CURRICULUM

OF

MATHEMATICS

Revised 2008

HIGHER EDUCATION COMMISSION
Islamabad

CURRICULUM DIVISION, HEC

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Composed by Convener, NCRC (Mathematics)

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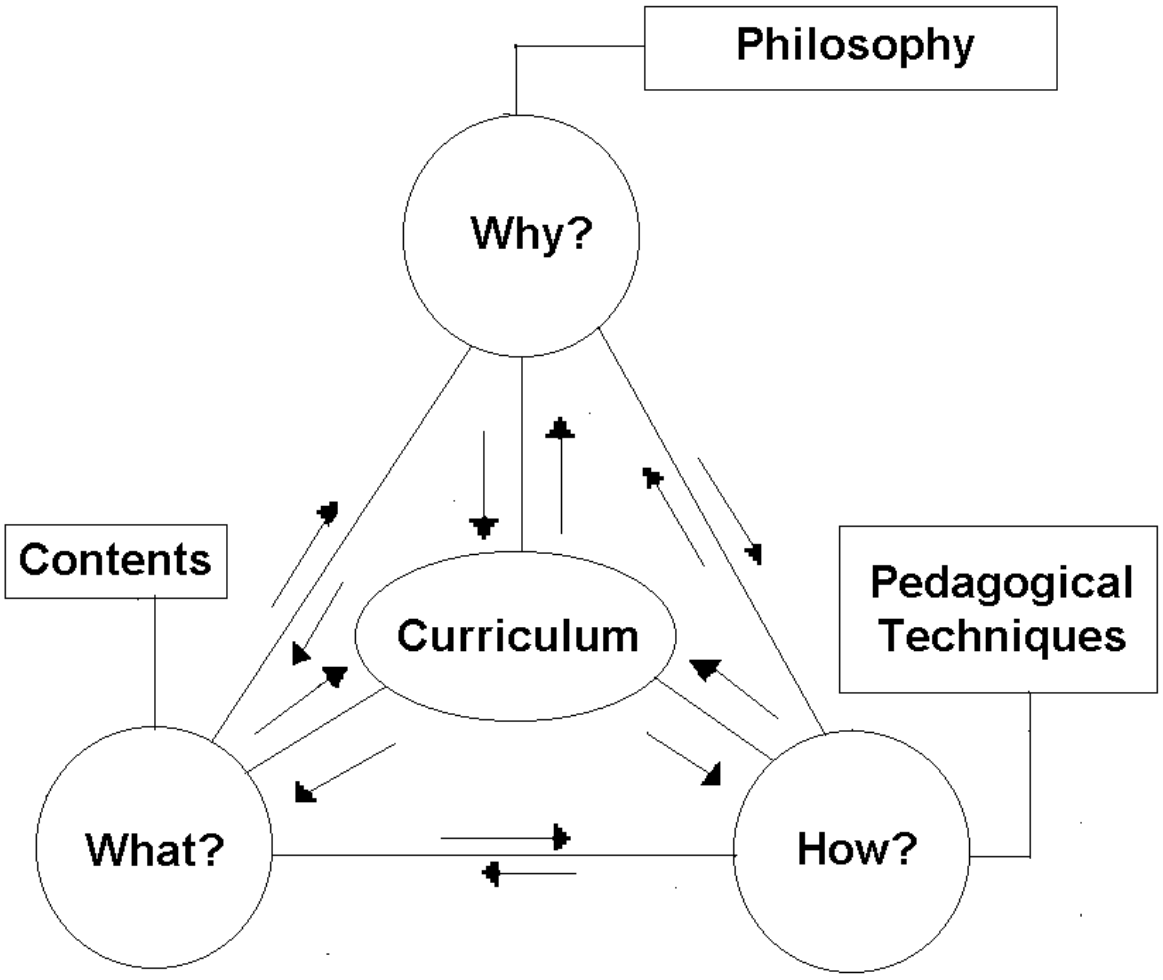

Fig. 1. Philosophy, contents and pedagogical techniques — their relationship with curriculum development

PREFACE

Curriculum of a subject is "the planned and the organized learning experiences, formulated through the systematic reconstruction of knowledge and experience, under the auspices of the institution, for the learner's continuous and willful growth in personal-social competence" (Daniel Tanner & Laurel N. Tanner, *Curriculum Development: Theory into Practice*). By looking at the curriculum, one can judge the state of intellectual development as well as the state of progress of a nation. The world has turned into a global village. It is, therefore, imperative that curriculum development must be a "process" not an "event".

In exercise of the powers conferred by Sub-Section (1) of Section 3 of the Federal Supervision of Curricula Textbooks and Maintenance of Standards of Education Act 1976, the Federal Government vide notification number D773/76-JEA (Cur.), dated December 4, 1976, appointed University Grants Commission (now, Higher Education Commission) as the Competent Authority to look after the curriculum-revision work beyond class XII at bachelor's level and onwards for all degrees, certificates and diplomas awarded by degree colleges, universities and other institutions of higher education.

In pursuance of the above decisions and directives, Higher Education Commission (HEC) is, continually, performing curriculum revision in collaboration with universities. According to decision of special meeting of Vice-Chancellors' Committee, curriculum of a subject must be reviewed after every 3 years. For this purpose, various committees are constituted at the national level comprising of senior teachers nominated by universities. Teachers from local degree colleges and experts from user organizations, where required, are also included in these committees. The National Curriculum Revision Committee (NCRC) for Mathematics in its special meeting held on February 15 & 16, 2008 at the HEC Regional Center, Lahore revised the curriculum after due consideration of the comments and the suggestions received from universities and colleges, where the subject under consideration is taught, in particular, modifying the BS scheme of studies according to the standard template agreed upon during the meeting of conveners of various NCRCs on April 30, 2007. The final draft prepared by NCRC, duly approved by the Competent Authority, is being circulated for implementation by architectural institutions.

PROF. DR. RIAZ-UL-HAQ TARIQ
Member, HEC

July 2008

CURRICULUM DEVELOPMENT

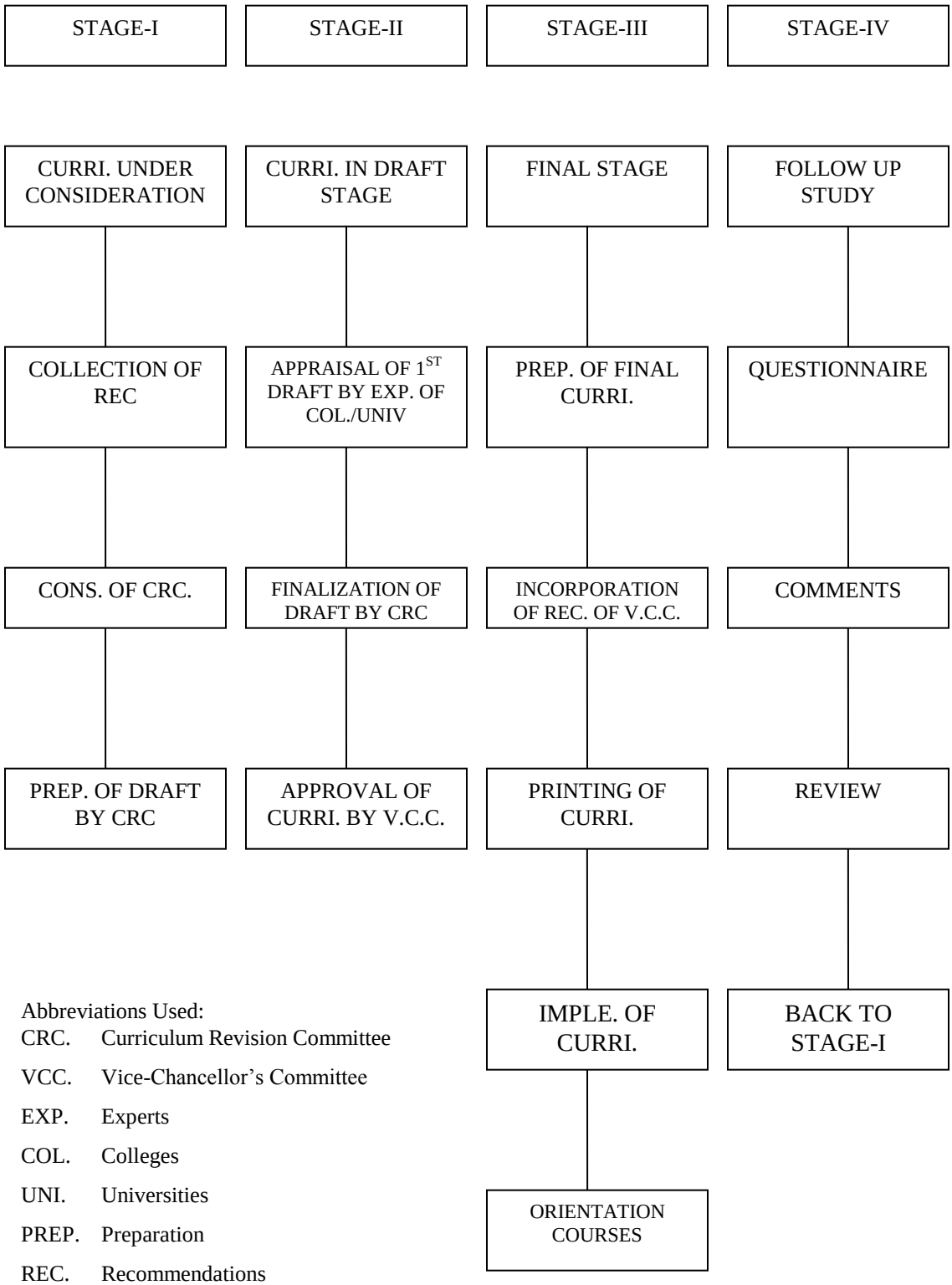

Fig. 2. Curriculum-development exercise

MINUTES OF THE NATIONAL CURRICULUM REVISION COMMITTEE (SPECIAL) FOR MATHEAMTICS

A final-special meeting of National Curriculum Revision Committee (NCRC) for Mathematics was held at HEC Regional Center, Lahore on February 15 & 16, 2008, which was a follow-up of preliminary meeting held at C&T Building, HEC, H-8, Islamabad on October 26 & 27, 2007. Earlier, curricula for BS, MS and PhD were prepared in the NCRC meetings held at HEC Regional Center, Lahore during January 31 – February 2, 2005 and June 9-11, 2005. The following experts attended the NCRC special meetings):

Convener

1. **Professor Dr. Syed Arif KAMAL**
   Professor
   Dept. of Mathematics
   University of Karachi
   Karachi 75270.
   Homepage: http://ngds-ku.org/kamal
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Members

(listed in alphabetical order according to last name)

P: attended preliminary meeting only
F: attended final meeting only

2. **Professor Dr. Ahsanullah BALOCH**
   Professor
   Dept. of Basic Sciences and Related Studies, Mehran University of Engineering and Technology, Jamshoro, Sindh.
3. **Professor Dr. Barbu BERCEANU (P)**
   Professor
   The Abdul Salam School of Mathematical Sciences (SMS), Lahore.
4. **Professor Dr. Anwar CHAUDHRY (F)**
   Director
   Center for Advanced Studies in Pure and Applied Mathematics (CASPAM), Bahauddin Zakariya University
   Multan.
5. **Professor Dr. Lal CHAND**
   Professor and Chairman
   Dept. of Mathematics, Shah Abdul Latif University, Khairpur.
6. **Professor Dr. Karamat HUSSAIN (F)**
   Professor and Chairman
   Dept. of Mathematics, GC University, Lahore.
7. **Professor Dr. Faqir MUHAMMAD (F)**
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   Dept. of Math. and Statistics, Allama Iqbal Open University, Islamabad.
8. **Professor Dr. Islam NOOR (F)**
   Professor and Chairman
   Dept. of Mathematics, University of Peshawar, Peshawar.
9. **Dr. Malik Muhammad YOUSAF (F)**
   Assistant Professor
   Dept. of Mathematics, Quaid-é-Azam University, Islamabad.

