EEG IN WEIGHTLESSNESS - A THEORETICAL ESTIMATE

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ABSTRACT

Effects of weak gravitational field on global electrocortical activity are considered in the framework of generalized coupling model developed earlier. First order shifts in EEG frequencies are calculated.

Keywords: Global electrocortical activity, magnetoencephaography, neuromagnetic response

INTRODUCTION

EEG (Electroencephalogram can be considered as a superposition of various electrical signals produced in the synaptic gap because of firing of neurons. These signals could be modeled as driven harmonic oscillators. Wright & Kydd (1984) developed a linear model for global electrocortical activity and its control by lateral hypothalamus. They proposed that the properties of the electrocortical waves be clearly distinguished from the microscopic and nonlinear interactions, which underlie them. The model was generalized (Kamal, Siddqui & Husain, 1989) to include magnetic fields. In this covariant model we wrote the equations of electrical potential in the dendritic trees in the comoving frame of signal. Group structure of this model was also explored (Siddqui *et al.*, 1993). A generalized coupling was also suggested (Kamal, 1989; Kamal & Siddiqui, 1997; Siddiqui & Kamal, 1992) in which the electromagnetic potentials not only depended on the neighboring potentials but also on their rates of change.

An external magnetic field ($\sim 10^{-16}$ tesla) produces a shift in the EEG frequencies. First-order corrections were estimated based on the covariant modal (Kamal *et al.*, 1992*a*) and generalized coupling model (Kamal *et al.*, 1992*b*). These may provide tests for the models.

In this paper we are presenting calculations of first-order shift in frequencies because of a weak gravitational field (*e. g.*, the gravitational field of earth). The inverse calculations give an estimate of EEG in weightlessness.

The Covariant Model

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The essential features of covariant model (Kamal *et al.*, 1989) are summarized. The equations for the time variation of potential of a dendritic tree were written in the comoving frame of the signal. The comoving frame was fixed to the potential wavefront of the dendrite. When this equation was transformed into the laboratory frame a magnetic vector potential appeared alongwith the electrostatic potential.

In the comoving frame of the signal a mass of unit sources coupled to each other may be represented by the following set of equations

(1)
$$\frac{d^2\Phi_i}{d\tau^2} + D_i(\tau)\frac{d\Phi_i}{d\tau} + N_i^2(\tau)\Phi_i = \sum_j K_i^j(\tau)\Phi_j$$

where $D_i(\tau)$, $N_i^2(\tau)$ and $K_i^j(\tau)$ are 4×4 matrices with eigenvalues $D_i^\mu(\tau)$, $N_i^\mu(\tau)$ and $K_i^{j\mu}(\tau)$ respectively. In the covariant model the eigenvalues corresponding to $\mu = 1, 2, 3$ were taken as zero. These numbers may be considered to be free parameters analogous to damping coefficients, natural frequencies and coupling constants. Φ is a 4×1 column vector with the first entry as the only nonzero entry representing the electrical potential φ. The quantity τ is time in the comoving frame of the signal, whereas t represents time as measured in the laboratory frame. Eq. (1) is transformed in the laboratory frame under the Lorentz transformation[3] λ. Under this similarity transformation the four-dimensional spacetime vector field and matrices take the form

(2a)
$$\Phi_i \Rightarrow A_i = \lambda_i \Phi_i$$

(2b)
$$D_i(\tau) \Rightarrow \varDelta_i(\tau) = \lambda_i D_i(\tau)\tilde{\lambda}_i$$

(2c)
$$N_i(\tau) \Rightarrow \eta_i(\tau) = \lambda_i N_i(\tau)\tilde{\lambda}_i$$

(2d)
$$K_i^j(\tau) \Rightarrow \kappa_i^j(\tau) = \lambda_i K_i^j(\tau)\tilde{\lambda}_i$$

