

## EFFECTIVE DECISION MAKING FOR THE PRESENCE OF SCOLIOSIS<sup>¶</sup>

Syed Arif Kamal\*, Maqsood Sarwar and Urooj A. Razzaq<sup>§</sup>

*SF Growth-and-Imaging Laboratory, the NGDS Pilot Project and Anthromathematics Group,  
Department of Mathematics, University of Karachi, Karachi 75270, Pakistan; e-mail: profdrakamal@gmail.com*

---

### ABSTRACT

Scoliosis, a body-disfiguring disease, is associated with lateral curvatures and rotations of a person's spine. It is, generally, detectable around the age of 8 years. A two-minute-stripped-orthopedic examination of students, in the age group 7-10-years, may alert the health-care provider to early-warning signals, which are expressed as a mathematical index, named as 'Cumulative-Scoliosis-Risk Weightage (CSRW)'. CSRW is based on family history, age, statuses of being tall and/or wasted, forward-bending tests, non-alignment of plumb-line, shoulder drooping, uneven scapulae, shape of midline of back, unequal body triangles, uneven spinal dimples and positive moiré. A high CSRW calls for further examination before sending the child for X rays. Effective methods are needed to eliminate need for unnecessary X rays, which damage bone marrow of children. A mathematical model is proposed and tested on 7- and 8-year-old students of a local school. Four tests were conducted, visual (standing), visual (sitting), forward bending (standing) and forward bending (sitting) — postural problem suspected through positive visual examinations (standing and sitting), indicated through positive mid-stretching test; leg-length in-equality suspected through positive visual and forward-bending tests (both standing), indicated through uneven spinal dimples; hip weakness suspected through positive visual and forward-bending tests (both sitting), indicated through positive Tredelenburg sign; spinal rotation suspected through positive forward-bending tests (standing and sitting), indicated through positive moiré. This paper reports effectiveness of CSRW in predicting lateral curvatures and spinal rotations.

**Keywords:** Modeling of spinal column, spinal-deformity screening, cumulative-scoliosis-risk weightage

---

### LIST OF ABBREVIATIONS

*mm*: millimeter(s) • *cm*: centimeter(s) • *kg*: kilogram(s)

<b>AP:</b> Anteroposterior	<b>SF:</b> The Syed Firdous Growth-and-Imaging Laboratory, University of Karachi
<b>CSRW:</b> Cumulative-Scoliosis-Risk Weightage	<b>SGPP:</b> Sibling Growth Pilot Project — a subproject of the NGDS Pilot Project
<b>NGDS:</b> National Growth and Developmental Standards for the Pakistani Children	

### INTRODUCTION

From the ancient civilization man was interested in maintaining an upright posture. Military parades and exercises, also, serve the same purpose. The original meaning of 'orthopedics' is straight (ortho) child (pedics). Hence, surveillance of scoliosis in primary-school children is of utmost importance (Persson-Bunke *et al.*, 2012; Saikia *et al.*, 2002). Scoliosis is defined as lateral curvatures and rotations of the spinal column and is commonly checked through visual examination of back in the attention position and Adam's forward-bending test (Anderson, 2007; Sengupta and Webb, 2010).

The above-mentioned tests generate a large number of false positives in young children under the age of 11 years. These children are, then, subjected to X rays to either confirm or rule out scoliosis. Unnecessary X-ray exposure to children, who have a delicate bone marrow, is of concern to clinicians and they have been looking for alternate methods to limit the number of children, who are sent for X rays. Moiré fringe topography is one such technique, which highlights minimal left-right asymmetries on the back of child using ordinary light (non-ionizing radiation). In fact, AP-X ray of the spinal column and moiré fringe topography of the entire back provide different sets of information about the spinal column — the first one shows lateral curvatures and the second one rotations. Hence, it is recommended that both of these techniques should be used in conjunction to obtain a complete picture.

The world of orthopedic surgeons is going beyond study of localized curvatures to modeling of the entire spinal column, all the way to vertebral and sub-vertebral levels. This paper lists some attempts in this direction as well as mathematical modeling for scoliosis-risk indicators and a decision matrix to select cases for follow up using moiré fringe topography, rasterstereography and other imaging techniques.

<sup>¶</sup>Main contribution of PhD dissertation of the second author, registered from Department of Mathematics, University of Karachi.

\*PhD (Mathematical Neuroscience); MA, Johns Hopkins, Baltimore, MD, United States; Project Director, the NGDS Pilot Project; Associated Professor in Orthopedic Surgery, Malmö General Hospital, Sweden (1988); Research Associate in Orthopedic Surgery, James Whitecomb Riley Hospital for Children, Indianapolis, IN, United States (1980); Head, Anthromathematics Group, University of Karachi; *paper mail*: Professor and Chairman, Department of Mathematics, University of Karachi, Karachi 75270, Pakistan; *telephone*: +92 21 9926 1300-15 ext. 2293; *homepage*: <http://www.ngds-ku.org/kamal>; *project URL*: <http://ngds-ku.org>

<sup>§</sup>MPhil Candidate (Mathematics), Research Center for Mathematical Sciences, Federal Urdu University of Arts Sciences and Technologies (FUUAST), Gulshan-é-Iqbal Campus, Karachi 75330, Pakistan.

## MODELING OF THE SPINAL COLUMN

Anthromathematics of the human spinal column is going to be one of the most active areas of research in this century, which should involve modeling of the spinal column (Kamal *et al.*, 2013c).

### 2-D Modeling

2-D-spinal-column models should be able to generate frontal view from projections of spine obtained from AP-X rays or back moiré topographs in the attention position (Oxborrow, 2000). An attempt was made to determine Cobb angles from moiré topographs of back (Kamal, 1982a; El-Sayyad and Kamal, 1981).

### Need for 3-D Modeling

Spinal column is a structure, which exists in three dimensions. AP-X rays of the spinal column show the entire spinal column from the external auditory meatus to hip joint (patient in the attention position). These X-ray pictures exhibit spinal projection only in the frontal plane. Hence, they are not capable of showing kyphosis or lordosis. 3-D-spinal-column models should have the ability to synthesize full view from projections of spine in the frontal and the sagittal planes, generated from AP- and lateral-X-ray pictures or moiré topographs of back in the attention position (Kamal 1982b). Recently, Bella *et al.* (2014) have tried to define shape of spine using moiré method.

### 3-D-Static Modeling

3-D-static modeling was started simultaneously during 1982 in Germany (Hierholzer and Lüxmann, 1982) and in the United States (Kamal, 1982b; 1983 a; b). Natural curvatures of the spinal column, as seen in the sagittal-plane projection, were incorporated in a later work (Kamal, 1987). A comprehensive model was published in 1996, which is, briefly, described here (Kamal, 1996c). From X-ray or moiré measurements, a parabolic curve was generated, relating  $x$ ,  $y$  and  $z$ , where  $x = x(\xi)$ ,  $y = y(\xi)$ ,  $z = z(\xi)$ , best fitted to discrete measurements performed at different locations represented by the parameters,  $\xi_i$ ;  $i = 1, \dots, 33$ ; corresponding to 33 vertebrae of backbone, consisting of cervical, thoracic, lumbar and sacral regions. The parabolic curve was represented by

$$(1) \quad x = f(y, z) = \frac{1}{2}ay^2 + byz + \frac{1}{2}cz^2$$

The cross term ( $yz$ ) vanished if the coördinate system was rotated through an angle  $\alpha$  given by

$$(2) \quad \alpha = \frac{1}{2} \tan^{-1} \frac{2b}{c-a}$$

Double the coefficients of squares of  $y$  and  $z$ , then, gave curvatures. Degree of correction of spinal deformity was defined in terms of these curvatures, taking into account normal curvatures of the human spinal column. Trunk deformity was classified as ‘severe’, ‘intermediate’ or ‘mild’ depending on the value of degree of correction of spinal deformity in the ranges 0-33.33, 33.34-66.66, 66.67-100, respectively. This 3-D-static model was found to be useful in the study of posture of children.

