A 3-D STATIC MODEL OF THE HUMAN SPINAL COLUMN

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ABSTRACT

A 3-D static model of the human spinal column put forward by the author is refined. A method is given to generate outline of spine in three dimensions from moiré topograph of back.

Keywords: Moiré fringe topography, scoliosis, spine

INTRODUCTION

Spinal column is a three-dimensional problem. Ordinary anteroposterior X rays only show the spine in frontal plane. Such an X ray would not show kyphosis or lordosis which could only be observed in the sagittal plane. The biomechanics researchers have recognized this and developed spinal models in three dimen-sions (Hierholzer & Lüxmann, 1982; Kamal, 1982; 1983; 1987). These models could generate the human spinal column from projections of spine in the frontal and the sagittal planes obtained from X rays.

Since X rays are harmful to a growing child, a technique is needed which could provide information aboutb the human spinal column without using ionizing radiations. Integrated Surface Imaging System (ISIS), moiré fringe topography and rasterstereography are such noninvasive, noncontact 3-D photogrammetric techniques which can quantify spinal deformities through surface measurements of the human back. In particular, moiré techniques are inexpensive, simple to operate, provide permanent record of the patient's condition, offer simple surface relationships to moiré contours, do not require the patient to be motionless unlike holographic techniques, and employ ordinary white light and cameras. The techniques have been used for the diagnosis, documentation and follow-up of spinal deformities especially scoliosis (Adair, van Wijk & Armstrong, 1977; Kamal & El-Sayyad, 1981; Kamal & Lindseth, 1980).

In the simplest words the technique of moiré topography consists of photographing the part of body through a specially constructed screen. Dark fringes are produced on body because of the presence of screen. If the light source and the camera lie on a line parallel to the plane of the moiré screen the fringes on human body are contours of constant distance from the screen. This mathematical property can be used for surface mapping and developing different algorithms for surface relationships.

In this paper refinement of a three-dimensional static model of the human spinal column developed earlier (Kamal, 1982; 1983; 1987) is presented. Using this model an outline of the spinal column in three dimensions may be generated from the moiré topographs of back in various positions.

Three-dimensional Model of the Spinal Column

To draw outline of the spinal column from a back moiré topograph consider Fig. 1. Using palpation a spinous process in the neck area, P, is found. This is, then, joined to a spinous process in the pelvic area, Q. To find the position of the spine at a given point draw a line perpendicular to PQ.

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Let this line intersect PQ at C and a particular moiré fringe at H and E such that E is always on the right side of H. The midpoint O of the line segment HE is assumed to give the position of the spine, provided the positioning during the X-ray and moiré examinations is identical. Several such points may be found along the line segment PQ, preferably 33 to correspond to the 33 vertebrae of the spinal column, and a spinal outline in the frontal plane (yz plane in Fig. 2) could be drawn as the best curve fitting these points. Since moiré topograph is a map of heights, spinal outline in the sagittal plane (xz plane in Fig. 2) may be drawn by plotting heights from a reference point, say, directly behind naval.

X rays of scoliosis patients were compared with the predictions of this model in Malmö General Hospital, Malmö, Sweden. Spinal outlines in the lateral plane were drawn on moiré topographs of backs of 10 scoliosis patients without looking at their X rays. In all cases the method gave the correct sign of curvature in all regions of the spinal column. Quantitatively, the differences observed in the values of the curvatures may be attributed to rotation, which is not incorporated in this model. The preliminary results indicate that this model works well in the absence of rotation. Hence, the model is ideal for drawing spinal outlines of normal children in various positions.

