

## THE HUMAN HEART AS A SYSTEM OF STANDING WAVES

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### ABSTRACT

The human heart is considered to be a system of standing waves. Acoustic properties of the human heart are modeled by applying the concepts of wave theory. The nature and the origin of various frequencies are considered. Some experimental suggestions are given.

**Keywords:** Heart models, heart sounds, standing waves, phonocardiogram, echocardiogram

### INTRODUCTION

The human heart may be considered as an engine performing work (Cameron & Skofronick, 1978). It may also be modeled as a vibrating system. The concept of standing waves could be used to calculate the possible frequencies of vibrations of the human heart. Previous models of the human heart consider heart as a sphere (Mazumdar & Woodard-Knight, 1984) or as a bullet. However, shape of the human heart may be better approximated if one considers the human heart as a deformed ellipsoid of revolution.

A good review of heart models is provided by Noble (2002). According to Noble (2002), there has been a considerable debate over the best strategy for biological simulation. One could start with the "top down", or the "bottom up". However, the consensus is that it should be "middle out". One starts modeling at the level(s), at which there are rich biological data and then reach up and down to other levels. In the case of heart, one notes that, in addition to the data-rich cellular level, there has also been data-rich modeling of the 3D geometry of the whole organ. Connecting these two levels must be an exciting venture.

Phonocardiography is graphic recording of the heart sounds. The electronic amplifiers used in phonocardiography have a much different response as compared to the human ear so that these recordings do not correspond well with what the cardiologist hears. In the same fashion, an electronically amplified stethoscope distorts the sounds that the physician is accustomed to hearing (Cameron & Skofronick, 1987). A phonocardiogram is basically a plot of the amplitude of sound versus time. A Fourier transform of the phonocardiogram shows peaks at two characteristic frequencies. The model presented in this paper attempts to relate ratio of these frequencies  $\omega_{\text{higher}}/\omega_{\text{lower}}$  to size of the human heart.

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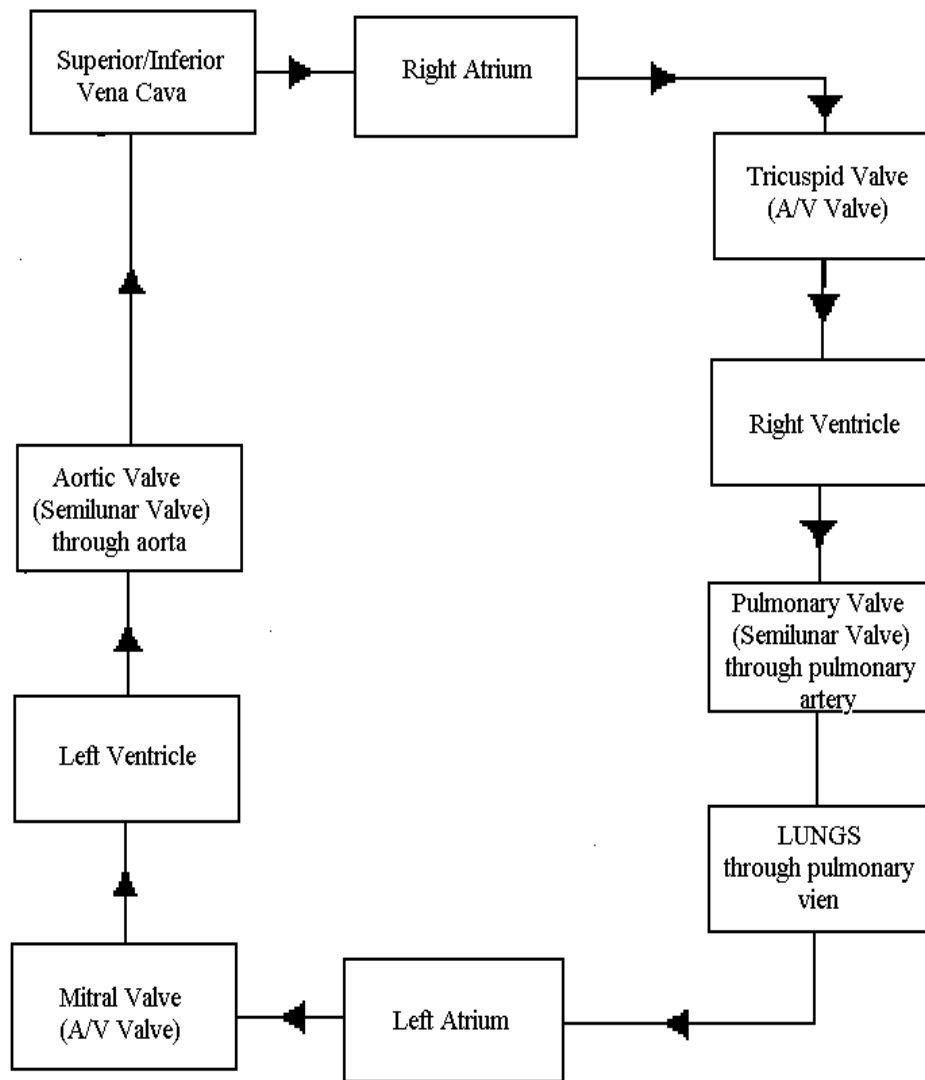