### 3-D-Dynamic Modeling

The dynamic model was a generalization of 3-D-static model, to study movement of the human spinal column during a gait cycle (Kamal, 1996a; Yosufzai *et al.*, 1995). The human spinal column in three dimensions was generated from moiré topographs of back in the attention position as well as during each of the four steps of the gait cycle (Kamal, 1996d). Spinal column in the attention position was, then, linked to first step through edge-based algorithm (Kamal, 1996b). Similarly, position in the second step was linked to the first step through edge-based algorithm and so on.

### 3-D-Crystal-Structure-Based Modeling

The human spinal column was considered as a crystal structure — a collection of vertebrae in the cervical, the thoracic, the lumbar and the sacral regions, which are located at specific distances from each other. The center-of-mass of each vertebra was expressed in terms of positional coördinates ( $x$ ,  $y$ ,  $z$ ) in the body-coördinate system. It could be considered as ‘form factor’, being used in crystallography. Study of vertebral-surface structure using moiré fringe topography, which provides height map of surface of child’s back (Kamal *et al.*, 2013b), rasterstereography, which gives local curvatures at various points of child’s back (Hackenberg *et al.*, 2003a; b; 2006; Kamal *et al.*, 2013a) or the enhanced version, dotted-rasterstereography, which generates better information of these curvatures (Wasim *et al.*, 2013) as well as combined moiré, raster and backscatter-X ray (Kamal, 2013; Kamal *et al.*, 2014b), which draws height and curvature maps on spinal-column surface. If rotational (in terms of Euler angles) and inter-

vertebral-spacing information is added, the analysis may be visualized as ‘structure factor’, used by solid-state physicists to study crystal structure (Kamal *et al.*, 2012).

### CUMULATIVE-SCOLIOSIS-RISK WEIGHTAGE (CSRW)

There is a dire need to study factors associated with scoliosis in school children (Baroni *et al.*, 2015). A mathematical index ‘Cumulative-Scoliosis-Risk Weightage’ (CSRW) was devised by our group, which assigned a weight to each early-warning signal, *e. g.*, family history (scoliosis in parents or one of the siblings increases the risk), age group, tallness, wasting, positive forward-bending tests, plumb-line non-alignment, positive indicators in visual examination of back (drooping shoulders, uneven scapulae, curved shape of midline of back, unequal body triangles, uneven spinal dimples) and positive moiré (front and back), with the weightage increasing if the condition existed for more than one checkup. Table 1 lists all these factors and their respective weightages for a single or multiple checkups (Kamal *et al.*, 2013d).

Differential-spinal-function testing (see the following section) should be performed to rule out scoliosis if CSRW is equal to or more than 5.5 after 1<sup>st</sup> checkup, 6.5 after 2<sup>nd</sup> checkup and 7.5 after 3<sup>rd</sup> checkup.

### DIFFERENTIAL-SPINAL-FUNCTION TESTING

A mandatory two-minute-stripped-scoliosis screening, which includes moiré examination of back, for school-going children in the age group 7-10 years and a follow-up of at-risk cases may prevent suffering of a lifetime (Akram and Kamal, 1991; Horn, 2012; Luk *et al.*, 2010; Kamal and Lindseth, 1980).

Table 1. Formulae for assigning Cumulative-Scoliosis-Risk Weightage (CSRW)<sup>§</sup>

<i>Scoliosis-Risk Weightage</i>	<i>A</i> <sup>³</sup>	<i>B</i> <sup>ε</sup>	<i>C</i> <sup>∩</sup>
01. Family history	2.0	2.0	2.0
02. Age [3, 6.5) <sup>∩</sup> years	0.5	0.5	0.5
03. Age [6.5, 7.5) <sup>∩</sup> years	1.0	1.0	1.0
04. Age [7.5, 8.5) <sup>∩</sup> years	1.5	1.5	1.5
05. Age [8.5, 11) <sup>∩</sup> years	2.0	2.0	2.0
06. Tall (above 50 <sup>P</sup> ) <sup>#</sup>	1.0	1.5	2.0
07. Tall (above 75 <sup>P</sup> ) <sup>#</sup>	1.5	2.0	2.5
08. Tall (above 97 <sup>P</sup> ) <sup>#</sup>	2.0	2.5	3.0
09. Wasted (more than 10%) <sup>#</sup>	1.0	1.5	2.0
10. Wasted (more than 20%) <sup>#</sup>	1.5	2.0	2.5
11. Wasted (more than 30%) <sup>#</sup>	2.0	2.5	3.0
12. FBT <sub>F</sub> (lumbar asymmetry)	1.0/1.5 <sup>∩</sup>	1.5/2.0 <sup>∩</sup>	2.0/2.5 <sup>∩</sup>
13. FBT <sub>B</sub> (thoracic asymmetry)	1.0/1.5 <sup>∩</sup>	1.5/2.0 <sup>∩</sup>	2.0/2.5 <sup>∩</sup>
14. Plumb-line non-alignment	1.0	1.5	2.0
15. Shoulder drooping	0.5	1.0	1.5
16. Uneven scapulae	0.5	1.0	1.5
17. Midline of back C-shaped	0.5/1.0 <sup>¥</sup>	1.0/1.5 <sup>¥</sup>	1.5/2.0 <sup>¥</sup>
18. Midline of back S-shaped	1.0/1.5 <sup>¥</sup>	1.5/2.0 <sup>¥</sup>	2.0/2.5 <sup>¥</sup>
19. Unequal body triangles	0.5	1.0	1.5
20. Uneven spinal dimples	0.5	1.0	1.5
21. Positive moiré (back)	1.0	1.5	2.0
22. Positive moiré (front)	0.5	1.0	1.5

<sup>§</sup>The student should be subjected to differential-spinal-function testing if CSRW is equal to or more than 5.5 after the first checkup, 6.5 after the second checkup and 7.5 after the third checkup.

<sup>³</sup>This value is applicable for 1<sup>st</sup> checkup or 2<sup>nd</sup> checkup or 3<sup>rd</sup> checkup.

<sup>ε</sup>This value is applicable for (1<sup>st</sup> + 2<sup>nd</sup>) checkups or (2<sup>nd</sup> + 3<sup>rd</sup>) checkups or (1<sup>st</sup> + 3<sup>rd</sup>) checkups.

<sup>∩</sup>This value is applicable for (1<sup>st</sup> + 2<sup>nd</sup> + 3<sup>rd</sup>) checkups.