$\tilde{\lambda} = g\lambda g$, where $g_{\mu\nu}$ is the metric tensor ($\mu, \nu = 0, 1, 2, 3$) with $g_{rr} = 1; r = 1, 2, 3$, $g_{00} = -1$, $g_{\mu\nu} = 0$ otherwise. Eq. (1), therefore, becomes

(3)
$$\frac{d^2 A_i}{d\tau^2} + \varDelta_i(\tau)\frac{dA_i}{d\tau} + \eta_i^2(\tau)A_i = \sum_j \kappa_i^j(\tau)A_j$$

The state transition matrix was constructed by defining new variables $Z_k = f(A_k, \frac{dA_k}{d\tau}); k = 1, ..., m$ where $m = 2n$ (n the number of dendritic trees considered in the model, usually $\sim 10^{15}$). Let us define a dimensionless parameter $t_{scale} = \frac{\tau}{\Omega}$ (Ω is a scaling parameter which may be taken as the average time of travel of a signal between two neurons). The coördinates are, therefore, defined as

(4a)
$$Z_k = A_k, \text{ if } k \text{ is an odd number}$$

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[3]One may think it is unnecessary to use Lorentz transformations when the velocities of the signal are ~ 10 *m/s*. However, this elegant formulation is necessary because it permits us to write equations in compact form and allows us to apply laws of electromagnetism. Remember that Maxwell equations do not retain their form under Galilean transformations, no matter how small velocity of the moving frame is.

(4b)
$$Z_k = \frac{dA_{k-1}}{dt_{scale}}, \text{ if } k \text{ is an even number}$$

In terms of Z_k, the system of equations (3) may be written as

(5)
$$\frac{d\mathcal{Z}}{dt} = \mathcal{A}_{cov}\mathcal{Z}$$

where $\mathcal{Z} = [Z_k]$ is a column vector and $\mathcal{A}_{cov}$ is covariant state transition matrix. The state transition matrix is a function of D's, N's and K's and Ω. Scaling parameter Ω is introduced to make all the elements of the state transition matrix dimensionless. The state transition matrix is a linear transformation, which connects the components of four-dimensional vector potential field to their rates of change.

Generalized Coupling Model

Generalized coupling model of global electrocortical activity has been described elsewhere (Kamal & Siddiqui, 1997; Kamal *et al.*, 1989; 1992b). Below are given the highlights. In the comoving frame of signal passing through a segment of dendritic tree, the electrical potential for a mass of unit sources coupled to each other may be represented by

(6)
$$\frac{d^2\varphi_i}{dt_2} + D_i(t)\frac{d\varphi_i}{dt} + N_i^2(t)\varphi_i = \sum_j \left[K_i^i(t)\varphi_j + M_i^j(t)\frac{d\varphi_j}{dt} \right]$$

The M_i^j's are free parameters given physiological meaning under the assumptions that they have a finite variance σ_M about $\overline{M}$, and they are stochastically independent. To set up a covariant formulation of generalized coupling model let us write the electrical potential variation for a mass of unit sources coupled to each other in the comoving frame of signal passing through a segment of dendritic tree as

(7)
$$\frac{d^2\Phi_i}{d\tau^2} + D_i(\tau)\frac{d\Phi_i}{d\tau} + N_i^2(\tau)\Phi_i = \sum_j \left[K_i^j(\tau)\Phi_j + M_i^j(\tau)\frac{d\Phi_j}{d\tau} \right]$$

where $M_i^j(\tau)$'s are 4 x 4 matrices having eigenvalues $M_i^{j\mu}(\tau)$; $\mu = 0, 1, 2, 3$. No particular type of distribution for $M_i^{j\mu}(\tau)$ is assumed. All other symbols are defined previously.