Fig. 1. The Cardiac Cycle

Details of working of the human heart (Figure 1 shows the cardiac cycle) may be found elsewhere (Bergel & Hunter, 1979; Chialvo & Jalife, 1987; Glass *et al.*, 1991; Guevara, 1984).

### ACOUSTIC MODEL OF THE HUMAN HEART

We modelled acoustic properties of the human heart to relate the phonocardiogram frequencies to the heart shape. The frequencies obtained in the Fourier transform of phonocardiogram may be calculated if we know the shape and the properties of surface of the human heart. The human heart may be considered as a system of standing waves (Kamal & Siddiqui, 1992). Everyone knows that standing waves with discrete frequencies exist in a bound system (Halliday & Resnick, 1988).

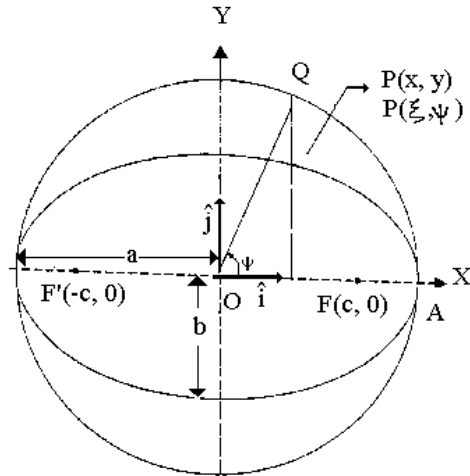


Fig.2. Elliptical co-ordinates

In order to accomplish this the human heart is modeled as a deformed ellipsoid of revolution about the major axis (by deformed ellipsoid we mean a union of two hemiellipsoids with different semi-major axes but same semi-minor axes - a term *hemiellipsoid* is coined to represent slicing of ellipsoid by a plane passing through the center of the ellipsoid of revolution normal to the major axis, the term *hemisphere* already exists in the mathematical literature). Using the concept of standing waves frequencies obtained in the Fourier transform of phonocardiogram may be calculated. We have introduced *the cardiac-coördinate mesh*, a modification of the ellipsoidal-coördinate mesh.

It is to be noted that even the deformed ellipsoid is invariant under rotation about the major axis. Hence, the corresponding angular momentum (the canonical momentum) shall be conserved. The human heart is assumed to be a deformed ellipsoid of revolution. Under these conditions there exists a rotational symmetry about the axis of revolution. Because of this symmetry we may drop one of the coördinates (the coördinate is cyclic), the three-dimensional problem reducing to a two-dimensional problem. We, now consider projection of the human heart on the frontal plane. Let us take the frontal plane as the xy plane. The anteroposterior axis, therefore, becomes the z axis.