<sup>∩</sup>[x, y) means x (3 years in the first entry) is included, but y (6.5 years) is not. Hence, a 6.5-year old student is rated according to criterion 03.

<sup>#</sup>The superscript P denotes percentile.

<sup>∩</sup>Second value is applicable, if the front and the back asymmetries are on the opposite sides.

<sup>¥</sup>Second value is applicable, if the deformity is not corrected upon asking the child to assume mild-stretching posture.

### ***Mathematical Concepts Involved in Scoliosis Screening***

Scoliosis screening would generate interest in the students to be screened (end users of this exercise), if they are taught in a classroom lesson (prior to being subjected to screening) to visualize anatomy of the spinal column as a geometrical figure, offering them opportunity to measure Cobb angles as part of their mathematics classes (Kurz *et al.*, 2015).

Kamal *et al.* (1998) discussed physics of scoliosis screening. Here, we list some of the mathematical concepts involved in scoliosis screening (Kamal, 2011):

**Symmetry:** Symmetry about sagittal plane (left-right) is the main concept behind screening for scoliosis — in the context of visual exam, the screener observed scapulae, body triangles, spinal dimples, shoulders/neck line, nipples and knee joints. If transverse axis were taken as the  $x$  axis, anteroposterior axis as the  $y$  axis and longitudinal axis as the  $z$  axis, sagittal plane could be identified as the  $yz$  plane. Mathematically, symmetry about the  $yz$  plane (described by the equation,  $x = 0$ ) was expressed as the condition — for every point  $(x, y, z)$ , which was on body surface, there existed a mirror point  $(-x, y, z)$  on the same surface.

**Inverse Problem:** The screener tried to determine properties of source (condition of spinal column) from the properties of field generated, *e. g.*, thermogram indicating crooked spine or determination of shape of organ from back-surface analysis using moiré fringe topography or rasterstereography

**Precedence Graph:** Precedence Graph illustrates the procedures, which must ‘precede’ the others, *e. g.*, heart was checked before Adam’s forward bending test. The later was omitted for a patient, who underwent multiple cardiac surgeries.

**Influence Graph:** Influence Graph shows various procedures ‘influenced’ by others. Visual examination of spine (in the attention position) did not give proper results, when the student was tired after vigorous activity, *e. g.*, right after recess or physical-education class.

### ***Decision Matrix for Presence of Scoliosis***

Figure 1 illustrates decision matrix to detect possible existence of lateral curvatures and rotation of the spinal column. The decision was made in two levels. In the first level 2 tests were conducted and the results compared to suspect a possible condition. Then a 3<sup>rd</sup> test was administered to indicate that condition. 4 tests were selected for this purpose: visual (standing), visual (sitting), forward bending (standing) and forward bending (sitting) — postural problem suspected through positive visual examinations (standing and sitting), indicated through positive mild-stretching test (if the deformity was not corrected after mild stretching, it was indicative of lateral curvatures); leg-length inequality suspected through positive visual and forward-bending tests (both standing), indicated through uneven spinal dimples; hip weakness suspected through positive visual and forward-bending tests (both sitting), indicated through positive Trendelenburg sign; spinal rotation suspected through positive forward-bending tests (standing and sitting), indicated through either positive moiré or positive forward-bending tests (back and front views) on opposite sides (Kamal *et al.*, 2014a).

## **MATERIAL AND METHODS**

### ***Data Collection***

In 1998, the NGDS (National Growth and Developmental Standards for the Pakistani Children) Pilot Project (<http://ngds.uok.edu.pk>) was launched after ‘Institutional Review Process’, which incorporated existing ethical and human-right standards applicable in Pakistan employing ‘opt-in policy’. Participation was possible after filled in and signed ‘Informed Consent Form’ [http://www.ngds-ku.org/ngds\\_folder/Protocols/NGDS\\_form.pdf](http://www.ngds-ku.org/ngds_folder/Protocols/NGDS_form.pdf) was received. Those students, who required further examinations, were called in SF Growth-and-Imaging Laboratory, along with their parents and siblings. ‘SGPP Participation Form’ [http://www.ngds-ku.org/SGPP/SGPP\\_form.pdf](http://www.ngds-ku.org/SGPP/SGPP_form.pdf) was filled out for detailed checkup — SGPP (Sibling Growth Pilot Project) is a subproject of the NGDS Pilot Project ([http://www.ngds-ku.org/ngds\\_URL/subprojects.htm#SGPP](http://www.ngds-ku.org/ngds_URL/subprojects.htm#SGPP)), in which Growth-and-Obesity Profiles of all siblings as well as Obesity Profiles of parents are generated (Kamal *et al.*, 2011). This paper reports analysis of spinal examinations performed on the students of a civilian school located in Karachi for the checkups performed during 2011-2013. Students studying in KG (4- to 5-year old) were enrolled in the study. They were followed up till they left class 2 (7- to 8-year old). Data of 68 boys and 65 girls are presented here. Posture examinations and brief scoliosis screening was performed in KG and Class 1 to find out cases of early onset of scoliosis (Fletcher and Brace, 2012; Tis *et al.*, 2012) and a detailed scrutiny of the spinal column performed in class 2 (Kamal *et al.*, 2014a).

Students were required to undress totally except briefs or panties. Spinal examinations were performed in standing position — visual and forward bending. Visual examination of back in the attention position consisted of

