (under-nutrition) might be the main concern. Tallness plus obesity, which amplifies tissue-synthesis rate and storage in body, may be termed as ON (over-nutrition). The remaining two possibilities, tallness associated with wasting and stunting associated with obesity, might show up due to energy-channelization (EC) problem in body, the first one termed as EC I and the second one as EC II. These conditions are caused by a large amount of micronutrients, all flowing through one channel of absorption. Stunting with obesity may be caused by storage of most micronutrients; whereas tallness with wasting, could result from micronutrients, mostly, involved in tissue synthesis. These nutritional statuses were determined using a child's Growth-and-Obesity Profile and become part of each child's Growth-and-Obesity Roadmap 2.0.

AM (acute malnutrition) is the limiting case of UN, when sum of scaled height and mass percentiles is less than 6. Such a condition demands immediate medical attention. It may be due to a chronic disease (cardiac, respiratory, renal or malabsorption). Puberty-Induced Energy-Channelization, also, termed as EC III, was introduced as limiting case of ON. This is characterized by height-gain leveling off combined with gain in mass as well as below-waist fat. On October 1 of this year, these 6 categories have been expanded into 10 categories in the context of Growth-and-Obesity Vector-Roadmap 2.1.

EC I (tallness + wasting) and EC II (stunting + obesity) may contribute to asymmetric distribution of mass about the sagittal plane causing generation of force and, hence, a torque about body's center of gravity during walking. Such a force may be the cause of abnormal gait in EC children. An unbalanced force about the sagittal plane may produce a torque, which may cause an EC child to exhibit spastic gait.

In a study by Kamal, Rajput and Ansari, published in 2016, data were collected and analyzed by computing Growth-and-Obesity Profile of each student to classify nutritional status. The probability of spastic gait in each gender was computed using the following equation:

$$\text{Spastic-Gait Probability} = 100 \frac{N_{\text{spastic}}}{N_{\text{spastic}} + N_{\text{normal}}}$$

where $N_{\text{spastic}}(N_{\text{normal}})$ = Number of children with spastic (normal) gait. The results indicate higher probability of spastic gait in EC children — 21.43% in EC I and 32% in EC II. Girls seem to have a higher risk of exhibiting spastic gait, may be due to their weak skeleton. The higher probability of spastic gait in EC children may be explained by considering that energy-channelization creates imbalance between the amount of mass a child has as compared to the youngster's height. Asymmetric distribution of mass about the sagittal plane may create a force and the resulting torque could be attributed to spastic gait in such children.



Question 4c

Why is it mandatory that the child should be in dress code 0/0.5 for a proper evaluation of gait?

Solution 4c

- i) To properly evaluate spastic gait (swaying of hips) child should be in underwear I could not check this aspect in the checkups on our children) — spastic gait is both in sagittal and transverse planes
- ii) To check limp, one may note that spinal dimples should perform simple-harmonic motion in the strap-down frontal plane (the frontal plane co-moving with the child) in the absence of limp — this should be additional to observation of gross movement in the frontal plane + sagittal plane
- iii) To better check, neck alignment with body
- iv) To rule out knee knocking

Question 5

Write short note on any one of the following:

- (a) Fitness of an athlete
- (b) Degree of correction of spinal deformity

Solution 5

(a) *Fitness of an athlete*

Fitness is defined as “the condition of being physically fit and healthy”. Alternately, it could be visualized as the quality of being suitable to fulfill a particular athletic activity. It could be classified into two categories — health-related fitness and skill-related fitness. Health-related fitness consists of 5 components of physical fitness, which correspond to good health, as opposed to athletic ability. These are cardiovascular endurance, flexibility, muscular strength, muscular endurance and body composition. Skill-related fitness consists of 6 components, which are important in performing the more technical aspect of sport. These are speed, reaction time, agility, balance, coordination and power. In order to get the most out of training the athlete must follow basic training principles, which are overload (putting bodies under more stress than normal to make adaptive changes), specificity (training imparted specific to sport or activity), reversibility (athletes need to keep up the training in order not to lose the benefits achieved) and variance (varying the training activities). The main purposes of fitness testing may be summarized as:

- i) To determine baseline level of fitness
- ii) To motivate
- iii) To set goals
- iv) To adapt an existing training program according to the specific needs of an athlete
- v) To gauge improvement
- vi) To compare with national averages
- vii) To pinpoint weakness or strengths
- viii) To re-assess training needs

There are limitations to fitness testing:

- i) Tests not sport specific
- ii) Uncompetitive nature of some tests (do not replicate competitive conditions)
- iii) Reliability issues with tests (tests used often lack validity⁸; do not replicate movements of activity)
- iv) Performer lacking motivation
- v) Reproducibility (not carried out under optimal conditions, *e. g.*, same time of day, same facilities) issues with tests

Issues to be considered before designing a fitness-testing program:

- i) Equipment needed to conduct a fair test
- ii) Rules and procedures to be followed
- iii) Selection of measurements
- iv) Scoring the tests
- v) Conclusion to be drawn from results

Some of the fitness tests employed are resting heart rate, heart rate after exercise, tests evaluating muscular endurance, strength, agility, speed, balance, flexibility, coordination and power.



(b) *Degree of correction of spinal deformity*

‘Degree of Correction of Spinal Deformity’ is a number between 0 and 100%, defined in terms of curvatures of each vertebra of the spinal column, in the attention position and mild-stretching position. It was introduced in 1982 by Kamal to classify spinal deformity as severe, intermediate or mild. Table 1 shows different classifications and recommended treatment:

Table 1. Severity of ‘Degree of Correction of Spinal Deformity’ (D) and recommended treatment

<i>Range of D</i>	<i>Severity Level</i>	<i>Recommended Treatment</i>
$0 \leq D < \frac{100}{3}\%$	Severe	Surgery
$\frac{100}{3}\% \leq D < \frac{200}{3}\%$	Intermediate	To be decided by orthopedic surgeon [£]
$\frac{200}{3}\% \leq D \leq 100\%$	Mild	Combination of exercises and brace

[£]The decision should depend on the location and the progression of scoliotic curve as well as the numerical value of D — how close the value is to $100/3$ (inclination towards surgical treatment) or $200/3$ (inclination towards a combination of exercises and brace)

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