



کراچی یونیورسٹی  
*University of Karachi*

**PROFORMA FOR THE SUBMISSION OF RESEARCH PROPOSAL TO  
FACULTY OF SCIENCE**

Math/Dean.Grant/2005-7471  
December 31, 2005

**Name and Address of the Principal Investigator:** Professor Dr. Syed Arif Kamal, Professor, Department of Mathematics, University of Karachi, Karachi 75270

**Title of the Project:** The Lambert Scheme for Steering a Satellite-Launch Vehicle

**Main Field of Study:** Astrodynamics

**Nature of the Project:** Applied  
(i. e., Basic OR Applied)

**Funds Requested:** Rs 77 000/= (Rupees Seventy Seven Thousand Only)

**Signatures of the Principal Investigator:** \_\_\_\_\_

**Endorsement by the Chairman/Director:** \_\_\_\_\_

Tel: 9926 1300/2380

Prof. Dr. S. Arif Kamal

Signatures \_\_\_\_\_

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**Title** **The Lambert Scheme for Steering a Satellite-Launch Vehicle (SLV)**

**Submitted to** Dean, Faculty of Science, University of Karachi

**Date of Submission** Saturday, December 31, 2005

**Diary Number** Math/Dean.Grant/2005-7471

**Principal Investigator** **Professor Dr Syed Arif Kamal**  
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Professor and Chairman, Department of Mathematics  
Project Director, the NGDS Pilot Project  
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**Introduction** **a) Background of the Problem:**  
A problem famous in astrodynamics, called “the Lambert Problem”, is based on the Lambert theorem. According to this theorem the orbital-transfer time depends only upon the semi-major axis, the sum of the distances of the initial and the final points of the arc from the center of force as well as length of the line segment joining these points. Based on this theorem a problem called the Lambert problem is formulated. This problem deals with determination of an orbit having a specified flight-time and connecting the two position vectors.

**b) Significance of the Problem**  
The Lambert problem is fundamental in developing guidance schemes for SLVs.

**c) Need for Solution:**  
If the problem is solved Pakistan would be able to develop and deploy its own satellite-launching business. Being near to equator, Pakistan is, ideal for such a venture.

**Statement of the Problem** Battin has set up the Lambert problem involving computation of a single hypergeometric function. Since transfer time (time-of-flight) computation is done on-board, it is desirable to use an algorithm involving as few computation steps as possible. The use of polynomials instead of actual expression and reduction of the number of degrees of freedom contribute towards the same goal. The formulation needs to be modified for practical implementation in SLV.

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**Proposed Solution**

Of the many techniques existing for solving this two-body, two-point, time-constrained orbital boundary-value problem, Gauss' and Lagrange's methods shall be combined to obtain an elegant algorithm based on Battin's work. This algorithm shall include detection of cross-range error.

**Methodology**

**a) Modeling of Data**

An elegant Lambert algorithm, presented by Battin, is to be scrutinized and omissions/oversights in his calculations pointed out. Battin's formulation, which highlights the main principles involved, is to be developed and expanded to present a set of formulae suitable for coding in the assembly language to be used as a practical scheme outside the atmosphere for steering the satellite-launch vehicle (SLV). These formulae are to be used to compute the velocity and the flight-path angle required at any intermediate time to be compared with the initial velocity and flight-path angle of the spacecraft. A spacecraft cannot reach the desired location if cross-range error is present. Battin's original work does not address this issue. A mathematical formulation is required to detect cross-range error.

**b) Simulation of Data**

Algorithms are to be developed and tested, which indicate cross-range error and suggest control action to correct it. In order to correct cross-range error velocity vector should be perpendicular to normal to the desired trajectory (*i. e.*, the velocity must lie, entirely, in the desired trajectory plane).

**c) Analysis of Results**

To verify the Lambert formulation a procedure is outlined, in which the radial coordinates,  $r_1$ , and,  $r_2$ , as well as the transfer angle are to be computed from the known orbital parameters. Further, the transfer time is to be calculated, directly, from the Kepler equation without using hypergeometric series. These quantities are to be compared with the corresponding quantities used in the Lambert scheme.

**Resources**

a) Necessary Facilities Required

- i) Powerful Computing
- ii) Scanning
- iii) Image Processing

b) Facilities Available

- i) Wordprocessing (secure system without internet connection)
- ii) Printing
- iii) e-mail
- iv) Internet

**Schedules**

**a) Start Date**

January 1, 2006

**b) Finish Date**

December 31, 2006

**c) Estimated Number of Hours to be Spent**

20 hours (may be modified at the time of submission of Feasibility Report)

**d) Submission of Reports**

Final Report (upon completion of project)

Audited Statement of Accounts (upon completion of project)

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**Estimated Budget**

<i>Description</i>	<i>Amount (in Rupees)</i>
Computer Upgrading and Accessories	Rs 20000.00
Image-Recording System	Rs 20000.00
Professional Society Membership	Rs 7000.00
Books/Journals/Literature	Rs 20000.00
Conference Registration/Stationery	Rs 10000.00
Total	Rs 77000.00
(Rupees Seventy Seven Thousand Only)0000	

**Benefits/Usefulness**

- i) To develop the skills of trajectory design and satellite-launching technology from our own indigenous resources
- ii) To provide assistance and technical expertise to R&D organizations, such as, SUPARCO, PMO, NDC
- iii) To give Professional Short Development Courses at Institute of Space Technology, Islamabad, College of Aeronautical Engineering, PAF Academy, Risalpur and Air Weapons Complex, Kamra
- iv) To be able to prepare students to do their MPhil/PhD in astrodynamics
- v) To be able to prepare students for graduate courses and research at ISPA

**References**

- R. H. Battin, "Lambert's problem revisited", *AIAA J.*, **15 (5)**, 707-713 (1977)
- R. H. Battin, *An Introduction to the Mathematics and (the) Methods of Astrodynamics*, AIAA Education Series, New York, United States, 1987, pp. 276-342, review: <https://www.ngds-ku.org/Papers/Battin.pdf>
- R. H. Battin & R. M. Vaughan, "An elegant Lambert algorithm", *J. Guid. Control*, **7 (6)**, 662-670 (1984)
- R. Deusch, *Orbital Dynamics of Space Vehicles*, Prentice Hall, Englewood Cliffs, New Jersey, United States, 1963, pp. 20-22
- H. Goldstein, *Classical Mechanics*, 2<sup>nd</sup> Ed., Addison Wesley, Reading, Massachusetts, United States, 1981
- S. A. Kamal, "Incompleteness of cross-product steering and a mathematical formulation of extended-cross-product steering", *Proceedings of the First International Bhurban Conference on Applied Sciences and Technologies*, Bhurban, KP, Pakistan, June 10-15, 2002, pp. 167-177, full text: <https://www.ngds-ku.org/Papers/C56.pdf>
- S. A. Kamal, "Dot-product steering: a new control law for spacecrafts and satellites", *Proceedings of the First International Bhurban Conference on Applied Sciences and Technologies*, Bhurban, KP, Pakistan, June 10-15, 2002, pp. 178-184, full text: <https://www.ngds-ku.org/Papers/C55.pdf>
- S. A. Kamal, "Ellipse-orientation steering: a control law for spacecrafts and satellite-launch vehicles (SLV)", *Space Science and Challenges of the Twenty-First Century*, ISPA-SUPARCO Collaborative Seminar (in connection with World Space Week), University of Karachi, Karachi, Pakistan, 2005, abstract: <https://www.ngds-ku.org/Presentations/Ellipse.pdf>

End of Document

Web address of this document: <https://www.ngds-ku.org/DFS/DFS2005.pdf>

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