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Two-Body Problem in the Elliptic-Astrodynamical-Coördinate Mesh

Syed Arif Kamal and Sabahat Latif*
Department of Mathematics, University of Karachi, Karachi 75270, Pakistan; *profdrakamal@gmail.com

Fig. 1. Prof. Dr. Syed Arif Kamal delivering his invited lecture during the Plenary Session, in which a new branch of mathematics was introduced, named as *astromathematics*

The plane-polar coördinates are not the natural choice for setting up two-body problem. The elliptic-astrodynamical coördinate mesh (ξ, E, z) — the first one is ellipse-shape coordinate, $\frac{1}{2ae}\ln\frac{1+\in}{1-\in}$ (a is semi-major axis of elliptical orbit, e is eccentricity, $\in = \sqrt{1-e^2}$), the second one is elliptic-eccentric anomaly, the third one is representing direction of angular momentum of the orbit — have been custom-designed to handle orbital problems based on central-force motion of two bodies (http://www.ngds-ku.org/pub/confabst.htm#C60:), which exhibited 12 degrees-of-freedom (3 translational and 3 rotational degrees-of-freedom for each of the two bodies). Mathematical framework available at present could not generate exact solutions for more than 2 degrees-of-freedom. The 12 degrees were reduced to 6 by considering each of the 2 bodies as point masses and neglecting structures. Further, setting up the problem in the center-of-mass (CM) frame-of-reference, separated the problem into 2 terms, the first one represented motion of CM and the second motion about CM. Realizing that CM was either at rest or moving with a uniform velocity in the absence of external forces, the first term was constant and, hence, played no role in the lagrangian equations, which, only, involved derivatives. Hence, the problem was reduced to 3 degrees-of-freedom. In the absence of external torques, angular momentum (a vector quantity having both magnitude and direction) was conserved. A fixed direction of angular momentum in space forced two-body orbits to lie in a plane, making the problem 2 dimensional. The orbital equation of motion was formulated using the elliptic-astrodynamical-coördinate mesh, evolved from the elliptic-cylindrical coördinates. This, further, reduced the degrees-of-freedom and the problem became a true one-parameter problem. Using a special substitution, the first integral of the dynamical equation (a nonlinear one) was obtained. Kepler's equation was shown to a particular solution http://www.ngds-ku.org/Papers/C56.pdf of the resulting dynamical equation. In addition, scale factors of the elliptic-astrodynamical-coördinate mesh were obtained as: $h_\xi = \frac{A}{B}$; $h_E = a\sqrt{1-e^2\cos^2 E}$; $h_z = 1$ ($A = 1-e\cos E$, $B = \frac{\xi}{a}$). This formulation brought out 3 constants of motion, instead of the customary 2. Control laws used in maneuvering targeted spacecrafts and satellites, the normal-component-dot-product steering http://www.ngds-ku.org/Papers/C55.pdf and the ellipse-orientation steering http://www.ngds-ku.org/pub/confabst.htm#C64:, were expressed using this formulation, which could be used to check and correct deviations from the guidance path using the cross-range-corrected- http://www.ngds-ku.org/Papers/C67.pdf and the multi-stage- http://www.ngds-ku.org/Papers/C72.pdf Lambert schemes as well as the multi-stage-Q system http://www.ngds-ku.org/Papers/C66.pdf. Technological implications of this formulation included *Air-Spacecraft of the Third Millennium*, a thematic aircraft designed after taking into account green-engineering principles http://www.ngds-ku.org/Presentations/ASTM.pdf. Mathematical structure of this formulation forms the basis of ***Astromathematics***, which focuses on geometry to study orbits from a kinematical perspective. In contrast to *astrodynamics*, the force expressions do not, explicitly, appear in the formulation of astromathamatics. Even if there appears a need to study force interactions, these were expressed as space-time-curvature equivalents. This formulation seems to be, generically, more suitable for accelerated frames governed by *geometrodynamics*, based on general theory of relativity.

Keywords: Two-body problem, keplarian motion, elliptic-astrodynamical-coördinate mesh, equation of motion
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*PhD; MA, Johns Hopkins, Baltimore, United States; Former Senior Scientific Officer, Control-Systems Laboratories and Consultant, Guidance, Navigation and Control Laboratories, both at SUPARCO (Plant), Pakistan Space and Upper Atmosphere Research Commission; Visiting Faculty, Department of Aeronautics and Astronautics, Institute of Space Technology; Interdepartmental Faculty, Institute of Space and Planetary Astrophysics, University of Karachi; *paper mail*: Professor, Department of Mathematics and Karachi 75270, Pakistan; *telephone*: +92 21 9926 1300-15 ext. 2293; *homepage*: http://www.ngds-ku.org/kamal