A similarity transformation under λ_i transforms the various four-dimensional spacetime vector fields and matrices [cf. Eq. (2a-d)]. The $M_i^j(\tau)$'s transform as

(2e)
$$M_i^j(\tau) \Rightarrow \mu_i^j(\tau) = \lambda_i M_i^j(\tau)\tilde{\lambda_i}$$

Eq. (7), therefore, becomes

(8)
$$\frac{d^2 A_i}{d\tau^2} + \Delta_i(\tau)\frac{dA_i}{d\tau} + \eta_i^2(\tau)A_i = \sum_j \left[\kappa_i^j(\tau)A_j + \mu_i^j(\tau)\frac{dA_i}{d\tau} \right]$$

Coördinate Transformation for Weak Gravitational Field

The Lorentz transformation matrix used as a similarity transformation in the models of electrocortical activity is valid only in flat spacetime of special theory of relativity. In order to calculate the effects of weightlessness or other effects related to gravitation on the electrocortical activity we must take into account the curvature of spacetime. In other words, we must find a transformation valid in curved spacetime (Ahmed, 1990).

Let's call the new transformation Λ. If Λ is a coördinate transformation then it must satisfy the following conditions

(9)
$$\Lambda_\alpha^\mu \Lambda_\beta^\nu g_{\mu\nu} = g_{\alpha\beta}$$

(10)
$$\Lambda_\alpha^\beta x^\alpha = x^\beta$$

where $g_{\alpha\beta}$ is the metric of that particular field.

Since we are only interested in determining the effects of gravity on the electrocortical activity in situations such as earth's gravitational field. We, therefore, may use weak field approximations of Einstein's general theory of relativity. Such transformations are called *background Lorentz transformations* in the literature of general theory of relativity because they are similar to Lorentz transformations of special relativity. They satisfy both conditions of the Lorentz transformations [*cf*. Eq. (9), (10)]. The only difference is that the minkowski metric ($g_{00} = -1$, $g_{ii} = 1$, $g_{\alpha\beta} = 0$ if $\alpha \neq \beta$; $\alpha, \beta = 0, 1, 2, 3$; $i = 1, 2, 3$) is replaced by a particular spacetime metric. The gravitational field of earth is termed as *weak* and may be described by the metric

(11*a-c*)
$$g_{00} = -(1 + 2\phi_{\text{Newton}}), \; g_{ii} = 1 - 2\phi_{\text{Newton}}, \; g_{\alpha\beta} = 0 \text{ if } \alpha \neq \beta$$

where ϕ_{Newton} is the newtonian potential correct up to first order, *i. e.*, $\phi_{\text{Newton}} \ll 1$. For a boost of velocity v in $+x$ direction we may find Λ_α^β easily using (9, 10). The nonzero elements are listed below

(12*a-c*)
$$\Lambda_0^0 = \omega = \Lambda_1^1, \; \Lambda_2^2 = 1 = \Lambda_3^3, \; \Lambda_1^0 = \rho v \omega = \Lambda_0^1$$

where $\rho = \dfrac{1 - 2\phi_{\text{Newton}}}{1 + 2\phi_{\text{Newton}}}$, $\omega = \dfrac{1}{\sqrt{1 - \rho v^2}}$. We have taken c (velocity of light in free space) = 1. These coördinate transformations may easily be generalized for a boost of velocity in an arbitrary direction. The general transformations are

(13*a-d*)
$$\Lambda_0^0 = \omega, \; \Lambda_i^0 = \omega v_i \rho, \; \Lambda_0^i = \omega v^i, \; \Lambda_j^i = \delta_j^i + v^i v_i \frac{\omega - 1}{v^2}$$

The new transformation is valid only for homogeneous, isotropic, non-time-varying and weak gravitational fields. The earth's gravitational field satisfies these conditions to a good degree of approximation.

EFFECTS OF WEIGHTLESSNESS ON EEG

With the background Lorentz transformations in hand we are now able to calculate the effects of different phenomena related to gravity on the electrocortical activity of brain. We shall perform our calculations on the covariant generalized coupling model (Kamal & Siddiqui, 1997). First we shall write the basic harmonic oscillator equations in comoving frame of the signal and then transform them to lab frame by background Lorentz transformation using it as similarity transformation.

