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A SYSTEMATIC WAY TO EXPRESS THE EQUATIONS OF STRAIGHT LINE IN TERMS OF THEIR DIRECTION RATIOS

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ABSTRACT

The equations of straight line are generally given as the intersection of two planes. A systematic way to express the straight-line equations in terms of their direction ratios is presented.

Keywords: linear modeling, straight-line equations

INTRODUCTION

The equations of straight line are very important when it comes to linear modeling (Pearson, 1983; West *et al.*, 1982). One is often required to manipulate and to program these equations to be able to predict unknowns in terms of the slope and the y intercept. Sometimes, the system requirement dictates the coördinate system to be chosen in such a way that the equations of straight line have to be expressed in three dimensions as the intersection of two planes. This form, however, is not the most convenient to be expressed as a set of parametric equations. The equations of straight line in terms of their direction ratios render themselves, immediately, to the parametric form. There is, therefore, a need to establish a formal method to be able to convert one form into the other.

THE PROBLEM

The equations of straight line in terms of three-dimensional analytic geometry may be written as (the intersection-of-planes form):

$$Ax + By + Cz + D = 0 \qquad (1a)$$

$$Px + Qy + Rz + S = 0 \qquad (1b)$$

Where *A*, *B*, *C*, *D*, *P*, *Q*, *R* and *S* are constants. Eq. (1*a*, *b*) define straight line as the intersection of two planes provided these planes are neither parallel nor identical, *i. e.*, the following condition does not hold**:**

$$\frac{A}{P} = \frac{B}{Q} = \frac{C}{R}$$

In other words,

$$AQ - BP \neq 0 \qquad (2a)$$

$$BR - CQ \neq 0 \qquad (2b)$$

$$CP - AR \neq 0 \qquad (2c)$$

The equations of straight line may also be written as (the direction-ratio form):

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n} \qquad (3)$$

Where *l*, *m*, *n* are the direction ratios of a line passing through the point (x_0, y_0, z_0). Eq. (1*a*, *b*) may be put in the form of Eq. (3) by the *trial-and-error method*. This can, sometimes, become very tedious. A systematic method is proposed to express the equations of straight line given in the form (1*a*, *b*) to be written in the form of Eq. (3).

THE METHOD

In order to express Equations (1*a*, *b*) in the form of Eq. (3) one sets $z = 0$ in (1*a*, *b*) and solves for *x* and *y*. This would give a point on the line, say, (x_0, y_0, z_0) where

$$x_0 = \frac{BS - DQ}{AQ - BP}; y_0 = \frac{DP - AS}{AQ - BP}; z_0 = 0 \qquad (4a\text{-}c)$$

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Condition (2*a*) insures that x_0 and y_0 are defined. Setting $x = a$ (where $a \neq x_0$, given above) in Eq. (1*a*,*b*) and solving for y and z gives another point on the line, say, $(x_1,\ y_1,\ z_1)$, where

$$x_1 = a \qquad (5a)$$

$$y_1 = \frac{(CS - DR) + a(CP - AR)}{BR - CQ} \qquad (5b)$$

$$z_1 = \frac{(DQ - BS) + a(AQ - BP)}{BR - CQ} \qquad (5c)$$

Condition (2*b*) insures that y_1 and z_1 are defined. The equation of straight line may, therefore, be written as:

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0} = \frac{z - z_0}{z_1 - z_0} = s \qquad (6)$$

Where s is a parameter. In parametric form this equation becomes**:**

$$x(s) = (x_1 - x_0)s + x_0 \qquad (7a)$$

$$y(s) = (y_1 - y_0)s + y_0 \qquad (7b)$$

$$z(s) = (z_1 - z_0)s + z_0 \qquad (7c)$$

or, in vector notation**:**

$$\boldsymbol{r}(s) = (\boldsymbol{r}_1 - \boldsymbol{r}_0)s + \boldsymbol{r}_0 \qquad (8)$$

It becomes tricky to use this method where one or more of the direction ratios are zero. For example, consider the set of planes $x = 0$ (the yz plane); $z = 0$ (the xy plane), whose intersection must be the y axis (direction cosines 0, 1, 0). In this situation, the above-mentioned method may be used after converting equations in the parametric form represented by Eq. (7).

Worked Example

Consider a set of planes represented by the following pair of equations:

$$3x + 4y + z = 0; 9x + 5y - z - 1 = 0$$

In these equations, $A = 3$, $B = 4$, $C = 1$; $P = 9$, $Q = 5$, $R = -1$. One notes that the factors:

$$AQ - BP = -1; BR - CQ = -9; CP - AR = 13$$

are all different from zero. Hence, the method described above may be applied. Setting z = 0 and solving for x and y gives a point on the straight line. Next, setting $x = 0$ and solving for y and z gives another point on the straight line. These two points

$$(\frac{4}{21}, -\frac{1}{7}, 0); (0, \frac{1}{9}, -\frac{4}{9})$$

determine the required straight line. The equations are:

$$\frac{x - 4/21}{0 - 4/21} = \frac{y + 1/7}{1/9 + 1/7} = \frac{z - 0}{-4/9 - 0}$$

or,

$$84x - 16 = -63y - 9 = 36z$$

DISCUSSION AND CONCLUSION

The method described in this paper may be used in linear modeling and numerical analysis to be able to convert one form of equations to the other. It may, easily, be generalized to more than three dimensions. A slight variation of the formulation may also be applicable to curves generated by the intersection of surfaces in three dimensions.

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