The elliptical coördinates (in two dimensions) are described in Appendix A, and drawn in Figure 2. These are adapted from the elliptic-cylindrical coördinates, well known in coördinate geometry. The cardiac coördinates, denoted by  $(\xi, \psi)$ , are basically extension of the elliptical coördinates, are listed in Appendix B and drawn in Figure 3. Let us consider the human heart as a union of two semi ellipses one of them having 'a'

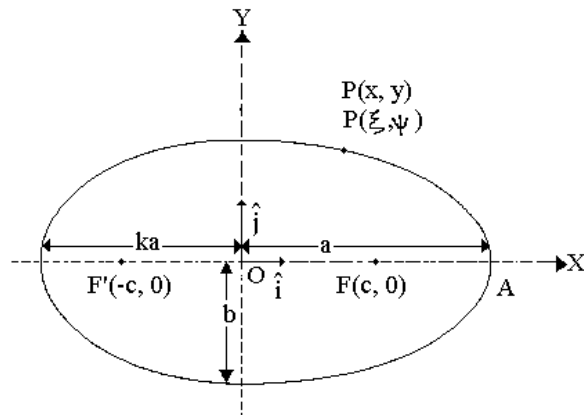


Fig. 3. Cardiac co-ordinates

and ‘ $b$ ’ as measures of the semi-major and the semi-minor axes respectively and the other having ‘ $ka$ ’ and ‘ $b$ ’ as measures of the semi-major and the semi-minor axes respectively ( $k > 0$ ). Let  $s(a, b, k)$  be the sum of circumferences of the two semi ellipses. The standing waves may be set up in a closed string if the path length,  $s(a, b, k)$ , is an integral multiple of wavelength. If this integral multiple is taken as unity

$$(1) \quad s(a, b, k) = \lambda_1$$

The frequency may, therefore, be expressed as

$$(2) \quad \omega_1 = 2\pi\nu_1 = \frac{2\pi u}{\lambda_1} = \frac{2\pi u}{s(a, b, k)}$$

$\nu_1$  is frequency corresponding to wavelength  $\lambda_1$  and ‘ $u$ ’ is wave velocity set up in outer layer of the heart membrane. Similarly, for a path normal to the major axis

$$(3) \quad 2\pi b = \lambda_2$$

$2\pi b$  is circumference of the circle having a radius ‘ $b$ ’ (Since we are considering heart surface as a surface of revolution, we are going to end up with a circle of radius ‘ $b$ ’ as a result of revolution). The frequency, therefore, becomes

$$(4) \quad \omega_2 = 2\pi\nu_2 = \frac{2\pi u}{\lambda_2} = \frac{u}{b}$$

From the above one notes that

$$(5) \quad \omega_1 < \omega_2$$

Using  $ds^2 = h_\xi^2 d\xi^2 + h_\psi^2 d\psi^2$ , we may write down for the ellipse circumference as [cf. Eq. (A.3)]

$$(6) \quad s = c \int_0^{2\pi} h d\psi$$

Using the cardiac coördinates [cf. Appendix B] we may write the expression for the circumference of surface of heart (Figure 3). Please note that the expression given in Kamal & Siddiqui (1992) has roles of ‘ $a$ ’ and ‘ $b$ ’ reversed by mistake.

$$(7) \quad s(a, b, k) = b \int_{-\pi/2}^{\pi/2} \frac{d\psi}{\sqrt{1 + (a^2/b^2 - 1)\sin^2 \psi}} + b \int_{\pi/2}^{3\pi/2} \frac{d\psi}{\sqrt{1 + (k^2 a^2/b^2 - 1)\sin^2 \psi}}$$

If we assume that  $b < c$  (which is valid if the eccentricity,  $e = \frac{c}{a} < 0.7071$ ), we may expand this integrand

using the binomial series. Upon integration we obtain

$$(8) \quad s(a, b, k) = \pi b \left( 2 + \frac{A_1}{4} - \frac{3A_2}{64} + \frac{5A_3}{256} - \dots \right)$$

where  $A_i = (a^2/b^2 - 1)^i + (k^2 a^2/b^2 - 1)^i$ , ( $i = 1, 2, 3, \dots$ ). Therefore,