From the measurements performed on the moiré topographs of back, a curve is generated, $x = x(\xi)$, $y = y(\xi)$, $z = z(\xi)$, which is a best fit to the discrete measurements performed at various locations represented by the parameters, $\xi_1, \xi_2, \ldots, \xi_n$; $n = 33$ corresponding to the 33 vertebrae of the backbone. The parameter ξ_i may be taken as representing the length measured along the spinal cord with origin in the neck region. In the neighborhood of any point on the spinal column, let us write

(1) $x = f(y,z) = (1/2)\, ay^2 + byz + (1/2)\, cz^2$

where $a = a(\xi_i)$, $b = b(\xi_i)$ and $c = c(\xi_i)$, the values of which could be found by solving simultaneous equations generated by eq. (1) using three neighboring values of x, y and z, that is

(2*a-c*) $x(\delta_i) = (1/2)\, a(\xi_i)\, y(\delta_i) + b(\xi_i)\, y(\delta_i)\, z(\delta_i) + (1/2)\, c(\xi_i)\, z(\delta_i)$

where $\delta_i = \xi_i - \Delta$ in (2*a*), $\delta_i = \xi_i$ in (2b) and di = xi + D in (2c) where D << ξ_i. It is assumed that the values of a, b and c are same for these closely located points. The values of x, y and z could be measured from the moiré topograph of back.

The coördinate x represents deviation of the curve from the yz plane. Rotating the y, z axes by an angle a clockwise about the x axis

(3*a*) $y = y_{rot} \cos\alpha + z_{rot} \sin\alpha$

(3*b*) $z = - y_{rot} \sin\alpha + z_{rot} \cos\alpha$

where

(4) $\alpha = (1/2) \tan^{-1} [2b/(c - a)]$

Eq. (3*a,b*) may, therefore, be written as

(5) $x = (1/2)\kappa_{1i}\, y_{rot}^2 + (1/2)\kappa_{2i}\, z_{rot}^2$

where

(6*a*) $\kappa_{1i} = a + c - 2b^2/[4b^2 + (c - a)^2]$

(6*b*) $\kappa_{2i} = a + c + 2b^2/[4b^2 + (c - a)^2]$

The patient is then asked to hang freely from a bar and the improvement in the deformity is noted. The curvatures are again measured after guarded graduated passive correction as κ'_{1i}, κ'_{2i}; i = 1,2,....,n. The degree of correction of trunk deformity is defined as

(7) $D = (50/n)\,\Sigma_i\,[(1 - \kappa'_{1i}/\kappa_{1i})^2 + (1 - \kappa'_{2i}/\kappa_{2i})^2]$

The trunk deformity is classified as *severe*, intermediate or *mild* if D lies between 0.00-33.33, 33.34-66.66, 66.67-100.00 respectively. Geometrically if

$\kappa'_{1i} = \kappa_{1i}, \kappa'_{2i} = \kappa_{2i}; i = 1,2,.....,n$

there is no correction and D = 0. On the other hand, if

$\kappa'_{1i} = \kappa'_{2i} = 0; i = 1,2,...,n$

the deformity is completely corrected and D = 100.

Refinement of the Mathematical Model

The model presented above is assuming that the normal spine is straight like a straight line. This is not the actual case. In reality there are natural curvatures of spine. If these curves are accentuated they may result in kyphosis or lordosis. Considering the natural curvatures of spine eq. (5) for a normal child becomes

(8) $x = (1/2)K_{1i}\,y_{rot}^2 + (1/2)K_{2i}\,z_{rot}^2$

where K_{1i} and K_{2i} are the natural curvatures of a normal child. If K'_{1i} and K'_{2i} are the curvatures in the hanging position, the modified degree of correction of spinal deformity is defined as

(9) $D = (50/n)\,\Sigma_i\,[(\kappa_{1i} - \kappa'_{1i})^2/(\kappa_{1i} - K'_{1i})^2 + (\kappa_{2i} - \kappa'_{2i})^2/(\kappa_{2i} - K'_{2i})^2]$

The trunk deformity is classified as severe, intermediate or mild if D lies between 0.00-33.33, 33.34-66.66, 66.67-100.00 respectively. Geometrically if

$\kappa'_{1i} = \kappa_{1i}, \kappa'_{2i} = \kappa_{2i}; i = 1,2,.....,n$

there is no correction and $D = 0$. On the other hand, if

$\kappa'_{1i} = K'_{1i}, \kappa'_{2i} = K'_{2i}; i = 1,2,...,n$

the deformity is completely corrected and $D = 100$.

CONCLUSION

An extension of this model for application to dynamic studies is desirable if we want to study spinal column during highly coördinated motor skills like gymnastic performance. For such studies a projection type variable focus FM-80 moiré camera will be needed. Moiré techniques will make possible studies which could not be done using X rays because of ethical considerations.

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REFERENCES

Adair, I. V., van Wijk, M. C. & Armstrong, G. W. D. *Clin. Orthop.* **129**: 165-171(1977).

Hierholzer, E. & Lüxmann, G. *J. Biomech.* **15**: 583-598 (1982).

Kamal, S. A. *Bull. Am. Phys. Soc.* **27**: 301 (1982).

Kamal, S. A. In: Moiré Fringe Topography and Spinal Deformity, Proceedings of the 2nd International Symposium, Münster, Germany, (ed. B. Drerup, W.Frobin & E. Hierholzer), Stuttgart & New York, Gustav Fischer, pp. 117-126 (1983.

Kamal, S. A. In: ***Surface Topography and Spinal Deformity, Proceedings of the 4th International Symposium***, Mont Sainte Marie, Québec, Canada, (ed. I. A. F. Stokes, J. R. Pekelsky & M. S. Moreland), Stuttgart & New York, Gustav Fischer, pp. 43-50 (1987).

Kamal, S. A. & El-Sayyad, M. M. *Med. Phys.* **8**: 549 (1981).

Kamal, S. A. & Lindseth, R. E. ***Periodic Structures, Gratings, Moiré Patterns and Diff-raction Phenomena***, Proc. Soc. Photo-Opt. Instr. Eng. **240**: 293-295 (1980).

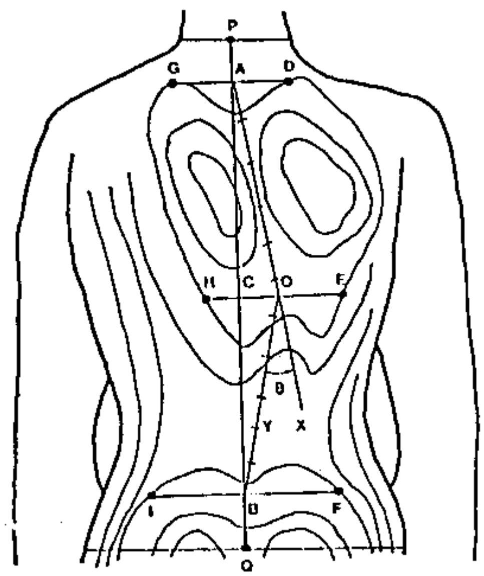

Fig. 1. Moiré topograph of the human back.

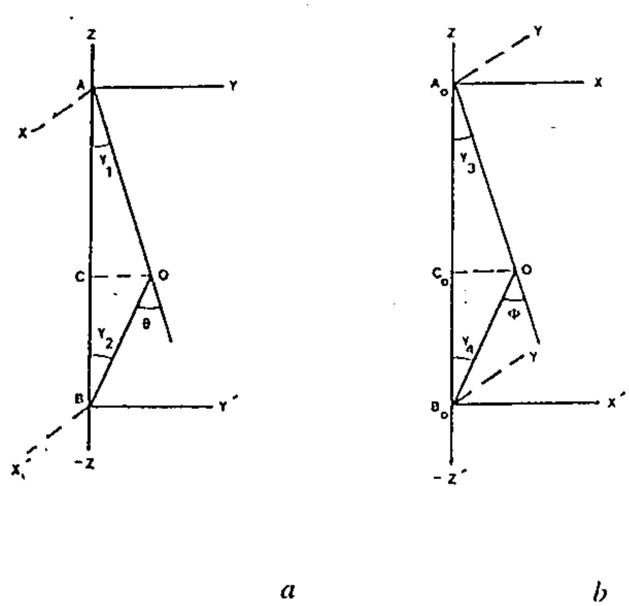

Fig. 2a,b. Projections of a point O on the spine in (*a*) yz (frontal) and (*b*) xz (sagittal) planes.

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