



UNIVERSITY OF KARACHI
Department of Chemistry (Evening Program)

MATH 401(S) • Mechanics and Geometry I

Course Supervisor: Professor Dr. Syed Arif Kamal

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Time Allowed: 3 hours • Maximum Marks: 100 • Date: Saturday, December 10, 2007

Student's Name _____ (in CAPITAL LETTERS using a "marker") • Paper Format A

Attempt **Question 1** and 5 other questions. Each part of **Question 1** is of 4 marks & of **Questions 2-7** is of 8 marks. Note down the time spent on solving each part of question and time spent on revision by making the following "Time Chart" on the answerbook front page (2 marks). Time spent on revision must be at least 10% of the total time

Question No.	1a	1b	1c	1d	2a	2b	3a	3b	4a	4b	5a	5b	6a	6b	7a	7b	Revision
Time (minutes)																	

a) DO NOT TURN PAPER AROUND unless the invigilator says: *Start* now

b) This is a closed book examination. Deposit all BOOKS, NOTES, MOBILE PHONES (switched off), DIGITAL DIARIES and LAPTOPS in the designated area. Remove everything from your desk except markers, pens, pencils, stapler and calculator.

c) If you want to use a calculator, **it must bear a "sticker"** displaying your NAME and your SEAT NUMBER large enough so that it is visible from a distance of 5 meters. **Absolutely, NO sharing of calculators.**

d) Use your own material. **Nothing can be borrowed from or given to** a friend.

e) The papers may be of different formats. Therefore, work on your own **without consulting anyone** (We have a record of your seating arrangement).

f) Write your "NAME" on all pages of **your question paper** (5 marks shall be deducted for failing to comply) [NOTHING ELSE SHOULD BE WRITTEN ON THE QUESTION PAPER] and "**PAPER FORMAT**" on the front page of **your answerbook** (the upper right-hand corner) and **the Yellow Sheet** using a "**marker**". Start your work from **Page 2** of your answerbook. The only thing that could be written on the front page is the "**Time Chart**" (see above) and the "**Honor Statement**" (see below).

g) The following statement must be copied on the front page of your answerbook and signed (2 marks): "*My signatures below testify that I am the person whose name and photograph appear on the Admit Card. Upon my honor, I declare that the following work is my own, completed without giving or receiving unacknowledged help, without copying, or the use of any unfair means.*" Signatures _____

h) This paper contains TWO PAGES (This page and the back page). On invigilator's signal (*Start now*) turn paper around, check if you have the back

page printed correctly. Last line of the second page is: <END>. Start working on the paper immediately.

i) Put your pens down and your papers turned (so that this page is facing you) and the FRONT PAGE of your answerbook should be facing you as soon as you hear "**ALL PENS DOWN**". Failure to do as directed shall result in "deduction of 5 marks" from your score.

j) If you use extra copies, it is "your responsibility" to write YOUR NAME, COPY NUMBER and all OTHER INFORMATION on each copy used. All the extra copies must be stapled with the main copy before turning in your paper (you may wish to bring in a stapler with you for this purpose).

k) If you have a question of "Fill in the blanks" in your paper you must write the complete sentence with the filled word underlined.

l) Nobody is allowed to leave the examination hall, **for whatsoever reason**, once the examination has started. Bring your own DRINKING WATER.

m) Students are not allowed to LEAVE THEIR SEATS or STAND UP during the examination. If you have a query, "raise your hand" and someone will help you.

n) All work, including rough work, must be on the official answerbook. No extra sheet may be used.

o) Students are *not* allowed to use RED anywhere. All work (except figures) must be in pen or ballpoint.

p) The result shall be displayed on my homepage on **Saturday, January 12, 2008 at 0900h**. DO NOT contact the Course Supervisor. Students are not permitted to see the answerbooks.

q) Anyone found cheating in the examination should be facing disciplinary action, which may result in **EXPLUSION** or **SUSPENSION** for 2 or more years. **Absolutely, no conversation among students. DON'T TURN THE PAPER, YET. Wait for "signal" from the invigilator.**

Student's Name _____ (in CAPITAL LETTERS using a "marker")

- 1-a) The necessary and the sufficient conditions for three vectors to be _____ (perpendicular to each other, coplanar, linearly independent) is that their scalar-triple product vanishes. Write down the scalar-triple product of vectors $\hat{A}$, $\hat{B}$ and $\hat{C}$ in terms of ϵ_{ijk} notation.
- 1-b) Describe (using equations) coordinate surfaces and coordinate curves for the cylindrical-coordinate mesh.
- 1-c) Write down different forms of equations of straight line (slope,x intercept; 2 points; 1 point, slope; 1 point, x intercept)
- 1-d) In the following problems from physical chemistry, which coordinate system should be used to solve the problem with least effort (cartesian, spherical polar, plane polar, cylindrical)
- (i) Water is flowing through a capillary tube of uniform diameter. You have to calculate velocities of streamlines at various distances from the central axis.
- (ii) An electron is moving in a circular orbit around a proton. You have to calculate the energy of electron in the first Bohr orbit.
- 2-a) Represent the vector $\hat{A} = 2\hat{y} - \hat{z} + 3\hat{k}$ in cylindrical coordinates and determine A_r , A_θ and A_z
- 2-b) Using the spherical-polar-coordinate formulation show that the surface area of a sphere of radius a comes out to $4\pi a^2$ and volume $\frac{4}{3}\pi a^3$.
- 3-a) Show that any arbitrary vector $\hat{A}$ could be resolved into 2 components along and perpendicular to a unit vector $\hat{e}$ in some fixed direction as: $\hat{A} = \hat{e}(\hat{A} \cdot \hat{e}) + \hat{e} \times (\hat{A} \times \hat{e})$
- 3-b) Prove the following relations: (i) $d_{ij} \epsilon_{ijk} = 0$; (ii) $\epsilon_{ijk} \epsilon_{ijk} = 6$
- 4-a) Determine a unit vector perpendicular to the plane formed by $\hat{A} = 2\hat{i} + a\hat{j} - \hat{k}$ and $\hat{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ (Hint: If $\hat{D}$ is perpendicular to this plane $\hat{D} \cdot \hat{A} = \hat{D} \cdot \hat{B} = 0$ OR evaluate $\hat{A} \times \hat{B}$)
- 4-b) Find the projection of the vector $\hat{A} = \hat{i} - 2\hat{j} + \hat{k}$ on the vector $\hat{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$.
- 5-a) Find the equation of a straight line, which passes through 2 given points A and B having position vectors $\hat{a}$ and $\hat{b}$ with respect to an origin O.
- 5-b) Using the result of angle between 2 lines, prove that the lines $y = m_1x + c$ and $y = m_2x + c$ are perpendicular if $m_1m_2 = -1$
- 6-a) Using ϵ_{ijk} notation prove: $(\hat{A} \times \hat{B}) \times \hat{C} = \hat{B}(\hat{C} \cdot \hat{A}) - \hat{A}(\hat{C} \cdot \hat{B})$
- 6-b) Express torque, $\hat{N}$, angular momentum, $\hat{L}$, and centrifugal force $\hat{F}_{centrifugal}$, each first as a vector-triple product, and then, as a difference of 2 vectors using BAC-CAB rule.
- 7-a) Determine the value of a , so that $\hat{A} = 2\hat{i} + a\hat{j} - \hat{k}$ and $\hat{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ are perpendicular.
- 7-b) Convert the following equations in terms of equations of straight line expressed in terms of 1 point and direction ratios:

$$4x + 3y + z = 0; 5x + 9y - z - 1 = 0$$