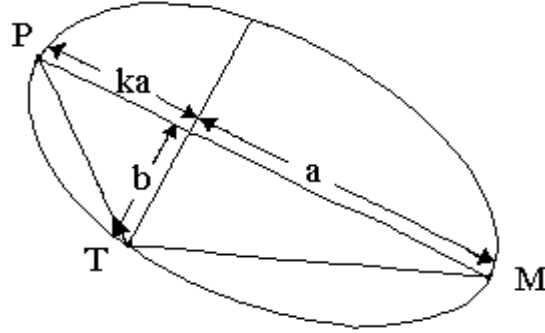
$$(9a) \quad \frac{\omega_1}{\omega_2} = \frac{s(a, b, k)}{2\pi b}$$

which may be expressed as

$$(9b) \quad \frac{\omega_{\text{higher}}}{\omega_{\text{lower}}} = \frac{\omega_1}{\omega_2} = 1 + \frac{A_1}{8} - \frac{3A_2}{128} + \frac{5A_3}{512} - \dots$$

Hence, we have evaluated ratio of the frequencies appearing in Fourier transform of the phonocardiogram. This model, therefore, relates ratio of these frequencies to the human heart size.

One way to check this model would be to determine ratio of these frequencies in a population having different heart sizes, which is, obviously, the pediatric population. In the next section we describe how to calcu-



**Fig. 4. Determination of the human heart parameters**

calculate this ratio from Eq. (9b). Comparison of  $(\omega_1/\omega_2)_{\text{calculated}}$  with  $(\omega_1/\omega_2)_{\text{experimental}}$  obtained from the phonocardiogram would indicate how closely the model describes dynamics of the human heart. A 2-D plot with  $(\omega_1/\omega_2)_{\text{experimental}}$  on the  $x$  axis and  $(\omega_2/\omega_1)_{\text{calculated}}$  on the  $y$  axis may be visually examined. The linear correlation coefficient of the two variables would provide a quantitative evaluation. The closer the correlation coefficient to unity, the better the fit.

#### *Determination of the Human Heart Parameters*

The ratio  $(\omega_2/\omega_1)_{\text{calculated}}$  may be determined from the location of heart sounds in the children and the adults. The values of 'a', 'b' and 'k' may be calculated by noting down the position of *PMI* (Point of Maximum Intensity). *PMI* is a point where the sound heard is the loudest for sounds generated by the mitral valve (*M*), the tricuspid valve (*T*) and the pulmonary valve (*P*). In the triangle *PTM* (Figure 4), let us call  $PM = \beta$ ,  $PT = \alpha$ ,  $TM = \gamma$ ,  $\angle TPM = \theta$ . From the cosine law

$$(10a) \quad \gamma^2 = \alpha^2 + \beta^2 - 2\alpha\beta \cos\theta$$

which may be rearranged as

$$(10b) \quad \cos\theta = \frac{\alpha^2 + \beta^2 - \gamma^2}{2\alpha\beta}$$

The heart parameters may be calculated using the relations

$$(11a) \quad b = \alpha \sin\theta$$

$$(11b) \quad a = \beta - \alpha \cos\theta$$

$$(11c) \quad k = \frac{\alpha \cos\theta}{\beta - \alpha \cos\theta}$$

The preliminary data suggest that location of the human heart sounds may provide an estimate of the heart shape. However, there was some difficulty in locating *PMI*. Therefore, it is suggested to estimate shape of the human heart using alternate methods.

## **DISCUSSION AND CONCLUSIONS**

A model of the human heart was presented in this paper. This model, based on a modified ellipsoid, appeared to be closer to reality as compared to the spherical or the bullet model. However, this model employed simplifications, *e. g.*, it assumed the human heart as an ellipsoid of revolution, which may not be the case in reality. Further, there are deviations from the ellipsoidal shape in the actual heart. The shape, also, varies from

person-to-person and during various ages.