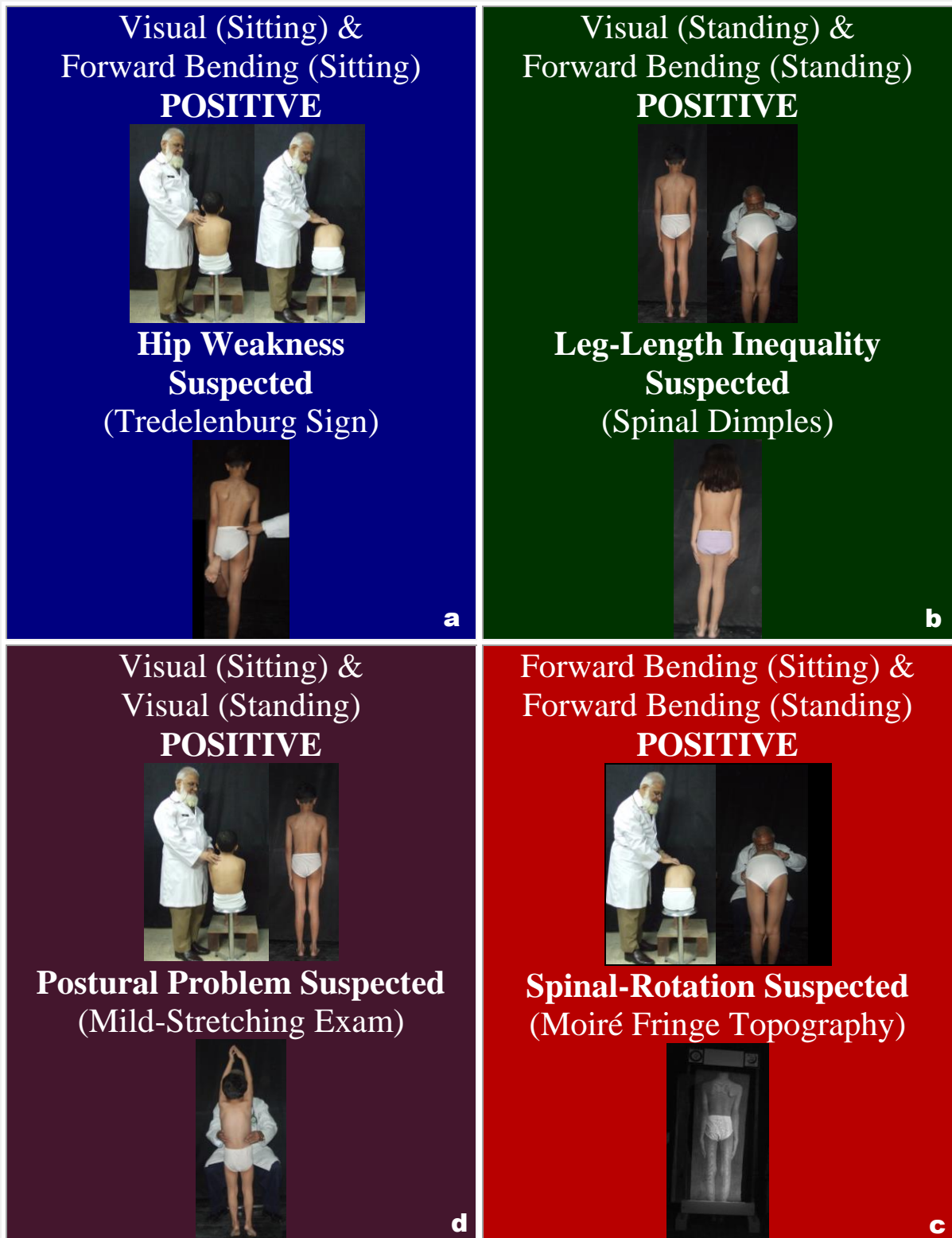


Fig. 1a-d. Decision matrix to detect possible presence of spinal rotation, which may contribute to scoliosis, based on 4 tests, (a) visual (sitting), (b) visual (standing), (c) forward bending (sitting) and (d) forward bending (standing)



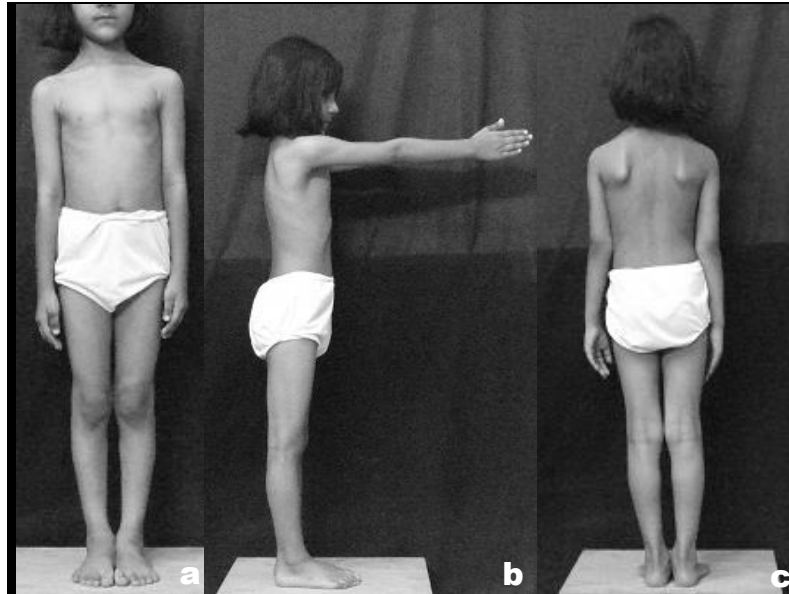


Fig. 2a-c. Visual examination from (a) front — drooping shoulder, nipples not level, (b) side and (c) back — drooping shoulder, uneven scapulae

noting presence of drooping shoulders, level of scapulae, symmetry of body triangles, level of spinal dimples and shape of midline of back — straight, C (most probably due to postural problem) or S (pathological — may be indicative of scoliosis). In addition, visual examination of child facing the examiner was conducted to confirm shoulder drooping, unequal body triangles (if observed during back examination) and shape of sternum as well as level of nipples and knee joints (Figures 2a-c). Body alignment was checked by placing a plumb line along midline of back. Body as considered to be aligned from back if the plumb line passed through midpoint of spinal dimples. From the front, plumb line was aligned with sternum. Body as considered to be aligned from front if the plumb line passed through navel (Figures 3a, b). Forward-bending test (standing position) was performed both with the student facing the examiner and with back towards the examiner. Student was asked to touch (or try to touch) toes with palms together and without flexing knees. The former brought into light curves in the lumbar and the sacral regions, whereas



Fig. 3a, b. Photograph on left illustrates body-alignment check of back using plumb line, body aligned, however, C curve is noticeable in the spinal outline and body triangles are unequal, photograph on right shows body-alignment check from front, nipples not level, body-triangles unequal and body not aligned

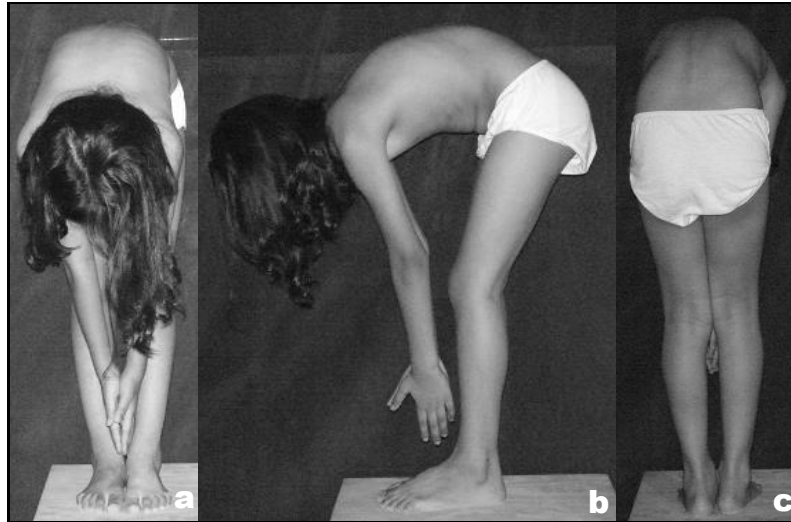


Fig. 4a-c. Forward-bending test from front (lumbar region highlighted) — left side elevated, side and back (thoracic region highlighted) — right side elevated; S curve suspected

as the later highlighted curves in the cervical and the thoracic regions. The asymmetry observed in forward-bending test (student facing the examiner or back towards the examiner) was confirmed by observing the student in forward-bending position observed from the side opposite to elevated back, *i. e.*, a student showing left side of back elevated was observed from the right side and vice versa. The side observation highlighted elevated portion (Figures 4a-c — on extreme left forward-bending test, with student facing the examiner, in which left side is elevated, on extreme right forward-bending test, with student's back towards the examiner, in which right side is elevated; strong indication of S curve and rotation of the spinal column) as well as indicated missing spinous process, if present. Students were asked to lift each foot for a count of 3 to check for hip weaknesses — the pelvis tilted downward on the side of unaffected hip, when weight was borne on weak hip abductors (positive Trendelenburg sign). Uneven spinal dimples indicated leg-length inequality.

