

Cross-Lattice Formulation Applied to Liquid-Crystal Structure

Syed Arif Kamal[¶], Muhammed Khalid Rajput[#] and Shakeel Ahmed Ansari[#]

[¶]Department of Mathematics and [#]Department of Physics, University of Karachi; profdrakamal@gmail.com

Reciprocal lattice is, widely, used in condensed-matter physics and materials science. The expressions of reciprocal-lattice vectors, having dimensions of inverse of length, are changed, <http://www.ngds-ku.org/pub/confabst2.htm#C03>: when different systems of units are used. In order to alleviate this problem, a modified lattice, termed as *cross lattice*, <http://www.ngds-ku.org/Papers/J07.pdf> was introduced. The cross-lattice vectors were constructed in such a way that they became independent of the coördinate system chosen. A coördinate-independent representation is a pre-requisite of representing generalized laws and forms the basis of principle of general covariance <http://www.ngds-ku.org/Presentations/CIIT.pdf> — this was the motivation to present the most general coördinate transformation. Let $\hat{a}_1, \hat{a}_2, \hat{a}_3$, be the basis vectors for ordinary lattice. Any vector L , may be represented in terms of these linearly-independent bases $L = L_k \hat{a}_k$ (repeated indices denote summation according to Einstein convention;



Fig. 1. Professor Dr. Syed Arif Kamal delivering his invited lecture during the Fourth Session, in which a new branch of mathematics was introduced, named as *condensed-matter mathematics*

all latin indices take up the values 1, 2, 3). The reciprocal-lattice vectors for a non-rectangular lattice, may be constructed, using the basis vectors (ϵ_{pqr} and ϵ_{klm} are Levi-Civita symbols)

$$\hat{g}_k = \epsilon_{klm} \frac{\hat{a}_l \times \hat{a}_m}{\epsilon_{pqr} a_p a_q a_r}$$

as $G = G_k \hat{g}_k = 2\pi n_k \hat{g}_k$; the numbers n_k are integers, this condition is obtained by applying the periodicity condition $f(r + L) = f(r)$. The reciprocal lattice vectors satisfy the condition $\hat{a}_i \cdot \hat{g}_j = \delta_{ij}$; where δ_{ij} is Kronecker delta. A cross product of the direct- and the reciprocal-lattice vectors is dimensionless, and named as *cross lattice*, $\hat{e}_i = \hat{a}_i \times \hat{g}_i$ (no summation over i). The cross-lattice length can be written as $D = D_k \hat{e}_k = 2\pi n_k \hat{e}_k$. The following results can, easily, be derived from the above definition, $D \cdot L = D \cdot G = 0$; $\exp(iD \cdot L) = \exp(iD \cdot G) = 1$ ($i = \sqrt{-1}$). The eigenfunctions of cross

lattice can be obtained by considering an anisotropic potential, which depends on angle, θ . The band must be well separated to make non-diagonal matrix elements vanish with respect to band index. Defining a modified Brillouin zone to a Wigner-Seitz cell in cross lattice, and letting D take all values inside this zone, all the coefficients vary and become functions of D . However, because of the periodicity in the momentum space, they are all identical functions of D , <http://www.ngds-ku.org/pub/confabstA.htm#C91>: translated in cross-lattice space. Wannier functions are useful for systems, which are *isotropic*, but *inhomogeneous*. The cross eigenfunctions, belonging to eigenstates of angular momentum in cross-lattice representation, are useful for systems, which are *homogeneous*, but *anisotropic*. Such systems are found in liquid crystals. We, now, have the necessary techniques available to study these systems. Future work may involve mathematical derivation of eigenfunctions, which are both *inhomogeneous* and *anisotropic*. Application of mathematical techniques to problems in solid-state physics may bring out new avenues of research, forming the basis of *Condensed-Matter Mathematics*.

Keywords: Reciprocal lattice, cross lattice, momentum eigenfunctions, liquid crystals, materials science

Conflict of Interest Statement: No potential conflict of interest is identified for this work

Grant Sponsor: Dean's (Science) Research Grant, University of Karachi, number DFSR/2009

Web address of this document: <http://www.ngds-ku.org/Presentations/BZU.pdf>

HTML version: <http://www.ngds-ku.org/pub/confabstA.htm#C95>:

[¶]PhD (Mathematical Neuroscience); MA, Johns Hopkins, Baltimore, MD, United States; MS, Indiana, Bloomington, IN, United States; MSc (Mathematical Physics), *summa cum laude*, BSc (Honors), *summa cum laude*, University of Karachi; Head, Mathematical Physics Group, University of Karachi • *paper mail*: Professor and Chairman, Department of Mathematics, University of Karachi, PO Box 8423, Karachi 75270, Pakistan • *telephone*: +92 21 9926 1300-15 ext. 2293 • *homepage*: <http://www.ngds-ku.org/kamal> • life member PIP (Pakistan Institute of Physics)