It is, therefore, recommended that after comparing this model with the actual phonocardiogram frequencies heart shapes in the various age groups and the various populations should be studied to determine a better set of parameters to be used in the model. Validity of the assumption  $b < c$  should, also, be explored in all the age groups. It may also be interesting to find out if area of the human-heart-sound location triangle  $PTM$  varies when the child is standing, mild stretching, lying or squatting. Further, velocity of sound in the membrane and shift in the frequencies because of movement of the heart walls may be studied using the echocardiogram.

Acoustic properties of the human heart may also be studied by considering heart as a system of coupled masses and evaluating the vibrational modes similar to the model developed for the cerebral cortex (Ahmed *et al.*, 1997, Kamal, 1989; Kamal & Siddiqui, 1997; Kamal *et al.*, 1989; 1992a; b 1992; Siddiqui & Kamal, 1992; Siddiqui *et al.*, 1993).

The model presented above may lead to a better understanding of the human heart, help in developing better cardiac care (Figures 5a, b) and, eventually, lead to the development of an artificial heart, which simulates the real heart.

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Fig. 5a, b. Auscultation of heart — a proper auscultation using LITTMAN Classic II may discover any cardiac problems; undressing is mandatory to look for clubbing and cyanosis

### APPENDIX A: The Elliptical-Coördinate Mesh

Equation of an ellipse with ‘ $a$ ’ and ‘ $b$ ’ as measures of the semi-major and the semi-minor axes and center located at the origin may be written as (Figure 2):

$$(A.1) \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ or } x = a \cos \psi, y = b \sin \psi$$

Where  $(x, y)$  are the cartesian coördinates of a point  $P$  on an ellipse with respect to a two-dimensional coördinate system with origin at the center of ellipse and the  $x$  and the  $y$  axes coinciding with the semi-major and the semi-minor axes respectively. The unit vectors along these axes are represented as  $\hat{\mathbf{e}}_x$  and  $\hat{\mathbf{e}}_y$ . Coördinates of the foci are  $(-c, 0)$  and  $(c, 0)$ ;  $c^2 = a^2 - b^2$ . Let us write

$$a = c \cosh(c\xi), b = c \sinh(c\xi)$$

Hence, the elliptical coördinates  $(\xi, \psi)$  are related to the cartesian coördinates  $(x, y)$  by

$$(A.2) \quad x = c \cosh(c\xi) \cos \psi, y = c \sinh(c\xi) \sin \psi$$

where  $\psi$  is obtained by drawing a circle with center located at the ellipse center and radius equal to measure of the semi-major axis. Draw a normal from the point  $P(x, y)$  on the major axis. Extend normal on the other side to meet the circle at the point  $Q$ . The angle  $\psi$  is the angle between the major axis and the line joining  $Q$  and center of the circle. This angle is measured as the positive angle from the major axis to the line segment. One may identify the angle  $\psi$  as the eccentric anomaly in Astronomy (Marion, 1970; Goldstein, 1981).

Now  $d\mathbf{r} = h_\xi d\xi \hat{\mathbf{e}}_\xi + h_\psi d\psi \hat{\mathbf{e}}_\psi$  where  $\hat{\mathbf{e}}_\xi$  and  $\hat{\mathbf{e}}_\psi$  are unit vectors in the directions of increasing  $\xi$  and  $\psi$  respectively. The scale factors, ‘ $h$ ’ may be expressed as

$$(A.3) \quad h_\xi = c^2 h, h_\psi = ch$$

where  $h = [\sinh^2(c\xi) + \sin^2 \psi]^{1/2}$ . Note that  $\xi$  has the dimensions of  $(\text{length})^{-1}$ . The transformation of unit vectors may be written as