In the sitting position visual and forward-bending examinations were performed by asking the child to sit on a stool with back straight. To make sure that the thighs were positioned perpendicular to back and feet were not hanging, wooden planks were placed under the feet to stabilize sitting posture. Body triangles could not be in the sitting-position-visual examination.

Moiré examination (Figures 5a-d) was performed by asking the child to stand behind a shadow-type moiré frame of opening  $57 \times 133 \text{ cm}$  (grating dimensions  $52 \times 108 \text{ cm}$ ) constructed from fishing line of black nylon thread of diameter  $0.85 \text{ mm}$  wound along the longer side with spacing maintained through a spring of pitch  $0.75 \text{ mm}$ . Distance between the camera and the moiré grating was kept as  $170 \text{ cm}$  and that from the light source and the camera

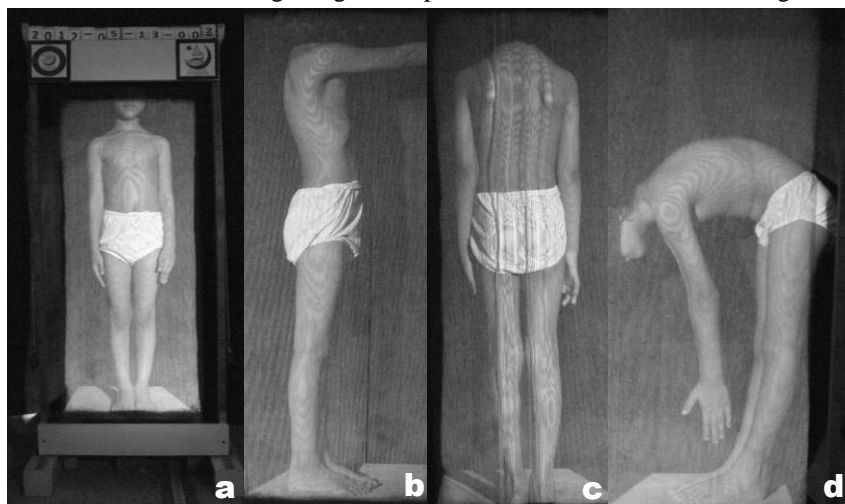


Fig. 5a-d. Use of moiré fringe topography to look for symmetric curves about the sagittal plane for possible detection of scoliosis (a, c) — compare a-c with Fig. 2a-c and d with Fig. 4b

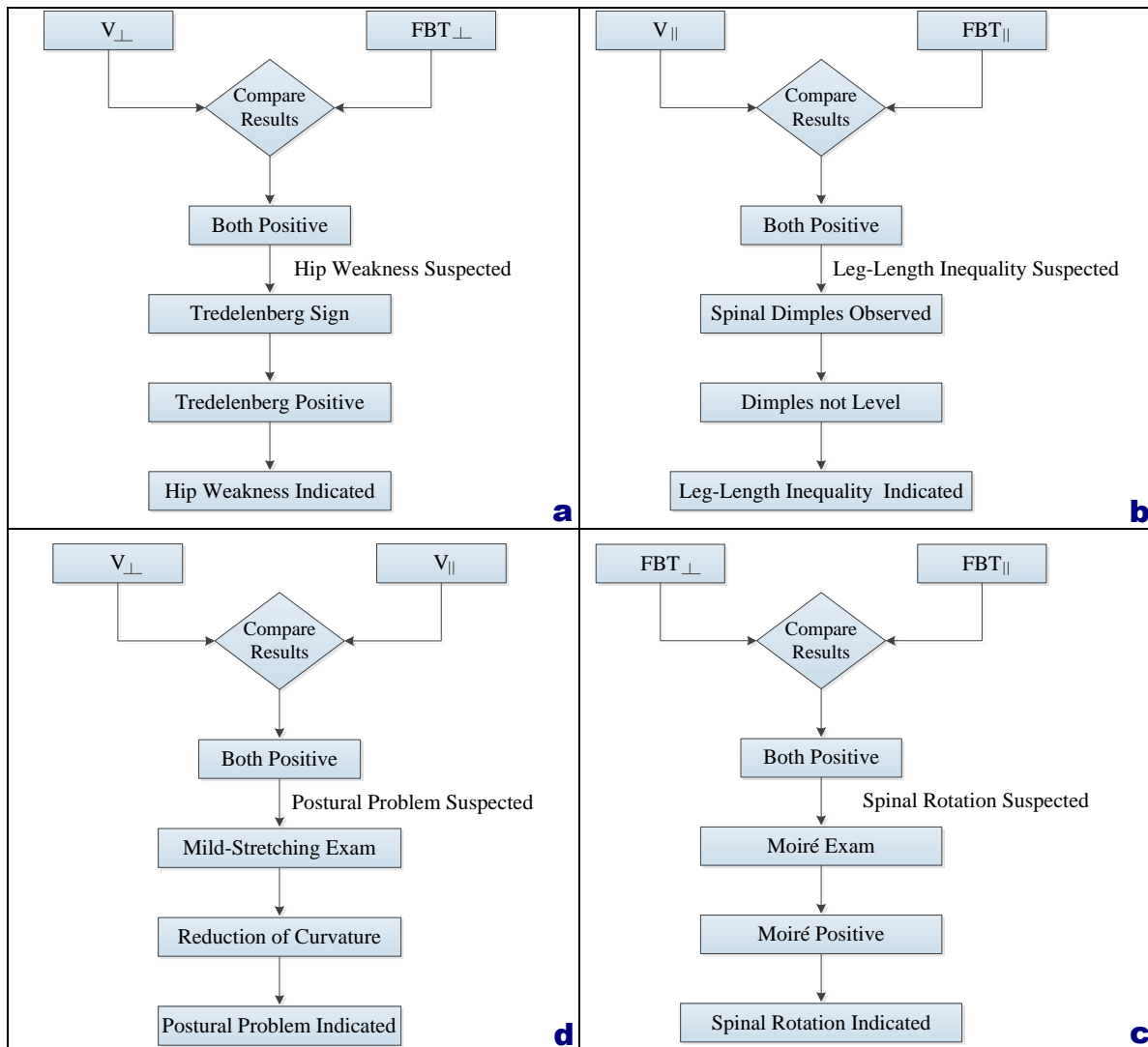


Fig. 6a-d. Flowchart for decision matrix, which may be used to plan for efficient detection and effective treatment of scoliosis —  $V_{\parallel}$  ( $V_{\perp}$ ) denotes visual examination performed, when the student was standing (sitting);  $FBT_{\parallel}$  ( $FBT_{\perp}$ ) represents forward-bending test conducted, when the pupil was standing (sitting)

as 70 cm (Akram, 1989). Overhead projector was used as light source. A transparency was placed next to moiré grating to focus the light beam. The transparency was, later removed and the child was instructed to stand in the attention position touching the moiré grating.

Heights and masses of students were measured by reproducible anthropometrists to accuracies of 0.01 cm and 0.01 kg, in the morning hours, as per protocols described elsewhere (Kamal *et al.*, 2011; 2015). Equipments were calibrated using a standard 100-cm ruler and a standard 2-kg mass. Zero errors were determined before starting each session and subtracted from the measured values.