Secretary

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On October 26, 2007, the preliminary-special meeting started with recitation of verses from the Holy Quran by Professor Dr. Ahsanullah Baloch. Member, HEC, Professor Dr. Riaz-ul-Haq Tariq, welcomed the participants and briefed them about the curricula-development exercise taken by HEC. He, also, requested the participants to develop 4-year BS program according to the standardized template agreed upon during the meeting of conveners of NCRC on April 30, 2007 as well as review MS and PhD programs. During this short talk Professor Tariq expressed the desire to develop communication and problem-solving skills in the students of mathematics.

The Committee, unanimously, appointed Professor Dr. Syed Arif Kamal as convener and Professor Dr Shahid S. Siddiqi as secretary. Professor Kamal, in his opening remarks, thanked the participants for working tirelessly with him to develop BS, MS and PhD schemes of studies. The following items were discussed:

Item No. 1: Look into the need and the necessity of compulsory courses, Mathematics I, Mathematics II and Introduction to Computers for mathematics majors.

Resolution No. 1: It was resolved that Introduction to Computers is needed for mathematics majors. The contents of Mathematics I and Mathematics II should be different for students majoring in mathematics because they have a much intensive program of studies. The students of mathematics should study computational mathematics aspects in the above courses. Mathematics I should focus on programming languages. The students should know one language in depth. Mathematics II should focus on computing-software applications. The students should learn one package in detail. The committee, also, proposed course contents for Mathematics I and Mathematics II to be taken by students from disciplines other than mathematics.

Item No. 2: Out of 4 electives, how many should be in the field of specialization and how many outside the field of specialization?

Resolution No. 2: It was resolved that 3 electives should be in the field of specialization and 1 outside the field of specialization.

Item No. 3: Prioritize the Core Courses into the following categories:

a) Must know (Discipline-Specific-Foundation Courses)
b) Should know (Major Courses)
c) Nice to know (Electives)

Resolution No. 3: The courses were classified into the following categories:

a) Foundation Courses:
Algebra I, Algebra II, Algebra III, Calculus I, Calculus II, Calculus III, Complex Analysis, Ordinary-Differential Equations, Real Analysis I, Real Analysis II

b) Major Courses:
Classical Mechanics, Discrete Structures, Functional Analysis, Mathematical Modeling and Simulation, Mathematical Spaces, Mathematical Statistics, Number Theory, Numerical Analysis, Numerical Computing, Optimization Theory, Partial-Differential Equations, Vector and Tensor Analysis

c) Electives:
Listed separately near the end of BS scheme.

Item No. 4: Look into the need and the necessity of 3-4 courses of Physics for mathematics majors (as physics majors are required to study 4 courses of mathematics in some universities).

Resolution No. 4: It was resolved that physics may be helpful for those, who opt for general courses in the area of physical sciences, but not for other areas, *e. g.*, natural sciences, management sciences, and social sciences

Item No. 5: Establish the depth and the breadth in terms of flow chart for the BS scheme:

a) Precedence Graphs (Depth: Pre-requisites)
b) Influence Graphs (Breadth: Co-requisites)

Resolution No. 5: A subcommittee headed by Professor Dr. Syed Arif Kamal was designated the task to prepare the flow chart. . The flow chart was discussed in the meeting and approved.

Item No. 6: Learning outcomes at the end of each year of BS and MS programs of study.

Resolution No. 6: A subcommittee headed by Professor Dr. Barbu Berceanu was given the task to prepare the learning outcomes. The prepared outcomes were discussed in the meeting and approved.

Item No. 7: Credit and grades for mathematics teaching as part of MS and PhD programs of study. Is MS program of 33 credit hours adequate for 2 years?

Resolution No. 7: It was resolved that mathematics teaching at the MS and PhD be given practical credits so that the time spent may be accounted for. To improve the quality of teaching and promote competitiveness, teaching should be graded according to at least two of the following evaluation criteria:

a) Feedback from students
b) Lecture/Tutorial/Problem-solving session observation by senior teachers
c) Videotaped lecture/tutorial/problem-solving session evaluation by experts

It was, further, resolved that MS program of 33 credit hours in not adequate for a period of 2 years. The minimum credit hours for a 2-year program should be 48 and the maximum should not exceed 60 hours.

The convener complied the schemes of studies, in particular setting the BS scheme according to the agreed-upon template, in the light of discussion of the committee. The schemes were uploaded on the homepage of convener.

On February 15, 2008, the final-special meeting started with recitation from the Holy Quran by Professor Baloch. Member HEC, Professor Tariq, in his opening remarks suggested that the BS scheme might include 2 social-science courses, instead of one. He, further, proposed that the compulsory-mathematics courses might be customized for different disciplines, *e. g.*, mathematics for biological scientists, mathematics for chemists, mathematics for linguists, mathematics for physicists and mathematics for social scientists. A similar suggestion was made in a paper by convener [Kamal SA and Siddiqui KA, *Physics Education (India)*, April-June 1989, pp. 53-61]. The participants, critically, scrutinized and finalized the BS scheme of studies, which was uploaded on website of convener. The draft was, also, e-mailed to all participants. Suggestions received were incorporated in the final version.

NCRC expressed the feelings that much more than mere curriculum development is needed to bring mathematics teaching at par with the developing countries, like Korea, Malaysia, Singapore, and others:

Teachers' Training

To handle such a demanding curriculum teachers need to be trained. Teachers should be prepared to teach all core courses. This can be achieved through refresher and special courses for college and university teachers. The teachers should maintain office hours to help students overcome their difficulties, guide and counsel students for further studies and career.

The fundamental responsibility of a mathematics teacher is to first enhance the teaching of basic theory, to execute closely such education of scientific methodology as induction, contrast, analogy and imitation.

Teaching Load

Teaching load should be balanced so that a teacher can find time to do research. The class size should not exceed 25 students. Each senior instructor should be provided at least two junior instructors (teaching assistants), one to teach tutorial sessions and the other to grade problems. In courses having laboratory, an additional teaching assistant is needed to conduct the laboratory.

Textbooks and Teaching Aids

To teach effectively there is a need to make available textbooks at cheaper prices. These textbooks should be supplemented with solution manuals for instructors and study guides for students (these are not the “keys”, or the “guides” available in the market) as well as softwares, slides, films, charts, demonstrations, models and journals. Students should be encouraged to make drawings, models and softwares, which illustrate difficult concepts.

Getting the Best Students

In Pakistan, the brightest students compete for places in engineering and medical colleges as well as computer science and information technology programs. Those unable to find places in the professional institutions come to Mathematics Departments. In order to attract the very best to mathematics one needs to make effective contact with the prospective students by organizing open houses, summer programs for high-school seniors, lectures in schools and colleges, *etc.* Mathematics should be presented as fun through games and simulations, dramatic demonstrations, activities done outdoors and illustrations of mathematics in sports. Usefulness of mathematics can be demonstrated by daily-life applications and as preparation of a scientific career. The goal is to enable the students to see the bridge leading them from mathematics to technology passing through physics and engineering.

Students’ Feedback

To promote effective learning and teaching there should be continuous feedback from students — much more than the confidential-evaluation form filled by each student at the end of a semester. End-of -the- term-confidential evaluation shall help students of the following batch, not the batch studying at that time.

Regulating Body

There is need for a regulating body, which could maintain quality control over the choice of textbooks and journals published within the country. Such a body may be ‘Pakistan Mathematics Society’, which could publish one or a series of journals, *e. g.*, Pakistan Journal of Mathematics, instead of every university bringing out its own journal.

Schemes of Studies

The schemes of studies for 4-year BS, 3-year MS and 3-year PhD programs are given on the following pages:

THE BACHELORS PROGRAM IN MATHEMATICS

Degree Awarded: **Bachelor of Studies (BS): Four-Year Program**

Rationale: In the present-day-civilized society, numeracy (knowledge of basic mathematics) is the single most important skill needed for survival next to communication skills.

Objectives: The BS scheme of studies aims to establish the base for lifelong education by creating essential concepts and equipping the student with necessary techniques, needed to start a career of research, development, teaching or financial applications involving mathematics. The equations of *mathematics* are made to speak through *physics*, which is the formulation of general laws applying, mainly, inductive logic. *Engineering* is modeling from the general laws to create practical systems. *Technology* is the implementation and the adaptation of a laboratory model to create a working system, which could be mass-produced. The journey from *mathematics* to *technology* could be considered as a journey from the abstract to the concrete, *mathematics* being in books, in the minds of philosophers, *physics* making contact with outside world, *technology* becoming the stage, where one enjoys the blessings. In other disciplines, like, economics, life sciences, a similar route is followed to transform the abstract formulae and equations developed by a pure mathematician to live and vibrating applications in everyday world. The training of mathematicians should inculcate creative thinking and make them capable to critically analyze a problem, taking them from various stages of concept building to equip them with problem-solving skills. Problem solving in the classroom should prepare them to problem solving in the laboratories, and, eventually, problem solving in the industry. After completing the BS program, the students should:

a)	be able to experience mathematics.
b)	appreciate importance of mathematics and visualize its applications in every walk of life.
c)	understand and use the basic structures applicable in modern sciences.
Entrance Requirements:	HSC (Pre-Engineering Group) or equivalent
Duration of the Program:	4 years (8 semesters, each semester comprising of 18 weeks); students NOT from the Pre-Engineering Group have to attend intensive courses in Mathematics, Physics and Chemistry during the semester breaks of first and second years depending on their deficiencies. Students cannot get the degree of BS before completing 4 years of study.
Total Credit Hours:	133 (maximum 18 hours per semester); 1 credit hour (theory) means 1 contact hour per week throughout the semester; 1 credit hour (laboratory) means 3 contact hours of practical work.
Compulsory Courses:	9 (25 credit hours) [listed in *Italics*]
General Courses:	8 (24 credit hours); 6 to be chosen from physical sciences (one course must be physics), natural sciences, social sciences or management sciences (18 credit hours); one course "Introduction to Psychology" (3 credit hours) and another "Introduction to Sociology" (3 credit hours)
Core Courses (Prioritization):	These are prioritized into the following categories:
a)	Must know (Discipline-Specific-Foundation Courses) 10 (28 credit hours) [listed in **Bold**]
b)	Should know (Major Courses) 12 (44 credit hours, including guest/students' seminars) [listed in ***Bold Italics***]
c)	Nice to know (Electives) 4 (12 credit hours); listed separately near the end of BS scheme (Page 39).
Electives A:	3 (9 credit hours) [Electives in Specialization]

Electives B:	1 (3 credit hours); mathematics courses outside the field of specialization [Free Electives]
Project:	3 credit hours, with accompanying written report and presentation (may be taken in lieu of fee elective)
Core Courses (Depth & Breadth):	The depth and the breadth is illustrated in terms of flow chart for the BS scheme (Figure 3, Page 24):
a)	Precedence Graphs (Depth: Pre-requisites)
b)	Influence Graphs (Breadth: Co-requisites)
Academic Standards:	In order to bring the standard of education at par with the developed countries, the notion that anyone who gets admitted to a university shall end up with a degree should be abolished.
a)	Those, who fail in more than two courses in a semester, are, automatically, dismissed.
b)	Those, who fail in one or two courses, may retake the examination during the 40-day grace period.
c)	Those, who have not cleared all courses after the grace period are dismissed.
d)	There shall be no choice in quizzes, hourlies and final examinations. All questions given have to be attempted.
Class Size:	50 (maximum) for lectures; 25 (maximum) for tutorial and laboratory session; course supervisor assisted by one or more associate instructors
Minimum Lectures:	Before final examination could be conducted minimum number of lectures to be delivered must be at least 13 per credit hour (*i. e.*, 26 for 2-hour course; 39 for 3-hour course; 52 for 4-hour course)

Attendance Requirement:	Students are required to maintain 75% attendance in order to sit in the final examination. Dean, under special circumstances, may condone attendance between 60-74%. Attendance registers, bearing the list of students, should be provided to the instructors on the first day of classes. Instructors take the attendance at the start of every class. At the end of each class associate instructor keys in the hard-copy attendance record to a centralized database. On the first of every month list of students having attendance less than 75% is displayed on the notice board. Those having attendance lass than 60% should be required to attend fresh classes.
Summer Activity:	Students could be assigned independent study. Third- and fourth-year students may be placed in local industry and financial institutions for internships during semester breaks. During the fourth year the students should receive career counseling for suitable placement after completion of BS degree.
Seminars:	During each semester student must participate in Guest Seminar (presentations by eminent mathematicians on theoretical aspects and applications) and Students' Seminar (presentations by students); Guest Seminars and Students' Seminars are held fortnightly during the same time slot (8 credit hours) — graded on the basis of presentation and write up by the student, a weekly quiz on the contents of Guest/ Students' Seminar held during the previous week
Comprehensive Viva:	Conducted by senior faculty members of the department at the end of eighth semester
Qualifications for Course Supervisors (Instructors):	Faculty holding PhD or MS degree is entitled to teach lecture session of a course
Qualifications for Associate Instructors (Teaching Assistants):	Associate Instructors (Teaching Assistants) must hold BS, MSc or MA and may handle tutorial, laboratory as well as assignment, quiz and problem grading
Syllabus Breakdown:	Course Supervisor is required to distribute syllabus breakdown into 14 units, each unit completed within a week.

Pedagogical Techniques:	Lecture sessions of each unit (normally, 2-3 lectures) are followed by a discussion session (reinforcing the concepts taught through examples, alternate derivations and proofs) as well as a problem-solving session (teaching skills of problem formulation, qualitative analysis and finding solutions), each of these sessions conducted, separately, by the Associate Instructor at the end of lecture session of every unit. In addition, a review session should be arranged prior to each monthly test (hourly, called because the test is of one-hour duration) and a comprehensive review before the final examination, both sessions conducted by the Course Supervisor.
Continuous-Evaluation Report:	Course Supervisor (Instructor) is required to prepare a progress report after each hourly mentioning tentative grade (cumulative grade based on all hourlies, quizzes, problem sets and assignments taken to date) and attendance record. It may, also, include qualitative description of student's weaknesses and areas needing special attention. Laboratory and theory portions are separate passing heads. This report is discussed with the student as well as student's parents, if unsatisfactory.
Grading of Problem Sets and Assignments:	An Associate Instructor (A Teaching Assistant) is available to mark problem sets (weekly), quizzes (weekly) and assignments (one or two during the semester).
Withdrawal Policy:	Student shall have the option to withdraw from a course during a period of 45 days from the start of course. This is possible, only, after the student submits the required forms in the Semester-Examination Section through the Chairman of Department. The first progress report (mentioning tentative grade of the student after the first hourly) is supposed to be available before the expiry of withdrawal period. A grade of 'W' is to be assigned in such a course, and it is not counted in the computation of GPA (Grade-Point Average).

Final Examinations: Final Examination (Part I) shall be set up and graded by the Course Supervisor. Final Examination (Part II) shall be set up and graded by a faculty member designated by Board of Studies. The student is supposed to attempt Part I and Part II on separate answer books.

Marks Breakdown: Passing grade is set at 50% (65% in foundation and 60% in major courses). Table 1 lists marks breakdowns for courses without laboratory (3 + 0) and courses with laboratory (2 + 1), respectively. Table 2 gives the equivalence of numerical and alphabetical grades.

Table 1. Marks Breakdown for Courses

Item	Maximum Marks for courses without Laboratory (3 + 0)	Maximum Marks for courses with Laboratory (2 + 1)
Monthly Tests (2-3) [a]	30	15
Quizzes (10-14) [b]	10	10
Problem Sets (10-14) [c]	10	10
Assignments [d]	10	5
Laboratory	---	20
Final Examination	40	40
Total	100	100

[a] 3 monthly tests (hourlies) for courses without laboratory, best 2 counted; 2 monthly tests (hourlies) for courses with laboratory, best of the 2 counted

[b] All quizzes are surprise and conducted at the beginning of class/seminar; best 10 counted

[c] One problem set given each week; best 10 counted

[d] 2 for courses without laboratory; 1 for courses with laboratory

Table 2. Numerical and Alphabetical Grades

Numerical Grade	Alphabetical Grade	Grade Point
90-100	A^+	4.00
85-89	A	4.00
80-84	A^-	3.67
75-79	B^+	3.33
71-74	B	3.00
68-70	B^-	2.67
64-67	C^+	2.33
60-63	C	2.00
57-59	C^-	1.67
53-56	D^+	1.33
50-52	D	1.00
0-49	F	Zero

Table 3. The BS Scheme of Studies: Structure

#	Categories	No. of Courses	Credit Hours
1	Compulsory Requirements (no choice)	9	25
2	General Courses (to be chosen from other departments)	8	24
3	Discipline-Specific-Foundation Courses	10	28
4	Major Courses (including seminars)	12	44
5	Electives (including project in lieu of free elective)	4	12
	Total	**43**	**133**

Table 4. The BS Scheme of Studies: Layout

Compulsory Requirements		**General Courses**	
Title	*Cr. Hr.*	*Title*	*Cr. Hr.*
1. English I (English Structure I)	3	1. General-A I	3
2. English II (English Structure II)	3	2. General-A II	3
3. English III (Communication Skills for Mathematicians)	3	3. General-A III	3
		4. General-B I	3
4. English IV (Technical Writing)	3	5. General-B II	3
5. Introduction to Computers	3	6. General-B III	3
6. Islamic Studies	2	7. Introduction to Psychology	3
7. Mathematics I (Programming Languages for Mathematicians)	4	8. Introduction to Sociology	3
8. Mathematics II (Computing Tools for Mathematicians)	2		
9. Pakistan Studies	2		
Total	**25**		**24**

Foundation Courses		**Major Courses + Seminars**		**Electives + Project**	
Title	*Cr. Hr.*	*Title*	*Cr. Hr.*	*Title*	*Cr. Hr.*
1. Algebra I	3	1. Discrete Structures	3	***Within the field of specialization***	
2. Algebra II	3				
3. Algebra III	3	2. Number Theory	3		
4. Calculus I	3	3. Probability Theory	3		
5. Calculus II	3	4. Vector & Tensor Analysis	3	1. Elective-A I	3
6. Calculus III	3			2. Elective-A II	3
7. Complex Analysis	3	5. Classical Mechanics	3	3. Elective-A III	3
8. Ordinary-Diff. Eqns.	3	6. Mathematical Spaces	3	***Outside the field of specialization***	
9. Real Analysis I	2	7. Mathematical Statistics	3		
10. Real Analysis II	2	8. Numerical Analysis	3	4. Elective-B OR Project	3
		9. Partial-Diff. Eqns.	3		
		10. Functional Analysis	3		
		11. Optimization Theory	3		
		12. Modeling & Simulation	3		
		Seminars	8		
Total	**28**		**44**		**12**

Table 5. The BS Scheme of Studies: Semester-Wise Breakdown

FIRST YEAR

First Semester

#	*Course Title*	*Cr. Hr.*
1	**Calculus I**	3 + 0
2	***Discrete Structures***	3 + 0
3	*English Structure I*	3 + 0
4	General-A I	2 + 1
5	General-B I	2 + 1
6	Guest/Students' Seminar I	1 + 0
7	*Islamic Studies*	2 + 0
	Total	**16 + 2**

Second Semester

#	*Course Title*	*Cr. Hr.*
1	**Calculus II**	3 + 0
2	General-A II	2 + 1
3	General-B II	2 + 1
4	Guest/Students' Seminar II	1 + 0
5	***Number Theory***	3 + 0
6	*Pakistan Studies*	2 + 0
7	*Introd. to Computers*	2 + 1
	Total	**15 + 3**

Learning Outcomes: After completing first year, the student should be able to utilize basic tools for computations in order to solve problems of mathematics.

SECOND YEAR

Third Semester

#	*Course Title*	*Cr. Hr.*
1	**Calculus III**	3 + 0
2	*Communication Skills for Mathematicians*	2 + 1
3	General-A III	2 + 1
4	General-B III	2 + 1
5	Guest/Students' Seminar III	1 + 0
6	*Prog. Languages for Mathematicians*	3 + 1
	Total	**13 + 4**

Fourth Semester

#	*Course Title*	*Cr. Hr.*
1	**Algebra I**	3 + 0
2	*Computing Tools for Mathematicians*	1 + 1
3	Introd. to Psychology	2 + 1
4	Introd. to Sociology	2 + 1
5	Guest/Students' Seminar IV	1 + 0
6	***Probability Theory***	3 + 0
7	*English Structure II*	3 + 0
	Total	**15 + 3**

Learning Outcomes: After completing second year, the student should be able to understand proof and to write a formal proof for theoretical applications.

THIRD YEAR

Fifth Semester

#	*Course Title*	*Cr. Hr.*
1	**Algebra II**	3 + 0
2	***Vect. & Tensor Anal.***	3 + 0
3	Guest/Students' Seminar V	1 + 0
4	***Math. Statistics***	3 + 0
5	***Numerical Analysis***	3 + 0
6	**Ordinary-Diff. Eqns.**	3 + 0
7	**Real Analysis I**	2 + 0
	Total	**18 + 0**

Sixth Semester

#	*Course Title*	*Cr. Hr.*
1	**Algebra III**	3 + 0
2	***Classical Mechanics***	3 + 0
3	**Complex Analysis**	3 + 0
4	Guest/Students' Seminar VI	1 + 0
5	***Mathematical Spaces***	3 + 0
6	***Partial-Diff. Eqns.***	3 + 0
7	**Real Analysis II**	2 + 0
	Total	**18 + 0**

Learning Outcomes: After completing third year, the student should know the main branches of mathematics, *i. e.*, analysis, topology, differential equations, mechanics, numerical analysis, probability and statistics and to apply these theories in practical problems.

FOURTH YEAR

Seventh Semester

#	*Course Title*	*Cr. Hr.*
1	Elective-A I	3 + 0
2	Elective-A II	3 + 0
3	***Functional Analysis***	3 + 0
4	Guest/Students' Seminar VII	1 + 0
5	*Technical Writing*	2 + 1
	Total	**12 + 1**

Eighth Semester

#	*Course Title*	*Cr. Hr.*
1	Elective-A III	3 + 0
2	Elective-B OR Project	3 + 0
3	Guest/Students' Seminar VIII	1 + 0
4	***Modeling & Simulation***	2 + 1
5	***Optimization Theory***	3 + 0
	Total	**12 + 1**

Learning Outcomes: After completing fourth year, the student should choose the field of specialization and write research paper.

The BS Course Contents

Course contents for Communication Skills, English Structure, Islamic Studies, General A and General B (could be chosen from the list of subjects offered in the institution, *e. g.*, chemistry, economics, geology, physics, statistics), Pakistan Studies, Social Sciences Course as well as Technical Writing, are to be drafted by the respective National-Curriculum-Revision Committees. Course contents for the mathematics courses are, alphabetically, given below:

ALGEBRA I
Prerequisite(s): Mathematics at intermediate level
Credit Hours: 3 + 0

Specific Objectives of the Course: This is the first course in groups, matrices and linear algebra, which provides basic background needed for all mathematics majors, a prerequisite for many courses. Many concepts presented in the course are based on the familiar setting of plane and real three-space, and are developed with an awareness of how linear algebra is applied.