$$(A.4) \quad \mathbf{n}_1 = \frac{\mathcal{A} \mathbf{n}_2}{h}$$

where  $\mathbf{n}_1$  and  $\mathbf{n}_2$  are  $2 \times 1$  column vectors with their components given by  $(\mathbf{n}_1)_1 = \hat{\mathbf{e}}_x$ ,  $(\mathbf{n}_1)_2 = \hat{\mathbf{e}}_y$ ,  $(\mathbf{n}_2)_1 = \hat{\mathbf{e}}_\xi$ ,  $(\mathbf{n}_2)_2 = \hat{\mathbf{e}}_\psi$ . The transformation matrix,  $\mathcal{A}$ , is a  $2 \times 2$  matrix whose components are given by  $(\mathcal{A})_{11} = \sinh(c\xi) \cos \psi$ ,  $(\mathcal{A})_{12} = -\cosh(c\xi) \sin \psi$ ,  $(\mathcal{A})_{21} = \cosh(c\xi) \sin \psi$ ,  $(\mathcal{A})_{22} = \sinh(c\xi) \cos \psi$ . The inverse transformations may be expressed as

$$(A.5) \quad \xi = \frac{1}{2c} \ln \frac{a+b}{a-b}, \psi = \tan^{-1} \frac{ay}{bx}$$

The unit vectors transform as

$$(A.6) \quad \mathbf{n}_2 = \frac{\lambda \mathbf{n}_1}{h a b c}$$

The transformation matrix  $\lambda$  has the components  $(\lambda)_{11} = b^2 x$ ,  $(\lambda)_{12} = a^2 y$ ,  $(\lambda)_{21} = -a^2 y$ ,  $(\lambda)_{22} = b^2 x$ . The factor ‘ $h$ ’ may now be expressed in terms of  $x$  and  $y$  as

$$h = \frac{1}{c} \sqrt{\frac{b^2 x^2}{a^2} + \frac{a^2 y^2}{b^2}}$$

### APPENDIX B: The Cardiac-Coördinate Mesh

The human heart could be considered as a union of two semi ellipses (Figure 3), described by the following set of equation in cartesian coördinates ( $j = 0, 1$ ):

$$(B.1a, b) \quad \frac{x^2}{(\delta_{j0} + jk)^2 a^2} + \frac{y^2}{b^2} = 1$$

where  $(j - \frac{1}{2})\pi < \psi < (j + \frac{1}{2})\pi$  and  $\delta_{lm}$  is the Kronecker delta which is equal to unity if  $l = m$  and zero otherwise. The range of  $\psi$  remains the same in (B.3a-d), (B.4a-d), (B.5a, b) and (B.6a, b). The coordinates of foci of the two semi ellipses are  $(c, 0)$  and  $(-c, 0)$  respectively where

$$(B.2a, b) \quad c^2 = a^2 - b^2, c_1^2 = k^2 a^2 - b^2$$

The transformation of coordinates may be expressed as ( $c_0 = c$ )

$$(B.3a-d) \quad x = c_j \cosh(c\xi) \cos\psi, y = c_j \sinh(c\xi) \sin\psi$$

The inverse transformations may be written as

$$(B.4a-d) \quad \xi = \frac{1}{2c_j} \ln \frac{(\delta_{j0} + jk)a + b}{(\delta_{j0} + jk)a - b}, \psi = \tan^{-1} \frac{(\delta_{j0} + jk)ay}{bx}$$

The transformation of unit vectors is given by

$$(B.5a, b) \quad \mathbf{n}_1 = \frac{A_1 \mathbf{n}_2}{h}$$

Note that (B.5a) is the same as (A.4) and hence components of the vectors and the transformation matrix are already listed. Components of the transformation matrix  $A_1$  are obtained by replacing  $c_1$  in place of  $c$  for components of  $A$ . The inverse transformations are

$$(B.6a, b) \quad \mathbf{n}_2 = \frac{\lambda_1 \mathbf{n}_1}{habc}$$

Again (B.6a) is the same as (A.6) and hence components of the vectors and the transformation matrix are already listed. Components of the transformation matrix  $\lambda_1$  are obtained by replacing  $c_1$  in place of  $c$  for the components of  $\lambda$ .

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