### Data Processing

Figure 6 gives a flow chart of the decision-making process (Kamal *et al.*, 2014a). We would like to study correlation of those cases in which lateral curvatures, spinal rotations or both were indicated with CSRW. Let us denote

$A$  :set of students, in whom lateral curvatures of spinal column were indicated

$B$  :set of students, in whom rotations of spinal column were indicated

$A \cap B$  :set of students, in whom both lateral curvatures and rotations were indicated

$A - B$  :set of students, in whom lateral curvatures were indicated, but rotations were not

$B - A$  :set of students, in whom rotations were indicated, but lateral curvature were not



Table 2. Mean Cumulative-Scoliosis-Risk Weightage (CSRW) for students suspected of scoliosis

Differential-Spinal-Function-Testing Result	Number of Boys	Number of Girls	Total	Mean CSRW <sup>∇</sup>
Lateral curvatures indicated, no rotations	1	2	3	11.92
Rotations indicated, no lateral curvatures	27	50	77	8.98
Both lateral curvatures and rotations	0	1	1	7.50

<sup>∇</sup>Mean of individual CSRW scores of each student

From the above description, it could be easily concluded

$$(3) \quad A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$$

Arithmetic mean of CSRW was computed corresponding to elements of each set.

## RESULTS

Figure 7 shows the results of differential-spinal-function testing. Table 2 gives the number of students, who indicated lateral curvatures or rotations and the corresponding mean Cumulative-Scoliosis-Risk Weightage (CSRW). This analysis indicates that a higher CSRW increases the risk of student acquiring scoliosis — girls are approximately at a double risk of acquiring scoliosis as compared to boys.

## DISCUSSION AND CONCLUSION

The authors recommend posture examinations for boys and girls in the age group 4-7 years (Kamal and El-Sayyad, 1981) and scoliosis-screening examinations for children in the age group 7-10 years (Kamal *et al.*, 2013b) employing visual inspection and moiré fringe topography. Child should be totally stripped except short underpants and barefoot for these exams (Kamal *et al.*, 2015). At-risk cases, determined through high CSRW (5.5 after the first checkup, 6.5 after the second checkup and 7.5 after the third checkup), should be followed up till the end of growth period (An *et al.*, 2015; Schulte *et al.*, 2008).

Future work should be focused on formulating a coördinate system, which should reduce the degrees-of-freedom of spinal column from a total of 231 = (33)(3 + 3 + 1) to, possibly, one by employing techniques similar to

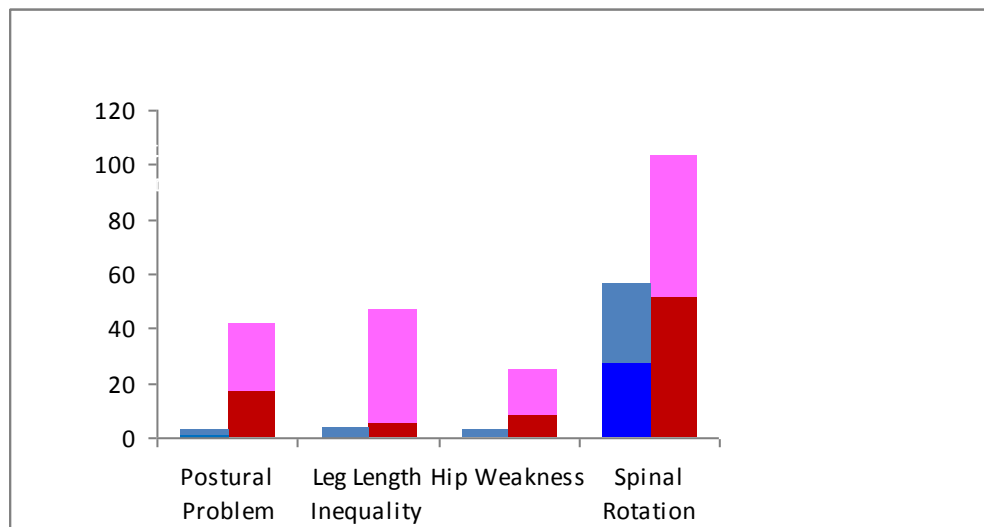


Fig. 7. Data analysis based on decision matrix, suspected conditions illustrated in indigo (boys) and pink (girls), indicated conditions in blue (boys) and red (girls) — postural problem suspected through positive visual examinations (standing and sitting), indicated through positive mid-stretching test; leg-length inequality suspected through positive visual and forward-bending tests (both standing), indicated through uneven spinal dimples; hip weakness suspected through positive visual and forward-bending tests (both sitting), indicated through positive Trendelenburg sign; spinal rotation suspected through positive forward-bending tests (standing and sitting), indicated through either positive moiré or positive forward-bending tests (back and front views) on opposite sides

those used in reducing degrees-of-freedom of two-body problem of planetary motion from 12 to one (Kamal, 1997). The product  $(33)(3 + 3 + 1)$  comes from multiplying the number of vertebrae in the spinal column (33) with the sum of positional degrees-of-freedom (3), rotational degrees-of-freedom (3) + inter-vertebral-spacing degree-of-freedom (1). The problem of human spinal column, one of the leading problems of 'orthopedics' (considered as a branch of medical science dealing with the study of bones, joints and skeletal deformations), therefore, is described by the branch of mathematics called 'algebraic topology'. One must realize that 'orthopedics' is derived from 'anatomy' (study of structures of human body), whereas 'algebra' is the understanding of mathematical structures and 'topology', studies invariance under deformations. 'Anthrotopology' may be the new branch of mathematics, which could deal with the mathematical framework related to spinal deformities (Kamal *et al.*, 2012).

## ACKNOWLEDGEMENTS

The authors would like to thank Prof. Dr. Anisuddin Bhatti, Professor of Orthopedic Surgery and Managing Director, Jinnah Postgraduate Medical Center, Karachi for visiting SF Growth-and-Imaging Laboratory, and taking a keen interest in our group's work. The protocols of differential-spinal-function testing were discussed with him on September 4, 2013, when the first author (SAK) gave Z. K. Kazi and M. A. Shah memorial lecture (Kamal *et al.*, 2013c). The authors are indebted to Mr. Sanaullah Kazi, Mrs. Azra Anwar Ahmed, Mrs. Yasmeen Salman, Mrs. Anis Hasan, Mrs. Farkhunda Ghufuran and Mrs. Nilofer Inam of Beacon Light Academy, 'O' Levels, Gulshan-é-Iqbal, Karachi, for providing data-collection facilities and our associate, Samira Sahar Jamil, for assisting in laboratory work. Shakeel Ahmed Ansari developed software, which was needed to compute CSRW. Muhammed Wasim helped in drawing diagrams for this paper. The authors declare that they don't have any financial/non-financial conflict of interest in the work presented in this paper. This work is dedicated to the loving memory of Stig Willner (1931-1999), Orthopedic Surgeon, who was host of the first author during the fall of 1988, when he visited Malmö General Hospital as part of research collaboration, which resulted in a publication (Kamal *et al.*, 1994) with the deceased.