Course Outline:

Group Theory: Basic axioms of a group with examples, abelian groups, center of a group, derived subgroup of a group, subgroups generated by subset of a group, system of generators, cyclic groups, cosets and quotient sets, Lagrange's theorem, introduction to permutations, even and odd permutations, cycles, lengths of cycles, transpositions, symmetric group, alternating groups, rings, finite and infinite fields (definition and examples), vector spaces, subspaces, linear span of a subset of a vector space, bases and dimensions of a vector space

Algebra of Matrices: Determinants, matrix of a linear transformation. row and column operations, rank, inverse of matrices, group of matrices and subgroups, orthogonal transformation, eigenvalue problem with physical significance

Recommended Books:

Anton H, *Linear Algebra with Applications* (8th edition), John Wiley, New York
Herstein IN, *Topics in Algebra* (2nd edition), John Wiley, New York
Hill RO, *Elementary Linear Algebra with Application* (3rd edition), 1995, Brooks/Cole
Leon SJ, *Linear Algebra with Applications* (6th edition), 2002, Prentice Hall, Englewood Cliffs, NJ, USA
Nicholson WK, *Elementary Linear Algebra with Applications* (2nd edition), 1994, PWS Publishing Co.

ALGEBRA II

Prerequisite(s): Algebra I

Credit Hours: 3 + 0

Specific Objectives of the Course: This is a course in advanced abstract algebra, which builds on the concepts learnt in Algebra I.

Course Outline:

Group Theory: Normalizers and centralizers of a subset of a group, congruency classes of a group, normal subgroup, quotient groups, conjugacy relation between elements and subgroups, homomorphism and isomorphism between groups, Homomorphism and isomorphism theorems, group of automorphisms, finite p-groups, internal and external direct products, group action on sets, isotropy subgroups, orbits, 1st, 2nd and 3rd Sylow theorems.

Ring Theory: Types of rings, matrix rings, rings of endomorphisms, polynomial rings, integral domain, characteristic of a ring, ideal, types of ideals, quotient rings, homomorphism of rings, fundamental theorem of homomorphism of rings.

Recommended Books:

Allenby RBJT, *Rings, Fields and Groups: An Introduction to Abstract Algebra*, 1983,Edward Arnold

Farleigh JB, *A First Course in Abstract Algebra* (7th edition), Addison-Wesley, Reading, Ma., USA

Macdonald ID, *The Theory of Groups,* 1975, Oxford Clarendon Press, Ma., USA

ALGEBRA III

Prerequisite(s): Algebra II

Credit Hours: 3 + 0

Specific Objectives of the Course: This is a course in abstract linear algebra. The majority of follow up courses in both pure and applied mathematics assume the material covered in this course.

Course Outline: Vector spaces; sums and direct sums of subspaces of a finite dimensional vector space, Dimension theorem, linear transformation, null space, image space of linear transformation, rank and nullity of a linear transformation, relation between rank, nullity and dimension of the domain of a linear transformation, matrix of linear transformation, change of basis, inner product spaces, projection of a vector along another vector, norm of a vector, Cauchy Schwartz inequality, Orthogonal and orthonormal basis, similar matrices and diagonalization of a matrix, Home (V,W), dimension and basis of Home (V.W), dual space and dual basis, annihilators.

Recommended Books:

Axle SJ, *Linear Algebra Done Right*, Undergraduate Texts in Mathematics, 1996, Springer, New York

Birkhoff G, Maclane S, *A Survey of Modern Algebra* (4th edition), AKP Classics

Perry WL, *Elementary Linear Algebra*, 1988, McGraw-Hill, New York

CALCULUS I

Prerequisite(s): Mathematics at intermediate level

Credit Hours: 3 + 0

Specific Objectives of the Course: This is the first course of the basic sequence, Calculus I-III, serving as the foundation of advanced subjects in all areas of mathematics. The sequence, equally, emphasizes basic concepts and skills needed for mathematical manipulation. Calculus I & II focus on the study of functions of a single variable.

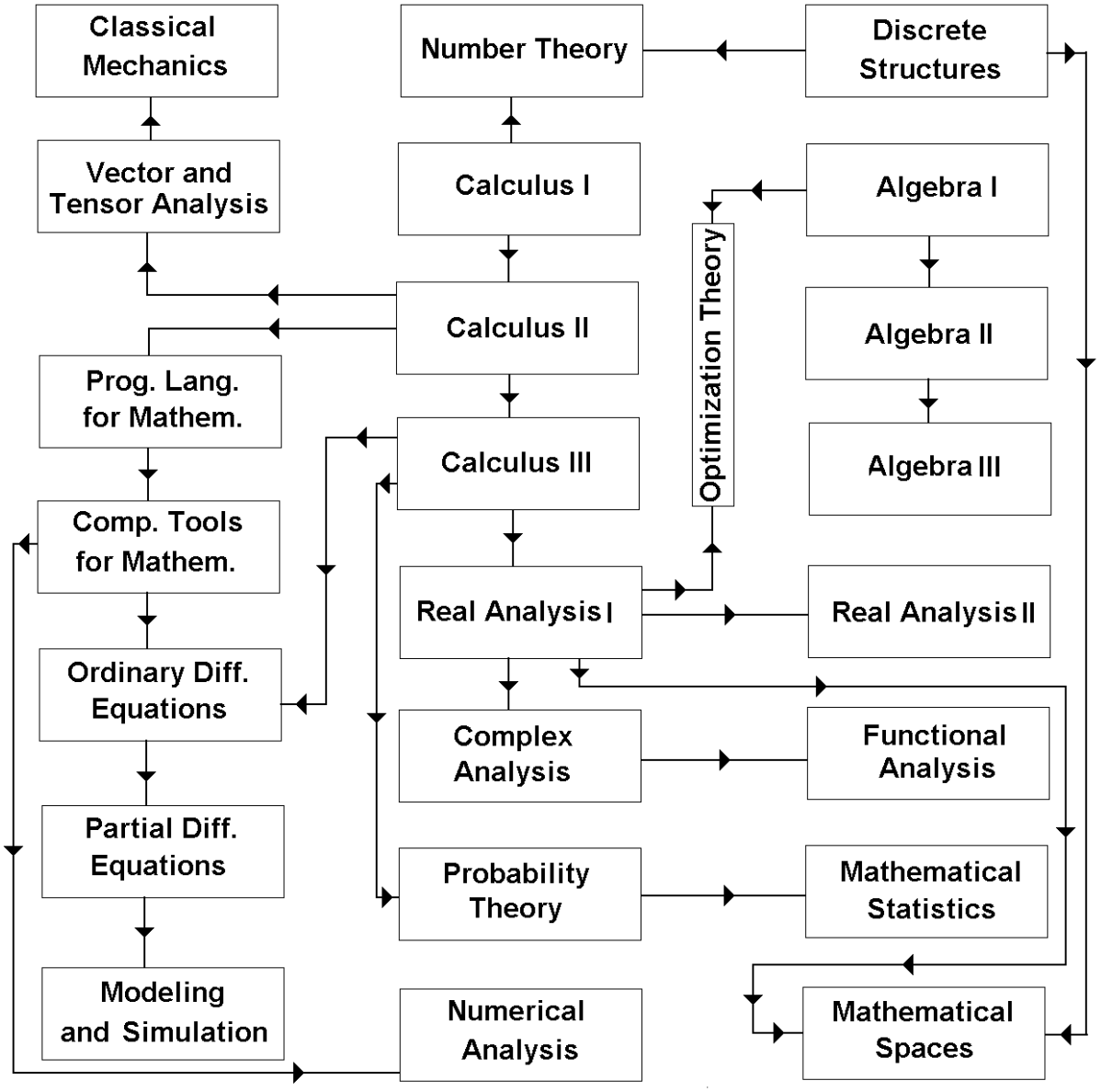

Fig. 3. Precedence and influence graphs representing pre-requisites and co-requisites for BS courses

Course Outline: Limits and continuity; derivative of a function and its applications; optimization problems; mean value theorem (Taylor's theorem and the infinite Taylor series with applications) and curve sketching; anti-derivative and integral; definite integral and applications; the fundamental theorem of calculus; inverse functions (Chapters 1-6 of the text)

Recommended Books:

Anton H, Bevens I, Davis S, *Calculus: A New Horizon* (8th edition), 2005, John Wiley, New York

Stewart J, *Calculus* (3rd edition), 1995, Brooks/Cole (suggested text)

Thomas GB, Finney AR, *Calculus* (11th edition), 2005, Addison-Wesley, Reading, Ma, USA

CALCULUS II

Prerequisite(s): Calculus I

Credit Hours: 3 + 0

Specific Objectives of the Course: This is the second course of the basic sequence Calculus I-III serving as the foundation of advanced subjects in all areas of mathematics. The sequence, equally, emphasizes basic concepts and skills needed for mathematical manipulation. As continuation of Calculus I, it focuses on the study of functions of a single variable.

Course Outline: Continuation of Calculus I: Techniques of integration; further applications of integration; parametric equations and polar coordinates; sequences and series; power series representation of functions (Chapters 7-10 of the text)

Recommended Books:

Anton H, Bevens I, Davis S, *Calculus: A New Horizon* (8th edition), 2005, John Wiley, New York

Stewart J, *Calculus* (3rd edition), 1995, Brooks/Cole (suggested text)

Thomas GB, Finney AR, *Calculus* (11th edition), 2005, Addison-Wesley, Reading, Ma, USA

CALCULUS III

Prerequisite(s): Calculus II

Credit Hours: 3 + 0

Specific Objectives of the Course: This is the third course of the basic sequence Calculus I-III serving as the foundation of advanced subjects in all areas of mathematics.

Course Outline: This course covers vectors and analytic geometry of 2 and 3 dimensional spaces; vector- valued functions and space curves; functions of several variables; limits and continuity; partial derivatives; the chain rule; double and triple integrals with applications; line integrals; the Green theorem; surface area and surface integrals; the Green, the divergence and the Stokes theorems with applications (Chapters 11-14 of the text)

Recommended Books:

Anton H, Bevens I, Davis S, *Calculus: A New Horizon* (8th edition), 2005, John Wiley, New York
Stewart J, *Calculus* (3rd edition), 1995, Brooks/Cole (suggested text)
Thomas GB, Finney AR, *Calculus* (11th edition), 2005, Addison-Wesley, Reading, Ma, USA

CLASSICAL MECHANICS

Prerequisite(s): Vector and Tensor Analysis
Credit Hours: 3 + 0
Specific Objectives of the Course: This course builds grounding in principles of classical mechanics, which are to be used while studying quantum mechanics, statistical mechanics, electromagnetism, fluid dynamics, space-flight dynamics, astrodynamics and continuum mechanics.
Course Outline: Particle kinematics, radial and transverse components of velocity and acceleration, circular motion, motion with a uniform acceleration, the Newton laws of motion (the inertial law, the force law and the reaction law), newtonian mechanics, the newtonian model of gravitation, simple-harmonic motion, damped oscillations, conservative and dissipative systems, driven oscillations, nonlinear oscillations, calculus of variations, Hamilton's principle, lagrangian and hamiltonian dynamics, symmetry and conservation laws, Noether's theorem, central-force motion, two-body problem, orbit theory, Kepler's laws of motion (the law of ellipses, the law of equal areas, the harmonic law), satellite motion, geostationary and polar satellites, kinematics of two-particle collisions, motion in non-inertial reference frame, rigid-body dynamics (3-D-rigid bodies and mechanical equivalence, motion of a rigid body, inverted pendulum and stability, gyroscope)

Recommended Books:

Bedford A, Fowler W, *Dynamics: Engineering Mechanics*, Addision-Wesley, Reading, Ma, USA
Chow TL, *Classical Mechanics*, 1995, John Wiley, New York
Goldstein H, *Classical Mechanics* (2nd edition), 1980, Addison-Wesley, Reading, Ma, USA
Marion JB, *Classical Dynamics of Particles and Fields* (2nd edition), 1970, Academic Press, New York (suggested text)
Synge JL, Griffith BA, *Principles of Mechanics*, McGraw Hill, New York

COMPLEX ANALYSIS

Prerequisite(s): Real Analysis I
Credit Hours: 3 + 0

Specific Objectives of the Course: This is an introductory course in complex analysis, giving the basics of the theory along with applications, with an emphasis on applications of complex analysis and especially conformal mappings. Students should have a background in real analysis (as in the course Real Analysis I), including the ability to write a simple proof in an analysis context.

Course Outline: The algebra and the geometry of complex numbers, Cauchy-Riemann equations, harmonic functions, elementary functions, branches of the logarithm, complex exponents. Contours and contour integrals, the Cauchy-Goursat Theorem, Cauchy integral formulas, the Morera Theorem, maximum modulus principle, the Liouville theorem, fundamental theorem of algebra. Convergence of sequences and series, the Taylor series, the Laurent series, uniqueness of representation, zeros of analytic functions. Residues and poles and the residue theorem, evaluation of improper integrals involving trigonometric functions, integrals around a branch point., the argument principle, the Roche theorem.

Recommended Text: Churchill RV, Brown JW: *Complex Variables and Applications* (5th edition), 1989, McGraw Hill, New York

COMPUTING TOOLS FOR MATHEMATICIANS

Prerequisite(s): Programming Languages for Mathematicians

Credit Hours: 1 + 1

Specific Objectives of the Course: The purpose of this course is to teach students the use of mathematical software like MATLAB, MAPLE, MATHEMATICA for solving computationally-difficult problems in mathematics. The student shall become well versed in using at least one mathematical software and shall learn a number of techniques that are useful in calculus as well as in other areas of mathematics.

Course Outline: The contents of the course are not fixed, however the following points should be kept in mind while teaching the course. The course should be taught in a computer lab setting. Besides learning to use the software, the students must be able to utilize the software to solve computationally difficult problems in calculus and other areas of mathematics. At the end of the course, the students should have a good command on at least two of the three programs mentioned above.

Recommended Books:

Etter DM, Kuncicky D, Hull D, *Introduction to MATLAB 6*, 2001, Prentice Hall, Englewood Cliffs, NJ, USA

Garvan F, *The Maple Book*, 2002, Chapman & Hall/CRC

Kaufmann S, *Mathematica As a Tool: An Introduction with Practical Examples,* 1994, Springer, New York

DISCRETE STRUCTURES
Prerequisite(s): Mathematics at intermediate level
Credit Hours: 3 + 0
Specific Objectives of the Course: This course shall assume background in number theory. It lays a strong emphasis on understanding and utilizing various strategies for composing mathematical proofs.
Course Outline:
Set and Relations: Basic notions, set operations, Venn diagrams, extended-set operations, indexed family of sets, countable and uncountable sets, relations, cardinality, equivalence relations, congruence, partitions, partial order, representation of relations, mathematical induction.
Elementary Logic: Logics of order zero and one, Propositions and connectives, truth tables, conditionals and biconditionals, quantifiers, methods of proof, proofs involving quantifiers.
Recommended Text: Rosen KH, *Discrete Mathematics and its Applications* (12^{th} edition), 1999, McGraw Hill, New York
Ross KA, Wright CRB, *Discrete Mathematics*, 2003, Prentice Hall, Englewood Cliffs, NJ, USA

FUNCTIONAL ANALYSIS
Prerequisite(s): Complex Analysis
Credit Hours: 3 + 0
Specific Objectives of the Course: This course extends methods of linear algebra and analysis to spaces of functions, in which the interaction between algebra and analysis allows powerful methods to be developed. The course will be mathematically sophisticated and will use ideas both from linear algebra and analysis.
Course Outline:
Metric Spaces: A quick review, completeness and convergence, completion.
Normed Spaces: Linear spaces, Normed spaces, Difference between a metric and a normed space, Banach spaces, Bounded and continuous linear operators and functionals, Dual spaces, Finite dimensional spaces, F. Riesz Lemma, The Hahn-Banach Theorem, The HB theorem for complex spaces, The HB theorem for normed spaces, The open mapping theorem, The closed graph theorem, Uniform boundedness principle and its applications
Banach-Fixed-Point Theorem: Applications in Differential and Integral equations
Inner-Product Spaces: Inner-product space, Hilbert space, orthogonal and orthonormal sets, orthogonal comple-ments, Gram-Schmidt orthogonalization process, representation of functionals, Reiz-representation theorem, weak and weak* Convergence.

Recommended Books:

Curtain RF, Pritchard AJ, *Functional Analysis in Modern Applied Mathematics,* Aademic Press, New York
Friedman A, *Foundations of Modern Analysis*, 1982, Dover
Kreyszig E, *Introductory Functional Analysis with Applications*, John Wiley, New York
Rudin W, *Functional Analysis*, 1973, McGraw Hill, New York

MATHEMATICAL SPACES

Prerequisite(s): Discrete Structures, Real Analysis I
Credit Hours: 2 + 0
Specific Objectives of the Course: This course is designed primarily to develop pure mathematical skills of students. Students will need some background in writing proofs. They will lean notions of spaces, metric, measure and topology
Course Outline:
Notion of Spaces: Example of set, group, field, ring, affine space, Banach space, normed space, Hilbert space (Simmon)

a) *Notion of Topology:* Calculus on manifolds, continuity of functions on spaces, neighborhoods, topological spaces, finer and weaker topologies, homomorphism, homomorphic spaces, compactness, connectedness, normal spaces, Urysohn's lemma (Munkres)
b) *Notion of Metric:* Metric space, complete metric space, Baire category theorem, metrization of spaces (Friedmann)
c) *Notion of Measure:* Spaces with measure, measurable function, idea of $\sigma-$fields (Holmos)

Recommended Books:

Friedmann A, *Foundations of Modern Analysis*, 1982, Dover
Holmos PR, *Measure Theory*, van Nostrand, New York
Munkres JR, *Topology: A First Course*, Prentice Hall, Englewood Cliffs, NJ, USA
Simmon GF, *Introduction to Topology and Modern Analysis*, 1963, McGraw Hill, New York

MATHEMATICAL STATISTICS

Prerequisite(s): Probability Theory
Credit Hours: 3 + 0
Specific Objectives of the Course: In the course "Probability Theory" the students learnt how to set up mathematical models of processes and systems that are affected by *chance*. In the present course the students would learn how to check these models against reality, to determine whether they are reliable/accurate enough for practical purposes or otherwise. This helps in making predictions and decisions

Course Outline: Sampling theory: sampling distributions; sampling procedures; estimation of parameters: estimation of mean, variance; confidence intervals; decision theory: hypothesis testing and decision making; types of errors in tests; quality control; control charts for mean, standard deviation, variance, range; goodness of fit, chi-square test. Regression analysis; method of least squares; correlation analysis

Recommended Books:

DeGroot MH, Schervish MJ, *Probability and Statistics* (3rd edition), 2002, Addison- Wesley, Reading, Ma, USA (suggested text)

Johnson RA, *Probability and Statistics for Engineers*, 1994, Prentice-Hall, Englewood Cliffs, NJ, USA

Papoulis A, *Probability, Random Variables, and Stochastic Processes,* (3rd edition), 1991, McGraw Hill, New York

Sincich T, *Statistics by Examples*, 1990, Dellen Publication Company

MODELING AND SIMULATION

Prerequisite(s): Partial-Differential Equations

Credit Hours: 2 + 1

Specific Objectives of the Course: Mathematics is used in many areas such as engineering, ecological systems, biological systems, financial systems, economics, etc. In all such applications one approximates the actual situation by an idealized model. This is an introductory course of modeling, consisting of three parts: modeling with ordinary differential equations and their systems; partial differential equations; and integral equations. The course will not be concerned with the techniques for solving the equations but with setting up the equations in specific applications. Whereas the first two types of equations have already been dealt with, the third type has not. Consequently, solutions of the former will be discussed but of the latter will barely be touched upon.

Course Outline: Concepts of model, modeling and simulation, functions, linear equations, linear-differential equations, nonlinear-differential equations and integral equations as models, introduction to simulation techniques

Ordinary-Differential Equations: Modeling with first order differential equations: Newton's law of cooling; radioactive decay; motion in a gravitational field; population growth; mixing problem; Newtonian mechanics. Modeling with second order differential equations: vibrations; application to biological systems; modeling with periodic or impulse forcing functions. Modeling with systems of first order differential equations; competitive hunter model; predator-prey model.

Partial-Differential Equations: Methodology of mathematical modeling; objective, background, approximation and idealization, model validation, compounding. Modeling wave phenomena (wave equation); shallow water waves, uniform transmission line, traffic flow, RC circuits. Modeling the heat equation and some application to heat conduction problems in rods, lamina, cylinders etc. Modeling the potential equation (Laplace equation), applications in fluid mechanics, gravitational problems. Equation of continuity.
Simulation: Techniques of simulation (students are required to simulate at least one system)

Recommended Books:

Giordano FR, Weir MD, *Differential Equations: A Modeling Approach*, 1994, Addison-Wesley, Reading, Ma, USA (suggested text)
Jerri AJ, *Introduction to Integral Equations with Applications*, 1985, Marcel Dekker, New York
Myint UT, Debnath L, *Partial Differential Equations for Scientists and Engineers* (3rd edition), 1987, North Holland, Amsterdam

NUMBER THEORY

Prerequisite(s): Calculus I, Discrete Structures
Credit Hours: 3 + 0
Specific Objectives of the Course: This course shall assume no experience or background in number theory or theoretical mathematics. The course introduces various strategies for composing mathematical proofs.
Course Outline: Divisibility, euclidean algorithm, GCD and LCM of 2 integers, properties of prime numbers, fundamental theorem of arithmetic (UFT), congruence relation, residue system, Euler's phi-function, solution of system of linear congruences, congruences of higher degree, Chinese remainder theorem, Fermat's little theorem, Wilson's theorem and applications, primitive roots and indices; integers belonging to a given exponent (mod p), primitive roots of prime and composite moduli, indices, solutions of congruences using indices., quadratic residues, composite moduli, quadratic residues of primes, the Legendre symbol, the Quadratic reciprocity law, the Jacobi symbol, Diophantine equations

Recommended Books:

Burton DM, *Elementary Number Theory*, Allyn and Bacon
Grosswald E, *Topics from the Theory of Numbers*, The Macmillan Company
LeVeque WJ, *Topics in Number Theory*, Vol.1, Addison-Wesley, Reading, Ma, USA
Niven I, Zuckerman HS, *An Introduction to The Theory of Numbers*, Wiley Eastern
Rosen KH, *Elementary Number theory and its Applications* (4th edition), 2000, Addison-Wesley, Reading, Ma, USA (suggested text)

NUMERICAL ANALYSIS

Prerequisite(s): Computing Tools for Mathematicians
Credit Hours: 3 + 0
Specific Objectives of the Course: This course is designed to teach the students about numerical methods and their theoretical bases. The students are expected to know computer programming to be able to write program for each numerical method. Knowledge of calculus and linear algebra would help in learning these methods
Course Outline: Computer arithmetic, approximations and errors; methods for the solution of nonlinear equations and their convergence: bisection method, regula falsi method, fixed point iteration method, Newton-Raphson method, secant method; error analysis for iterative methods. Interpolation and polynomial approximation: Lagrange interpolation, Newton's divided difference, forward-difference and backward-difference formulae, Hermite interpolation. Numerical integration and error estimates: rectangular rule, trapezoidal rule, Simpson's one-three and three-eight rules. Numerical solution of systems of algebraic linear equations: Gauss-elimination method, Gauss-Jordan method; matrix inversion; LU-factorization; Doolittle's, Crount's, Cholesky's methods; Gauss-Seidel and Jacobi methods
Recommended Books:

Atkinson KE, *An Introduction to Numerical Analysis* (2nd edition), 1989, John Wiley, New York (suggested text)
Burden RL, Faires JD, *Numerical Analysis* (5th edition), 1993, PWS Publishing Company
Chapra SC, Canale RP, *Numerical Methods for Engineers*, 1988, McGraw Hill, New York

OPTIMIZATION THEORY

Prerequisite(s): Algebra I, Real Analysis I
Credit Hours: 3 + 0
Specific Objectives of the Course: The main objective is to teach the basic notions and results of mathematical programming and optimization. The focus will be to understand the concept of optimality conditions and the construction of solutions. Students should have a good background in analysis, linear algebra and differential equations.

Course Outline: Linear programming: simplex method, duality theory, dual and primal-dual simplex methods. Unconstrained optimization: optimality conditions, one-dimensional problems, multi-dimensional problems and the method of steepest descent. Constrained optimization with equality constraints: optimality conditions, Lagrange multipliers, Hessians and bordered Hessians. Inequality constraints and the Kuhn-Tucker Theorem. The calculus of variations, the Euler-Lagrange equations, functionals depending on several variables, variational problems in parametric form, transportation models and networks

Recommended Books:

Elsgolts L, *Differential Equations and the Calculus of Variations*, 1970, Mir Publishers, Moscow

Gotfried BS, Weisman J, *Introduction to Optimization Theory*, 1973, Prentice Hall, Englewood Cliffs, NJ, USA

Luenberger DG, *Introduction to Linear and Non-Linear Programming*, 1973, Addision-Wesley, Reading, Ma, USA

ORDINARY-DIFFERENTIAL EQUATIONS

Prerequisite(s): Calculus III, Computing Tools for Mathematicians

Credit Hours: 3 + 0

Specific Objectives of the Course: This course provides the foundation of all advanced subjects in Mathematics. Strong foundation and applications of Ordinary Differential Equations is the goal of the course.

Course Outline: Introduction; formation, solution and applications of first-order-differential equations; formation and solution of higher-order-linear-differential equations; differential equations with variable coefficients; Sturm-Liouville (S-L) system and boundary-value problems; series solution and its limitations; the Frobenius method, solution of the Bessel, the hypergeometric, the Legendre and the Hermite equations, properties of the Bessel function

Recommended Text: Zill DG, Cullen MR, *Differential Equations with Boundary-Value Problems,* (3rd Edition), 1997, PWS Publishing Co.

PARTIAL-DIFFERENTIAL EQUATIONS

Prerequisite(s): Real Analysis I, Ordinary-Differential Equations

Credit Hours: 3 + 0

Specific Objectives of the Course: The course provides a foundation to solve Partial Differential Equations with special emphasis on wave, heat and Laplace equations. Formulation and some theory of these equations are also intended.

Course Outline: First-order-partial-differential equations; classification of second-order PDE; canonical form for second-order equations; wave, heat and the Laplace equation in Cartesian, cylindrical and spherical-polar coördinates; solution of partial differential equation by the methods of: separation of variables; the Fourier, the Laplace and the Hankel transforms, non-homogeneous-partial-differential equations

Recommended Text: Myint UT, *Partial Differential Equations for Scientists and Engineers* (3rd edition), 1987, North Holland, Amsterdam

PROBABILITY THEORY

Prerequisite(s): Calculus III

Credit Hours: 3 + 0

Specific Objectives of the Course: This course is designed to teach the students how to handle data numerically and graphically. If data are influenced by *chance* effect, the concepts and rules of probability theory may be employed, being the theoretical counterpart of the observable reality, whenever *chance* is at work.

Course Outline: Introduction to probability theory; random variables; probability distributions; mean, standard deviation, variance and expectation. Binomial, negative binomial, Poisson,, geometric, hypergeometric and normal distributions; normal approximation to binomial distribution; distributions of 2 random variables.

Recommended Books:

DeGroot MH, Schervish MJ, *Probability and Statistics* (3rd edition), 2002, Addison-Wesley, Reading, Ma, USA (suggested text)

Papoulis A, *Probability, Random Variables, and Stochastic Processes,* (3rd edition), 1991, McGraw Hill, New York

Sincich T, *Statistics by Examples*, 1990, Dellen Publishing Company

PROGRAMMING LANGUAGES FOR MATHEMATICIANS

Prerequisite(s): **Calculus II**

Credit Hours: 3 + 1

Specific Objectives of the Course: The purpose of this course is to introduce students to operating systems and environments

Course Outline: Introduction to operating systems, one Language (FORTRAN or C/C++), building blocks, variables, input/output, loops (FOR, WHILE, DO), decisions (IF, IF ELSE, ELSE IF) construct switch statement, conditional statement, function hat returns a value using argument to pass data to another function, external variable, arrays and strings, pointers, structure, files and introduction to object-oriented programming

Recommended Books:

Aho, AV, Ulman JD, Foundation of Computer Science, 1995, Computer Science Press, WH Freeman, New York

Hein JL, *Theory of Computation: An Introduction* (1st edition), Jones & Bartlett, Boston

Laffo R, *Introduction to Object-Oriented Programming*, McGraw Hill, New York

REAL ANALYSIS I

Prerequisite(s): Calculus III

Credit Hours: 2 + 0

Specific Objectives of the Course: This is the first rigorous course in analysis and has a theoretical emphasis. It rigorously develops the fundamental ideas of calculus and is aimed to develop the students' ability to deal with abstract mathematics and mathematical proofs.

Course Outline: Ordered sets, supremum and infimum, completeness properties of the real numbers, limits of numerical sequences; limits and continuity, properties of continuous functions on closed bounded intervals; derivatives in one variable; the mean value theorem; Sequences of functions, power series, point-wise and uniform convergence. Functions of several variables: open and closed sets and convergence of sequences in R^n; limits and continuity in several variables, properties of continuous functions on compact sets; differentiation in n-space; the Taylor series in R^n with applications; the inverse and implicit function theorems.

Recommended Books:

Bartle RG, Sherbert DR, *Introduction to Real Analysis* (3rd edition), 1999, John Wiley, New York

Brabenec RL, *Introduction to Real Analysis*, 1997, PWS Publishing Company

Gaughan ED, *Introduction to Analysis* (5th edition), 1997, Brooks/Cole

Rudin W, *Principles of Mathematical Analysis* (3rd edition), 1976, McGraw Hill, New York

REAL ANALYSIS II

Prerequisite(s): Real Analysis I

Credit Hours: 2 + 0

Specific Objectives of the Course: A continuation of Real Analysis I, this course rigorously develops integration theory. Like Real Analysis I, Real Analysis II emphasizes proofs.

Course Outline: Series of numbers and their convergence. Series of functions and their convergence. Dabroux upper and lower sums and integrals; Dabroux integrability. Riemann sums and the Riemann integral. Riemann integration in R^2, change of order of variables of integration. Riemann integration in R^3, and R^n. Riemann-Steiltjes integration. Functions of bounded variation. The length of a curve in R^n

Recommended Books:

Bartle RG, Sherbert DR, *Introduction to Real Analysis* (3rd edition), 1999, John Wiley, New York
Brabenec RL, *Introduction to Real Analysis*, 1997, PWS Publishing Company
Fulks W, *Advanced Calculus*, John Wiley, New York (suggested text)
Gaughan ED, *Introduction to Analysis* (5th edition), 1997, Brooks/Cole
Rudin W, *Principles of Mathematical Analysis* (3rd edition), 1976, McGraw Hill, New York

VECTOR AND TENSOR ANALYSIS

Prerequisite(s): Calculus II

Credit Hours: 3 + 0

Specific Objectives of the Course: This course shall assume background in calculus. It covers basic principles of vector analysis, which are used in mechanics

Course Outline: 3-D vectors, summation convention, kronecker delta, Levi-Civita symbol, vectors as quantities transforming under rotations with $\in_{ijk}$ notation, scalar- and vector-triple products, scalar- and vector-point functions, differentiation and integration of vectors, line integrals, path independence, surface integrals, volume integrals, gradient, divergence and curl with physical significance and applications, vector identities, Green's theorem in a plane, divergence theorem, Stokes' theorem, coördinate systems and their bases, the spherical-polar- and the cylindrical-coördinate meshes, tensors of first, second and higher orders, algebra of tensors, contraction of tensor, quotient theorem, symmetric and skew-symmetric tensors, invariance property, application of tensors in modeling anisotropic systems, study of physical tensors (moment of inertia, index of refraction, etc.), diagnolization of inertia tensor as aligning coördinate frame with natural symmetries of the system

Recommended Books:

Bourne DE, Kendall PC, *Vector Analysis and Cartesian Tensors* (2nd edition), Thomas Nelson
Shah NA, *Vector and Tensor Analysis*, 2005, A-One Publishers, Lahore
Smith GD, *Vector Analysis*, Oxford University Press, Oxford
Spiegel MR, *Vector Analysis*, 1974, McGraw Hill, New York

BS Compulsory Mathematics Courses
(for students not majoring in mathematics)

MATHEMATICS I (ALGEBRA)
Prerequisite(s): Mathematics at secondary level
Credit Hours: 3 + 0
Specific Objectives of the Course: To prepare the students, not majoring in mathematics, with the essential tools of algebra to apply the concepts and the techniques in their respective disciplines.
Course Outline:
Preliminaries: Real-number system, complex numbers, introduction to sets, set operations, functions, types of functions.
Matrices: Introduction to matrices, types, matrix inverse, determinants, system of linear equations, Cramer's rule.
Quadratic Equations: Solution of quadratic equations, qualitative analysis of roots of a quadratic equations, equations reducible to quadratic equations, cube roots of unity, relation between roots and coefficients of quadratic equations.
Sequences and Series: Arithmetic progression, geometric progression, harmonic progression.
Binomial Theorem: Introduction to mathematical induction, binomial theorem with rational and irrational indices.
Trigonometry: Fundamentals of trigonometry, trigonometric identities.
Recommended Books:
Dolciani MP, Wooton W, Beckenback EF, Sharron S, *Algebra 2 and Trigonometry*, 1978, Houghton & Mifflin, Boston (suggested text)
Kaufmann JE, College *Algebra and Trigonometry*, 1987, PWS-Kent Company, Boston
Swokowski EW, *Fundamentals of Algebra and Trigonometry* (6^{th} edition), 1986, PWS-Kent Company, Boston

MATHEMATICS II (CALCULUS)
Prerequisite(s): Mathematics I (Algebra)
Credit Hours: 3 + 0
Specific Objectives of the Course: To prepare the students, not majoring in mathematics, with the essential tools of calculus to apply the concepts and the techniques in their respective disciplines.
Course Outline:
Preliminaries: Real-number line, functions and their graphs, solution of equations involving absolute values, inequalities.
Limits and Continuity: Limit of a function, left-hand and right-hand limits, continuity, continuous functions.

Derivatives and their Applications: Differentiable functions, differentiation of polynomial, rational and transcendental functions, derivatives.
Integration and Definite Integrals: Techniques of evaluating indefinite integrals, integration by substitution, integration by parts, change of variables in indefinite integrals.

Recommended Books:

Anton H, Bevens I, Davis S, *Calculus: A New Horizon* (8th edition), 2005, John Wiley, New York
Stewart J, *Calculus* (3rd edition), 1995, Brooks/Cole (suggested text)
Swokowski EW, *Calculus and Analytic Geometry*, 1983, PWS-Kent Company, Boston
Thomas GB, Finney AR, *Calculus* (11th edition), 2005, Addison-Wesley, Reading, Ma, USA

MATHEMATICS III (GEOMETRY)

Prerequisite(s): Mathematics II (Calculus)
Credit Hours: 3 + 0
Specific Objectives of the Course: To prepare the students, not majoring in mathematics, with the essential tools of geometry to apply the concepts and the techniques in their respective disciplines.

Course Outline:

Geometry in Two Dimensions: Cartesian-coördinate mesh, slope of a line, equation of a line, parallel and perpendicular lines, various forms of equation of a line, intersection of two lines, angle between two lines, distance between two points, distance between a point and a line.
Circle: Equation of a circle, circles determined by various conditions, intersection of lines and circles, locus of a point in various conditions.
Conic Sections: Parabola, ellipse, hyperbola, the general-second-degree equation

Recommended Books:

Abraham S, Analytic Geometry, Scott, Freshman and Company, 1969
Kaufmann JE, College *Algebra and Trigonometry*, 1987, PWS-Kent Company, Boston
Swokowski EW, *Fundamentals of Algebra and Trigonometry* (6th edition), 1986, PWS-Kent Company, Boston

BS Customized Mathematics Courses
(for students majoring in various disciplines)

Following a suggestion by Member, HEC, Professor Dr. Riaz-ul-Haq Tariq, NCRC is suggesting that the following points be emphasized, while teaching compulsory mathematics courses to students majoring in biology, chemistry, linguistics, physics and social sciences:

MATHEMATICS FOR BIOLOGISTS

The course(s) should make the student:

a) Identify the essential mathematical tools to appreciate biological processes, *e. g.*, exponential functions to model cell growth, logarithmic functions to model ear response

b) Learn mathematical techniques in physiological modeling, *e. g.*, equation of straight line to predict height and weight of child and compute growth velocity

c) Be exposed to basic statistical tools, *e. g.*, concept of percentile to interpret growth curves

d) Mathematics of equilibrium, steady state and non-equilibrium (as rates of energy transfer as well as probability of occupation) to study physiological processes

MATHEMATICS FOR CHEMISTS

The course(s) should make the student:

a) Visualize of periodic table as a matrix with rows representing number of shells and columns representing valiancy

b) Appreciate that balancing a chemical equation as a problem of solving system of linear equations

c) Interpret the law of mass action (to determine reaction rates) as a special case of law of conservation of energy

d) Recognize structures (tetrahedron, hexagon, cube, cuboid) for the study of molecules and lattices

e) Use polynomial equations of order 3 or more to compute reaction rates

f) Develop understanding of curvilinear coördinates (cylindrical, spherical polar) to model various chemical processes, *e. g.*, flow in a pipe, computation of surface tension of a water drop

MATHEMATICS FOR LINGUISTS

The course(s) should make the student:

a) Appreciate symmetry in nature and recognize in rhyming

b) Understand circular functions and Fourier series to get the concept of harmonics in generating difference in sound produced by various instruments as well as recognition of speech of different persons

c) Get the basic knowledge of functional analysis to establish one-to-one correspondence between the written word and the spoken word.

MATHEMATICS FOR PHYSICISTS

The course(s) should make the student:

a) Realize that visualizing form of mathematical equation describing a physical system would give insight into the nature of the associated physical system or process, in particular, properties of spacetime, which are expressed in the form of scalar or tensor (relationship of eccentricity of orbit to energy and latus rectum to angular momentum in two-body problem, diagonilization of inertia tensor equivalent to rotating axes to align with axes of symmetry – principal axes of inertia, form of electric-susceptibility tensor for nonlinear, linear-isotropic, linear-isotropic-homogeneous dielectrics)

b) Appreciate the situations, where mathematics brings in new physics (negative eigenvalues of energy and suggestion of existence of positron, discovery of omega meson from group-theory prediction, experimental verification of bending of light near massive objects from a prediction of general theory of relativity, mathematical predication of expansion of universe, although not realized at that time), and where physics generates new mathematics (bosy numbers to model bosons, determinant of general tensor to model relativistic kinemetics, astrodynamical coördinates to model planetry orbits)

c) Understand that setting up a physical problem closer to natural symmetries of a system (using appropriate curvilinear coördinates) shall render the equation simple to visualize and to handle (*e. g.*, using plane-polar and spherical-polar coördinates for problems having circular and spherical symmetries, respectively)

d) Apply exponentials to study of radioactive decay, charging of a capacitor, etc.

e) Obtain solutions of the second-order-linear-differential equation to study damped and driven oscillations and appreciate the phenomenon of resonance

MATHEMATICS FOR SOCIAL SCIENTISTS

The course(s) should make the student:

a) Learn basic concepts of calculus to model changes in attributes and opinions

b) Know basics of statistics to design and to analyze surveys, *e. g.*, concept of percentile to interpret standardized tests

The BS Electives (Alphabetical Listing)

Abstract Algebra I
Advanced Calculus I
(Numerical Solutions of PDE)
Advanced Numerical Analysis I
Astronomy I
Differential Geometry I
Electromagnetism I
Fluid Dynamics I
Group Theory I
(Fundamentals)
History and Philosophy of Mathematics I
Measure Theory I
Modern Algebra I
(Galios Theory & Applications)
Nonlinear Systems I
Numerical Solutions of Partial-Differential Equations I
Operations Research I
Projective Geometry I
Quantum Mechanics I
Relativity I
Software Engineering I
(Design & Development)
Theory of Processes I
(Stochastic Processes)
Theory of Splines I
Topology I
(Topological-Dimension Theory)

Abstract Algebra II
Advanced Calculus II
(Integral Equations)
Advanced Numerical Analysis II
Astronomy II
Differential Geometry II
Electromagnetism II
Fluid Dynamics II
Group Theory II
(Study of Symmetries)
History and Philosophy of Mathematics II
Measure Theory II
Modern Algebra II
(Commutative Rings & Fields)
Nonlinear Systems II
Numerical Solutions of Partial-Differential Equations II
Operations Research II
Projective Geometry II
Quantum Mechanics II
Relativity II
Software Engineering II
(Analysis)
Theory of Processes II (Renewal Processes & Theory of Ques)
Theory of Splines II
Topology II
(Differential Topology)

THE MASTERS PROGRAM IN MATHEMATICS

Degree Awarded:	**Master of Studies (MS): Two-Year Program**
Rationale:	Numerical interpretation (ability to formulate a problem, mathematically) is one of the most desirable skills needed in the R&D projects and the academia.
Objectives:	The MS scheme of studies aims to enhance the concepts, developed in BS and refine the techniques, learnt in BS, at the same time inviting the student to explore conflicting ideas, thoughts and views. After completing the MS program, the students should:
a)	be able to formulate a problem, mathematically.
b)	be able to determine if the formulated problem can be solved or not (qualitative analysis).
c)	be able to see the relationship of mathematics with other disciplines.
Entrance Requirements:	BS (Mathematics) or equivalent; GRE (General); cutoff scores 50th percentile (each section); written test; personal interview
Duration of the Program:	2 years (4 semesters), students must submit thesis at the end of 2 years of study
Total Credit Hours:	60
Course Requirements:	24 credit hours [Research Methodology + 15-credit-hour elective courses in the area of specialization (Elective A) and 6-credit-hour free electives (Elective B) + Mathematics Teaching Techniques]; more courses could be prescribed by the supervisor depending on the results of masters examinations; students coming from allied disciplines have to complete additional courses; courses to be taken in consultation with the supervisor

Masters Examinations:	Masters' Examinations (written + oral) must be passed before starting thesis research; any student, who fails to qualify these examinations in 2 attempts, or by the end of first year of study, is asked to withdraw from the program. Masters' examinations are to be offered by the department 4 times during an academic year. Written examination may consist of selected questions from PhD Qualifying Examination, testing the student in core areas. Oral examination, testing the student in the area of specialization, is scheduled only after the student passes the written examination. During the oral examination the student is, also, required to describe the proposed research work. Committees formed by Board of Advanced Studies and Research conduct these examinations.
Thesis Requirements:	Student must write and publicly defend a thesis bearing original work, which should have enough substance to result in at least one publishable paper in a journal of international repute.
Mathematics Undergraduate Teaching:	Student acts as Associate Instructor (Teaching Assistant) in a BS course. This is an opportunity for a student to gain teaching experience at the university level. Each course taught by the student earns a practical-experience credit and is graded according to at least two of the following evaluation criteria: *a)* Feedback from students *b)* Lecture/Tutorial/Problem-solving session observation by senior teachers *c)* Videotaped lecture/tutorial/problem-solving session evaluation by experts
Summer Activity:	Students, who have passed Masters Examination, may start independent study to find thesis problem during the first year. During the second year the students shall be involved in thesis research.
Qualifications for Course Supervisors (Instructors):	Faculty holding PhD degree is entitled to teach lecture session; the electives offered are taken by both MS and PhD students

Qualifications for Associate Instructors (Teaching Assistants):	Associate Instructors (Teaching Assistants) must hold MS (Two-Year Program), and may handle laboratory, discussions as well as assignment, quiz and problem grading
Pedagogical Techniques:	Lecture sessions of each unit (normally, 2-3 lectures) are followed by a discussion session (reinforcing the concepts taught through examples, alternate derivations and proofs, innovative solutions to the problems), conducted by the Associate Instructor. In addition, a review session should be arranged prior to each monthly test and a comprehensive review before the final examination, both sessions conducted by the Course Supervisor.
Continuous-Evaluation Report:	Same as in BS Scheme of Studies with the difference that the report is discussed ONLY with the student and NOT with the student's parents.
Marks Breakdown:	Passing grade is set at 60%. Marks breakdown and GPA assignment same as in BS Scheme of Studies, with the difference that the numerical grade 0-59 corresponds to alphabetical grade F, with grade point zero. There is no D, D^+ or C^- for graduate courses.

Academic standards, attendance requirements, syllabus breakdown, guest/students' seminars, withdrawl policy, final grades and grading of problem sets same as in the BS Scheme of Studies. There shall be no choice in the papers of final examination for MS.

Table 6. The MS Scheme of Studies: Semester-Wise Breakdown

FIRST YEAR

First Semester

#	*Course Title*	*Cr. Hr.*
1	Elective-A I	3 + 0
2	Elective-A II	3 + 0
3	Elective-A III	3 + 0
4	Guest /Students' Seminar I	1 + 0
5	Masters-Examination Preparation I	Non-Credit
6	Math. Teaching Tech.	2 + 0
7	Mathematics Undergraduate Teaching I	0 + 3
	Total	**12 + 3**

Second Semester

#	*Course Title*	*Cr. Hr.*
1	Elective-A IV	3 + 0
2	Elective-A V	3 + 0
3	Guest /Students' Seminar II	1 + 0
4	Independent Study	2 + 0
5	Masters-Examination Preparation II	Non-Credit
6	Mathematics Undergraduate Teaching II	0 + 3
7	Research Methodology	3 + 0
	Total	**12 + 3**

Learning Outcomes: After first year the students should have a good knowledge in a specific area of pure/ applied mathematics. They should know in detail 2-3 books, which are classics in the field. They should be able to read and to understand 2-3 mathematical papers published in the well-known journals during the last 5 years.

SECOND YEAR

Third Semester

#	*Course Title*	*Cr. Hr.*
1	Elective-B I	3 + 0
2	Guest /Students' Seminar III	1 + 0
3	Mathematics Undergraduate Teaching III	0 + 3
4	Thesis Research I	8 + 0
	Total	**12 + 3**

Fourth Semester

#	*Course Title*	*Cr. Hr.*
1	Elective-B II	3 + 0
2	Guest/Students' Seminar IV	1 + 0
3	Mathematics Undergraduate Teaching IV	0 + 3
4	Thesis Research I	8 + 0
	Total	**12 + 3**

Learning Outcomes: After second year, the students should be able to write a paper as a result of their own investigations of a research problem.

Note: The students who have passed Masters' Examinations before the start of Second Semester shall not be attending "Masters-Examination Preparation II"

RESEARCH METHODOLOGY
Prerequisite(s): Mathematical Statistics
Credit Hours: 3 + 0
Specific Objectives of the Course: This course is supposed to introduce the student with research tools and methods and particularly geared for mathematics research, both pure and applied. Student is supposed to choose a research problem, prepare and present a research proposal, and write a review/research paper (8-10 pages) in format HEC-approved research journal in mathematics.
Course Outline:
Scientific Method: Scientific statement, hypothesis, model, theory and law, steps in problem solving, limitation of science, calibration (mathematical definition: conversion of output, which is not desired, to desired output through a linear/nonlinear model), sensitivity, least count, reproducibility (stability and objectivity), difference between accuracy and precision, role of mathematician in industrial, business, financial and health-care research, challenges of research in pure mathematics
Research Proposals: Identifying problem, literature search, defining problem, feasibility study, pilot projects/field trials, formal research proposal, budgeting and funding, progress report, final technical and fiscal report
Experiment Design: Purpose of an experiment, good, bad and inefficient experiments, null and alternative hypothesis, α and β errors and their relationship to sensitivity and specificity, designing efficient experiments, calculation of sample size, identifying variables of interest and their interactions, operating-characteristic curves, power curves, surveys and field trials
Publishing Original Paper: Submission, role of the editor, peer-review process, importance of citations, impact factor, plagiarism, protection of your work from misuse
Recommended Books:
Day RA, *How to Write and Publish a Scientific Paper*, 1979, ISI Press, Philadelphia, Pa., USA
Diamond WJ, *Practical Experiment Designs for Scientists and Engineers* (2nd Edition), 1989, John Wiley, New York
Harré R, *Great Scientific Experiments: Twenty Experiments that Changed the World*, 2002, Dover, New York

The MS Electives (Alphabetical Listing)

3-D Optical Imaging & Image Processing I
Advanced Astronomy I (Solar-System Astrophysics)
3-D Optical Imaging & Image Processing II
Advanced Astronomy II (Cosmology)

Advanced Differential Equations I	Advanced Differential Equations II
Advanced Fluid Dynamics I	Advanced Fluid Dynamics II
Advanced Functional Analysis I	Advanced Functional Analysis II
Advanced Group Theory I (Abelian Groups)	Advanced Group Theory II (Soluble & Nilpotent Groups)
Advanced Operational Research I	Advanced Operational Research II
Advanced Quantum Mechanics I	Advanced Quantum Mechanics II
Advanced Relativity I	Advanced Relativity II
Advanced Topology I (Topological-Dimension Theory)	Advanced Topology II (Differential Topology)
Almost Periodic Functions I	Almost Periodic Functions II
Astrodynamics I	Astrodynamics II
Atmospheric Study I	Atmospheric Study II
Biomathematics & Bioinformatics I	Biomathematics & Bioinformatics II
Classical Electrodynamics I	Classical Electrodynamics II
Classical Field Theory I	Classical Field Theory II
Combinatorics & Measure Theory I	Combinatorics & Measure Theory II
Continuum Mechanics I	Continuum Mechanics II
Control Theory I (Basic Principles)	Control Theory II (Design of Control System)
Differential Geometric Techniques I	Differential Geometric Techniques II
Experiment Designs I	Experiment Designs II
Fourier Series I	Fourier Series II
Gauge Theory of Gravitation I	Gauge Theory of Gravitation II
General Topology I	General Topology II
Homological Algebra I	Homological Algebra II
Lie Algebra I	Lie Algebra II
Near Rings I	Near Rings II (Special Classes)
Plasma Dynamics I	Plasma Dynamics II
Quantum Field Theory I	Quantum Field Theory II
Reliability Analysis I	Reliability Analysis II
Representation Theory I	Representation Theory II
Special Classes of Rings I	Special Classes of Rings II
Summability Theory I	Summability Theory II
System Analysis I	System Analysis II
Theory of Rings I	Theory of Rings II
Topics in Astronomy I (Stellar & Galactic Astronomy)	Topics in Astronomy II (Extra-Galactic Astronomy)
Topics in Numerical Analysis I	Topics in Numerical Analysis II
Topological Groups I	Topological Groups II

THE DOCTORAL PROGRAM IN MATHEMATICS

Degree Awarded:	**Doctor of Philosophy (PhD): Two-Year Program**
Rationale:	Ability to propose solution to problem, employing techniques of mathematics, is one of the most desirable skills needed in the R&D projects and the academia.
Objectives:	The PhD scheme of studies aims to be broad based, challenging the student to critically analyze the different ideas and develop one's own school of thought. After completing the PhD program, the students should:
a)	be able to propose innovative, efficient and effective solutions of problems using mathematical skills.
b)	be able to link mathematics with other disciplines.
c)	be prepared to take up careers in research and teaching in a university or other institution of advanced learning
Entrance Requirements:	MS in Mathematics or equivalent; GRE (General) and GRE (Mathematics); cutoff scores 50th percentile (each section for the former); personal interview
Duration of the Program:	2 years (4 semesters); students cannot submit dissertation before completing 2 years of study; admission shall be valid till 7 years of initial enrolment
Total Credit Hours:	60
Course Requirements:	18 credit hours [12-credit-hour elective courses in the area of specialization (Elective A) and 6-credit-hour free electives (Elective B)]; students coming from allied disciplines or possessing MPhil without course work/less than 8 courses have to complete additional courses.

Qualifying Examinations:	Qualifying Examinations (written + oral) must be passed before starting dissertation research; any student who fails to qualify these examinations in 2 attempts, or by the end of first year of study is asked to withdraw from the program. Qualifying examinations are to be offered by the department 4 times during an academic year. Written examination tests the student in core areas. Oral examination, testing the student in the area of specialization, is scheduled only after the student passes the written examination. The student is, also, required to describe the proposed research work. Committees formed by Board of Advanced Studies and Research conduct these examinations.
Dissertation Requirements:	Student must write and publicly defend a dissertation bearing original work, which should result in at least 2 publishable papers in journals of international repute (acceptance of papers required for award of degree)
Mathematics Graduate Teaching:	Student acts as Associate Instructor (Teaching Assistant) in an MS/a PhD course, which the student has, already, passed. This is an opportunity for a student to gain teaching experience at the university level. Each course taught by the student earns a practical-experience credit and is graded according to criteria described in MS scheme .
Independent Study:	Supervised by a faculty member, which may help student find dissertation problem
Summer Activity:	Students, who have passed Qualifying Examination, may start dissertation research during the first year. They will be engaged in full-time research during the subsequent summers to finish their degrees in time.

Class size, academic standards, attendance requirements, syllabus breakdown, guest/students' seminars, withdrawl policy, final grades and grading of problem sets same as in the BS Scheme of Studies.

Qualifications for Course Supervisors (Instructors), qualifications for Associate Instructors (Teaching Assistants), pedagogical techniques, continuous-evaluation report, marks breakdown and final examinations same as in the MS Scheme of Studies.

Table 1. The PhD Scheme of Studies: Semester-Wise Breakdown

FIRST YEAR

First Semester

#	*Course Title*	*Cr. Hr.*
1	Elective-A I	3 + 0
	Elective-A II	3 + 0
2	Elective-B I	3 + 0
3	Guest/Students' Seminar I	1 + 0
4	Independent Study I	2 + 0
5	Mathematics Graduate Teaching I	0 + 3
6	Qualifying-Examination Preparation I	Non-Credit
	Total	**12 + 3**

Second Semester

#	*Course Title*	*Cr. Hr.*
1	Elective-A III	3 + 0
	Elective-A IV	3 + 0
2	Elective-B II	3 + 0
3	Guest/Students' Seminar II	1 + 0
4	Independent Study II	2 + 0
5	Mathematics Graduate Teaching II	0 + 3
6	Qualifying-Examination Preparation II	Non-Credit
	Total	**12 + 3**

SECOND YEAR

Third Semester

#	*Course Title*	*Cr. Hr.*
1	Dissertation Research I	11 + 0
2	Guest/Students' Seminar III	1 + 0
3	Mathematics Graduate Teaching III	0 + 3
	Total	**12 + 3**

Fourth Semester

#	*Course Title*	*Cr. Hr.*
1	Dissertation Research II	11 + 0
2	Guest/Students' Seminar IV	1 + 0
3	Mathematics Graduate Teaching IV	0 + 3
	Total	**12 + 3**

Notes:

i) List of PhD electives is included with the MS Scheme of Studies.

ii) The students who have passed Qualifying Examinations before the start of Second Semester shall not be attending "Qualifying-Examination Preparation II"

RECOMMENDATIONS
(to make the curriculum-development effort effective)

National-Curriculum-Revision Committee in Mathematics, unanimously, made the following recommendations:

i) Every public/private sector university should have a mathematics department employing full-time faculty.

ii) HEC should pay for the membership of at least one international professional society membership and subscription of at least two international journals for each active faculty member.

iii) National conferences in mathematics should have a session devoted to oral and poster presentations of BS students.

iv) National conferences in mathematics should have a session devoted to pedagogical skills in mathematics.

v) BS students presenting paper in international conference should receive monetary incentives.

vi) MS/PhD students publishing paper in an international journal should receive monetary incentives.

viii) Every oral and poster presentation as well as journal publication in mathematics by multiple authors must, clearly, describe the nature and the type of contribution made by each author. The presentation or publication should, also, mention the names of study supervisor and study advisor(s), wherever applicable.

viii) Course load for Professor, Associate Professor, Assistant Professor and Lecturer should be 3-credit hours, 6-credit hours 9-credit hours and 12-credit hours, respectively.

ix) For selection/promotion, teaching experience should be counted after 16 years of schooling.

x) Existing faculty members should be selected/promoted to higher grades according to existing criteria. New faculty members may be inducted according to revised criteria.

xi) There should be equal salaries for Mathematics and IT teachers.

xii) Mathematics-research-impact factor should be evaluated separately from other disciplines.

xix) Top 3 position holders in BS (Mathematics) of public-sector universities should be awarded scholarships to pursue higher education in the country or abroad.

xiii) There should be a provision of teachers-training program (philosophy + contents + pedagogical techniques) to prepare manpower for BS, MS and PhD programs proposed in this document.

xiv) In order to make these programs successful, faculty satisfaction is essential. Hence, a committee should be formed to formulate career-development protocols for the faculty.

xiv) Textbook writing by Pakistani experts to be encouraged, monetary incentives and sabbatical leave to bring out quality publications; young faculty members should be compiling their lecture notes as e-books, which could come out regular hard-copy books after they have been teaching the course for 5 consecutive years.

xv) Evaluation of thesis should be by experts in the field, who may reside in or outside Pakistan. The competence of expert is not to be determined by the expert country of residence, but the number of publications and their impact factor in the area during the last 5 years.

xvi) Examination systems must also be modernized in order to maintain standards. A separate forum should chart out examination-conduct procedures, pre- and post-examination exercises to conduct the examinations smoothly, with efficiency and effectiveness.

xvii) Four-Year BS Program in Mathematics should, also, be implemented in Engineering Universities.

xviii) Effectiveness of the Four-Year BS Program implemented in various universities should be evaluated.

*Web address of this document (on convener's homepage)***:**
http://www.ngds-ku.org/hec/math-booklet-final-2008.pdf