At the surface of earth the damped harmonic oscillator equations, in the comoving frame of the signal are given by Eq. (7). This takes the form of Eq. (8) in the laboratory frame. Let us suppose that in the condition of weightlessness the four-dimensional-spacetime-vector-potential field A_i becomes $A_i^{'}$ and the natural frequencies $N_i^\mu(\tau)$ become $N_i^{'\mu}(\tau)$. If we assume that there is no shift in damping coefficients and coupling parameters we may write

$$(14) \qquad \frac{d^2\Phi_i^{'}}{d\tau^2} + \boldsymbol{D}_i(\tau)\frac{d\Phi_i^{'}}{d\tau} + N_i^2(\tau)\Phi_i^{'} = \sum_j \left[\boldsymbol{K}_i^j(\tau)\Phi_j^{'} + \boldsymbol{M}_i^j(\tau)\frac{d\Phi_j^{'}}{d\tau} \right]$$

In the lab frame Eq. (14) takes the form

$$(15) \qquad \frac{d^2A_i^{'}}{d\tau^2} + \varDelta_i(\tau)\frac{dA_i^{'}}{d\tau} + \eta_i^2(\tau)A_i^{'} = \sum_j \left[\kappa_i^j(\tau)A_j^{'} + \mu_i^j(\tau)\frac{dA_j^{'}}{d\tau} \right]$$

Subtracting (8) from (15) we get

$$\left(\frac{d^2A_i^{'}}{d\tau^2} - \frac{d^2A_i}{d\tau^2} \right) + \varDelta_i(\tau)\left(\frac{dA_i^{'}}{d\tau} - \frac{dA_i}{d\tau} \right) + \eta_i^2(\tau)\left(A_i^{'} - A_i \right) = \sum_j \left[\kappa_i^j(\tau)\left(A_j^{'} - A_j \right) + \mu_i^j(\tau)\left(\frac{dA_j^{'}}{d\tau} - \frac{dA_i}{d\tau} \right) \right]$$

Keeping in view the relation $\dfrac{dA_i^{'}}{d\tau} = \omega^{'}\dfrac{dA_i}{d\tau}$, where

$$(16a,b) \qquad \omega^{'} = \frac{1}{\sqrt{1 - \rho^{'}v^2}}, \quad \rho^{'} = \frac{1 - 2\phi_{\text{Newton}}^{'}}{1 + 2\phi_{\text{Newton}}^{'}}$$

$\phi_{\text{Newton}}^{'}$ being the newtonian potential in the condition of weightlessness. The shift in frequencies is, thus, given by

$$(17) \qquad \left(N_i^{'\mu} \right)^2 = \left[\frac{v^2}{2\omega}(\rho^{'} - \rho) + \frac{\omega}{\omega^{'}} \right]\left(N_i^{\mu} \right)^2 ; \mu = 0, 1, 2, 3$$

In the condition of weightlessness the gravitational potential is constant which may be taken to be zero. Using this condition and a value of signal velocity equal to 20 *m/s* we may calculate the shift in natural frequency as compared to the frequency at the surface of earth. The calculated shift is

$$(18) \qquad \frac{\Delta N}{N} = \frac{N^{'} - N}{N} = 2.515 \times 10^{-6}$$

Hence the shift comes out to be of the order of 2 parts per million.

CONCLUSION

The background Lorentz transformations calculated in this work rest on many assumptions. The question is to find out the invariants of this transformation. We have used the invariance of the metric $g_{\alpha\beta}$ of the linearized theory to determine the components of the new transformation Λ.

The calculated shifts in frequencies for the condition of weightlessness is very small (~2 parts per million). This was expected since significant changes in frequencies are observed only in large brain disorders and no such incidence has occurred to any astronaut traveling in space. One way to check this result experimentally is to compare the EEG taken in weightlessness with the one taken on earth using computerized matching techniques to possibly look for modulated amplitudes because of slight shift in frequencies.

This work, therefore, introduces curved spacetime methods for the estimation of EEG. This may, in future be generalized for medium strong and strong gravitational fields. The experiments done in artificial gravity may, then, provide a test for general relativity.

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