## REFERENCES

- Akram, M. (1989, May 28). Development of a shadow-type moiré apparatus. *MSc Project*, Department of Physics, University of Karachi, Karachi, Pakistan (unpublished)
- Akram, M. and S. A. Kamal (1991, April 16-20). Role of moiré fringe topography in the skeletal examination of school athlete. *The International Congress and Exposition on Sports Medicine and Human Performance*, Vancouver, Canada, p. 2, abstract: <http://www.ngds-ku.org/pub/confabst0.htm#C36>:
- An, K. C., D. H. Park, G. M. Kong, *et al.* (2015). Prevalence study of adolescent idiopathic scoliosis in ten-, eleven-year olds for 10 years. *Journal of Korean Orthopaedic Association*, 50 (1): 25-30
- Anderson, S. M. (2007). Spinal curves and scoliosis. *Radiologic Technology*, 79 (1): 44-65
- Balla, P., G. Manhertz and A. Antal (2014). Defining of the shape of the spine using moiré method in case of patients with Scheuermann disease. *World Academy of Science, Engineering and Technology International Journal of Medical, Health, Biomedical and Pharmaceutical Engineering*, 8 (6): 348-353
- Baroni, M. P., G. J. B. Sanchis, S. J. C. de Assis, *et al.* (2015). Factors associated with scoliosis in school children: A cross-sectional population-based study. *Journal of Epidemiology*, 25 (3): 212-220
- El-Sayyad, M. M. and S. A. Kamal (1981, September 21-23). Cobb's angle measurement by moiré topographs. *Proceedings of Thirty-Fourth Annual Conference on Engineering in Medicine and Biology*, Houston, Texas, United States, 23: 311, abstract#30.7: <http://www.ngds-ku.org/pub/confabst1.htm#C12>:
- Fletcher, N. D. and R. W. Brace (2012). Early onset scoliosis: Current concepts and controversies. *Current Reviews in Musculoskeletal Medicine*, 5 (2): 102-110
- Hackenberg, L., E. Hierholzer, V. Bullmann, U. Liljenqvist and C. Götze (2006). Rasterstereographic analysis of axial back surface rotation in standing versus forward bending posture in idiopathic scoliosis. *European Spine Journal*, 15 (7): 1144-1149
- Hackenberg, L., E. Hierholzer, W. Pötzi, C. Götze and U. Liljenqvist (2003a). Rasterstereographic back shape analysis in idiopathic scoliosis after anterior correction and fusion. *Clinical Biomechanics*, 18 (1): 1-8
- Hackenberg, L., E. Hierholzer, W. Pötzi, C. Götze and U. Liljenqvist (2003b). Rasterstereographic back shape analysis in idiopathic scoliosis after posterior correction and fusion. *Clinical Biomechanics*, 18 (10): 883-889
- Hierholzer, E. and G. Lüxmann (1982). Three-dimensional shape analysis of the scoliotic spine using invariant shape parameters. *Journal of Biomechanics*, 15 (8): 583-598
- Horn, P. (2012). Scoliosis: Early identification of affected patients. *Clinician Reviews*, 22 (1): 17-22
- Kamal, S. A. (1982a). Measurement of angle of spinal curvature by moiré topographs. *Journal of Islamic Medical Association (United States)*, 14 (4): 145-149, full text: <http://www.ngds-ku.org/Papers/J04.pdf>

- Kamal, S. A. (1982b, March 8-12). Moiré topography for the measurement of spinal curvature in three dimensions. *March Meeting of the American Physical Society*, Dallas, Texas, United States; *Bulletin of the American Physical Society*, 27: 301, abstract#GY15: <http://www.ngds-ku.org/pub/confabst1.htm#C16>:
- Kamal, S. A. (1983a, October 30-November 3). Moiré fringe topography and the spinal deformity. *The National Guard Eighth Saudi Medical Conference*, Riyadh, Kingdom of Saudi Arabia, p. 192, full text: <http://www.ngds-ku.org/Papers/C22.pdf>
- Kamal, S. A. (1983b). Determination of degree of correction of spinal deformity by moiré topographs. *Moiré Fringe Topography and Spinal Deformity (Proceedings of the Second International Symposium, Münster, Germany, September 12-15, 1982)*, edited by B. Drerup, W. Frobin and E. Hierholzer, Gustav Fischer, Stuttgart and New York, pp. 117-126, full text: <http://www.ngds-ku.org/Papers/C23.pdf>
- Kamal, S. A. (1987). Moiré topography for the study of multiple curves of spine. *Surface Topography and Spinal Deformity (Proceedings of the Fourth International Symposium, Mont Sainte Marie, Québec, Canada, September 27-30, 1986)*, edited by I. A. F. Stokes, J. R. Pekelsky and M. S. Moreland, Gustav Fischer, Stuttgart and New York, pp. 43-49, full text: <http://www.ngds-ku.org/Papers/C26.pdf>
- Kamal, S. A. (1996a). Gait analysis using moiré fringe topography and rasterstereography (simultaneous recording). *Karachi University Journal of Science*, 24 (2): 7-18, full text: <http://www.ngds-ku.org/Papers/J16.pdf>
- Kamal, S. A. (1996b). Combination of moiré contours and edge-based algorithm to study motion in the sagittal plane. *Karachi University Journal of Science*, 24 (2): 53-60, full text: <http://www.ngds-ku.org/Papers/J17.pdf>
- Kamal, S. A. (1996c). A 3-D-static model of the human spinal column. *Karachi University Journal of Science*, 24 (1): 29-34, full text: <http://www.ngds-ku.org/Papers/J18.pdf>
- Kamal, S. A. (1996d, June 27-July 11). 3-D-dynamic modeling of the human spinal column. *The Twenty-First International Summer College on Physics and Contemporary Needs*, Nathiagali, KP, Pakistan, abstract: <http://www.ngds-ku.org/pub/confabst0.htm#C42>:
- Kamal, S. A., (1997, March 30-April 3). Planetary-orbit modeling based on astrodynamical coordinates. *The Pakistan Institute of Physics International Conference (PIPC 1997)*, Government College, Lahore, Pakistan, pp. 28-29, abstract#P2: <http://www.ngds-ku.org/pub/confabst0.htm#C45>:
- Kamal, S. A. (2011, May 9, 10). Anthromathematics: A new branch of mathematics. *The Sixth Symposium on Computational Complexities, Innovations and Solutions (CCIS 2011)*, the COMSATS Institute of Information Technology, Abbotabad, KP, Pakistan, pp. 13, 14, abstract#1: <http://www.ngds-ku.org/Presentations/Anthromath.pdf>
- Kamal, S. A. (2013, May 30). 3-D-spinal-column-surface analysis (height and curvature maps) by combining moiré fringe topography and rasterstereography with backscatter-X-ray-scanning technology. *The First Undergraduate National Computing Conference*, Usman Institute of Technology, Karachi, Pakistan (keynote lecture), abstract: <http://www.ngds-ku.org/Presentations/Backscatter.pdf>
- Kamal, S. A., A. Haider and M. Sarwar (2013a, September 4, 5). Rasterstereography in scoliosis detection and management. *The First Conference on Anthromathematics in the Memory of (Late) Syed Firdous (ANTHROMATHEMATICS 2013)*, Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, p. 10, abstract#Anthro13-04: <http://www.ngds-ku.org/Presentations/Rasterstereography.pdf>
- Kamal, S. A., G. Benoni and S. Willner (1994). A study to test the reproducibility of moiré topographs. *Karachi University Journal of Science*, 22 (1&2): 67-74, full text: <http://www.ngds-ku.org/Papers/J15.pdf>
- Kamal, S. A. and M. M. El-Sayyad (1981, August 9-13). The use of moiré topographs for detection of orthopedic defects in children of age group four to seven years. *The Twenty-Third Annual Meeting of the American Association of Physicists in Medicine*, Boston, Massachusetts, United States — presented by title, *Medical Physics*, 8 (4): 549, paper#T7: <http://www.ngds-ku.org/pub/confabst1.htm#C11>:
- Kamal, S. A., M. Sarwar and A. Haider (2014a, March 20). Effective decision making for presence of scoliosis based on moiré fringe topography. *The Second Conference on Mathematical Sciences (CMS 2014)*, Department of Mathematics, University of Karachi, Karachi, Pakistan, p. 2, abstract#CMS14-05: <http://www.ngds-ku.org/Presentations/Decision.pdf>
- Kamal, S. A., M. Sarwar and M. K. Rajput (2012, December 27-29). Crystal-structure concepts applied to static and dynamic modeling of the human-spinal column. *The International Conference on Condensed-Matter Physics and Engineering*, the Bahauddin Zakaria University, Multan, Pakistan p. 81, abstract: <http://www.ngds-ku.org/Presentations/BZU1.pdf>
- Kamal, S. A., M. Sarwar and M. K. Rajput (2014b, September 4). Crystal-structure-concept-based modeling of the human spinal column validated through 3-D-bone scanning. *The Second Conference on Anthromathematics and Sport mathematics in the Memory of (Late) Hussain Ahmed Bilgirami (ANTHROMATHEMATICS 2014)*, Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, p. 8, abstract#Anthro14-04: <http://www.ngds-ku.org/Presentations/Spinal.pdf>
- Kamal, S. A., M. Sarwar and S. K. Raza (2013b, September 4, 5). Moiré fringe topography in scoliosis detection and management. *The First Conference on Anthromathematics in the Memory of (Late) Syed Firdous (ANTHROMATHE-*

