UNIVERSITY OF KARACHI



Department of Chemistry (Evening Program) MATH 401(S): Mechanics and Geometry I

Course Supervisor: Professor Dr. Syed Arif Kamal

Homepage: http://ngds.ku.edu.pk/kamal • *e-mail*: kamal(at the rate of)ngds-ku.org *Time Allowed*: 3 hours • *Maximum Marks*: 100 • *Date*: Saturday, December 23, 2006

| Student's Name | (in CAPITAL LETTERS using a "marker") | • Pa | mar Format A |
|------------------|---|-------|----------------------|
| Situati s Ivante | (III CAI ITAL LETTERS using a marker) | • 1 u | ipei Formai <u>A</u> |

Attempt Question 1 and 5 other questions. Each part of Question 1 is of 4 marks & of Questions 2-7 is of 8 marks. Note down the time spent on solving each part of question and time spent on revision by making the following "Time Chart" on the answerbook front page (2 marks). Time spent on revision must be at least 10% of the total time

| Ī | Question No. | 1a | 1b | 1c | 1d | 2a | 2b | 3a | 3b | 4a | 4b | 5a | 5b | 6a | 6b | 7a | 7b | Revision |
|---|----------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----------|
| | Time (minutes) | | | | | | | | | | | | | | | | | |

- a) DO NOT TURN PAPER AROUND unless the invigilator says: Start now
- b) This is a closed book examination. Deposit all BOOKS, NOTES, MOBILE PHONES (switched off), DIGITAL DIARIES and LAPTOPS in the designated area. Remove everything from your desk except markers, pens, pencils, stapler and calculator.
- c) If you want to use a calculator, it must bear a "sticker" displaying your NAME and your SEAT NUMBER large enough so that it is visible from a distance of 5 meters. Absolutely, NO sharing of calculators.
- d) Use your own material. **Nothing can be borrowed from or given to** a friend.
- e) The papers may be of different formats. Therefore, work on your own without consulting anyone (We have a record of your seating arrangement).
- f) Write your "NAME" on all pages of your question paper (5 marks shall be deducted for failing to comply) [NOTHING ELSE SHOULD BE WRITTEN ON THE QUESTION PAPER] and "PAPER FORMAT" on the front page of your answerbook (the upper right-hand corner) and the Yellow Sheet using a "marker". Start your work from Page 2 of your answerbook. The only thing that could be written on the front page is the "Time Chart" (see above) and the "Honor Statement" (see below).
- g) The following statement must be copied on the front page of your answerbook and signed (2 marks): "My signatures below testify that I am the person whose name and photograph appear on the Admit Card. Upon my honor, I declare that the following work is my own, completed without giving or receiving unacknowledged help, without copying, or the use of any unfair means." Signatures
- h) This paper contains TWO PAGES (This page and the back page). On invigilator's signal (*Start now*) turn paper around, check if you have the back

- page printed correctly. Last line of the second page is: **END**>. Start working on the paper immediately.
- i) Put your pens down and your papers turned (so that this page is facing you) and the FRONT PAGE of your answerbook should be facing you as soon as you hear "ALL PENS DOWN". Failure to do as directed shall result in "deduction of 5 marks" from your score.
- *j)* If you use extra copies, it is "your responsibility" to write YOUR NAME, COPY NUMBER and all OTHER INFORMATION on each copy used. All the extra copies must be stapled with the main copy before turning in your paper (you may wish to bring in a stapler with you for this purpose).
- *k)* If you have a question of "Fill in the blanks" in your paper you must write the complete sentence with the filled word underlined.
- *l)* Nobody is allowed to leave the examination hall, **for whatsoever reason**, once the examination has started. Bring your own DRINKING WATER.
- *m)* Students are not allowed to LEAVE THEIR SEATS or STAND UP during the examination. If you have a query, "raise your hand" and someone will help you.
- *n)* All work, including rough work, must be on the official answerbook. No extra sheet may be used.
- *o)* Students are *not* allowed to use RED anywhere. All work (except figures) must be in pen or ballpoint.
- p) The result shall be displayed on my homepage on **Saturday**, **January 6**, **2007** at **0900h**. DO NOT contact the Course Supervisor. Students are not permitted to see the answerbooks.
- q) Anyone found cheating in the examination should be facing disciplinary action, which may result in **EXPLUSION** or **SUSPENSION** for 2 or more years. **Absolutely, no conversation among students. DON'T TURN THE PAPER, YET.** Wait for "signal" from the invigilator.

Student's Name_____ (in CAPITAL LETTERS using a "marker")

- 1-a) List 5 types of coördinate transformations discussed in class.
- 1-b) What is value of the angle between a unit vector and the corresponding coördinate curve? In case of electric intensity, which one corresponds to equipotential surface and which one corresponds to line of force?
- 1-c) Write down different forms of equations of straight line (slope, y intercept; 2 points; 1 point, slope; 1 point, y intercept)
- 1-*d*) Fill in the blanks in the following table:

| Condition | Number of Solutions | Nature of Graph | Outcome (in case a |
|---------------------------------|---------------------|--------------------|------------------------|
| | | | solution is attempted) |
| | | Intersecting Lines | Unique Solution |
| $a/a' = b/b'$, but $\neq c/c'$ | 0 | | |
| | | Identical Lines | Identity |

- 2-a) In 30 words or less comment on the statement: Statistics is "present", but mathematics is "future".
- 2-b) List 2 properties of mathematics *NOT* possessed by other branches. Give an example of a problem, which could not be handled by the most sophisticated supercomputer available, but solved by techniques of mathematics.
- 3-a) For any two vectors \mathbf{A} and \mathbf{B} show that (μ is the angle between \mathbf{A} and \mathbf{B}):

$$|A + B|^2 = |A|^2 + |B|^2 + 2|A||B|\cos \mu$$

3-*b*) Prove the following relations:

(i)
$$d_{ii}A_{i} = A_{i}$$
; (ii) $\in_{iik} \in_{lmk} = d_{il}d_{im} - d_{im}d_{il}$

4) Prove the following using \in_{ijk} notation as well as physical arguments and give one application of each:

(a)
$$\nabla \times (\nabla f) = 0$$
; (b) $\nabla \cdot \nabla \times A = 0$

- 5-a) What is a soleniodal vector field? Can sources or sinks appear in a region where the vector field is solenoidal? How can you explain $\operatorname{div} \mathbf{B} = 0$ to a Class V student?
- 5-b) Using the result of angle between 2 lines, prove that the lines $y = m_1 x + c$ and $y = m_2 x + c$ are perpendicular if $m_1 m_2 = -1$
- 6-a) Using \hat{I}_{ijk} notation prove: $A \times (B \times C) = B(A.C) C(A.B)$
- 6-b) Express the vector field V(x, y) = x i + y j in plane-polar coördinates. Compute the divergence and the curl. Is it a source field, a sink field or a rotation field (clockwise/counter-clockwise)?
- 7-a) Compute the scale factors for the spherical-polar-coördinate mesh.
- 7-b) Convert the following equations in terms of equations of straight line expressed in terms of 1 point and direction ratios:

$$3x + 4y + z = 0$$
; $9x + 5y - z - 1 = 0$

Web address of this document: http://www.ngds-ku.org/Chemistry/Exams401-2/401-06_Paper.pdf