- MATICS 2013), Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, p. 16, abstract#Anthro13-10: <http://www.ngds-ku.org/Presentations/Moire.pdf>
- Kamal, S. A., M. Sarwar and U. A. Razzaq (2013c, September 4, 5). Anthromathematics of the human spinal column. *The First Conference on Anthromathematics in the Memory of (Late) Syed Firdous (ANTHROMATHEMATICS 2013)*, Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, p. 7 (Prof. Dr. Z. K. Kazi and Prof. Dr. M. A. Shah memorial lecture), abstract#Anthro13-01: <http://www.ngds-ku.org/Presentations/Scoliosis.pdf>
- Kamal, S. A., M. Sarwar and U. A. Razzaq (2013d, September 4, 5). Cumulative-Scoliosis-Risk Weightage (CSRW) — designing preventive strategies. *The First Conference on Anthromathematics in the Memory of (Late) Syed Firdous (ANTHROMATHEMATICS 2013)*, Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, p. 11, abstract#Anthro13-05: <http://www.ngds-ku.org/Presentations/CSRW.pdf>
- Kamal, S. A., Naseeruddin, M. Wasim and S. Firdous (1998). Physics of scoliosis screening in school-going children. *Karachi University Journal of Science*, 26: 5-12, full text: <http://www.ngds-ku.org/Papers/J22.pdf>
- Kamal S. A., N. Jamil and S. A. Khan (2011). Growth-and-obesity profiles of children of Karachi using box-interpolation method. *International Journal of Biology and Biotechnology*, 8 (1): 87-96, full text: <http://www.ngds-ku.org/Papers/J29.pdf>
- Kamal, S. A. and R. E. Lindseth (1980, July 29-August 1). Moiré topography for the detection of orthopedic defects. *Periodic Structures, Moiré Patterns and Diffraction Phenomena (Proceedings of the Society of Photo-Optical Instrumentation Engineers, San Diego, California, United States)*, edited by C. H. Chi, E. G. Loewen and C. L. O'Bryan III, 240: 293-295, full text: <http://www.ngds-ku.org/Papers/C08.pdf>
- Kamal, S. A., S. A. Ansari and S. S. Jamil (2015). Generating and validating Growth-and-Obesity Roadmaps for the Pakistani children. *International Journal of Biology and Biotechnology*, 12 (1): 47-61, full text: <http://www.ngds-ku.org/Papers/J35.pdf>
- Kurz, T. L., H. B. Yanik and M. Y. Lee (2015). The geometry of scoliosis. *Teaching Children Mathematics*, 21 (6): 372-375
- Luk, K. D., C. F. Lee, K. M. C. Cheung, *et al.* (2010). Clinical effectiveness of school screening for adolescent idiopathic scoliosis: A large population-based retrospective cohort study. *Spine*, 35 (17): 1607-1614
- Oxborrow, N. J. (2000). Assessing the child with scoliosis: The role of surface topography. *Archives of Diseases in Childhood*, 83 (5): 453-455
- Persson-Bunke, M., G. Häggglund, H. Lauge-Pedersen, W. Philippe and L. Westborn (2012). Scoliosis in a total population of children with cerebral palsy. *Spine*, 37 (12): 708-713
- Saikia, K. C., A. Duggal, P. K. Bhattacharya and M. Borgohain (2002). Scoliosis: An epidemiological study of school children in lower Assam. *Indian Journal of Orthopaedics*, 36 (4): 243-245
- Schulte, T. L., E. Hierholzer, A. Boerke, *et al.* (2008). Rasterstereography versus radiography in the long-term follow-up of idiopathic scoliosis. *Journal of Spinal Disorders and Techniques*, 21 (1): 23-28
- Sengupta, D. P. and J. K. Webb (2010). Scoliosis — The current concepts. *Indian Journal of Orthopaedics*, 44 (1): 5-8
- Tis, J. E., L. I. Karlin, B. A. Akbarnia, *et al.* (2012). Early onset scoliosis: Modern treatment and results. *Journal of Pediatric Orthopedics*, 32 (7): 647-657
- Wasim, M., S. A. Kamal and A. Shaikh (2013). A security system employing edge-based rasterstereography. *International Journal of Biology and Biotechnology*, 10 (4): 613-630, full text: <http://www.ngds-ku.org/Papers/J31.pdf>
- Yosufzai, M. A. K., S. A. Kamal and J. A. Zubairi (1995, January 2-5). Computer-based analysis of human gait using moiré fringe topography. *Proceedings of the Second International Workshop on Computer Vision and Parallel Processing*, edited by G. N. Khan, A. A. Naqvi and M. A. Shah, the Quaid-é-Azam University, Islamabad, Pakistan, pp. 60-71, full text: <http://www.ngds-ku.org/Papers/C41.pdf>

(Accepted for Publication: March 2015)

Web address of this document (on first author's homepage): <http://www.ngds-ku.org/Papers/J36.pdf>

Abstract: <http://www.ngds-ku.org/pub/jourabstB.htm#J36>: