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EFFECTS OF WEAK MAGNETIC FIELDS ON GLOBAL ELECTROCORTICAL ACTIVITY

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ABSTRACT

Effects of weak magnetic fields are considered on a recently proposed covariant model of global electrocortical activity. A method to calculate the ratio of components of signal velocities is given.

Keywords: Global electrocortical activity, magnetoencephalography, neuromagnetic response

Introduction

We have recently proposed a generalization (Kamal, Siddiqui, & Husain, 1989) of a linear model of Wright and Kydd (1984) for global electrocortical activity. In this covariant model we have written the equations of electrical potential in the dendritic trees in the comoving frame of the signal. Transformation to laboratory frame has generated magnetic fields (Recall that a stationary electron appears as current in the moving frame).

Magnetic fields of the brain are studied experimentally by many groups especially for localization of epileptic foci (Romani, 1987; Ricci, Romani, Salustri, Pizzella, Torrioli, Buonomo, Peresson, & Papanicolaou, 1987; Narici, Romani, Salustri, Pizzella, Torrioli, & Modena, 1987). It would be of interest to include the effects of weak magnetic fields in our covariant model. We are presenting here a generalization of the covariant model proposed earlier (Kamal Siddiqui & Husain 1989). It is shown that a weak magnetic field, modifies the frequencies.

Covariant Model

We summarize the essential features of our covariant model (Kamal, Siddiqui, & Husain, 1989). We have written the equation for the time variation of potential of a dendritic tree in the comoving frame of the signal. The comoving frame is fixed to the potential wavefront of the dentrite. When this equation was transformed into the laboratory frame a magnetic vector potential appeared along with the electrostatic potential.

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In the comoving frame of the signal a mass of unit sources coupled to each other may be represented by (j runs from 1 to n subject to the condition that $i \neq j$)

$$(1) \qquad \ddot{\phi}_i + D_i(\tau)\dot{\phi}_i + N_{i}^2(\tau)\phi_i = \sum_j \Re_i^{\,j}(\tau)\phi_j$$

where $D_i(\tau)$, $N_i(\tau)$ and $\Re_i^{\,j}(\tau)$ are 4 x 4 matrices. The only nonzero entries are at the intersection of first row and first column; which are $D_i(\tau)$ and $K_{ij}(\tau)$ respectively. These are free parameters analogous to damping coefficients, natural frequencies and coupling constants. ϕ_i is a 4 x 1 column vector with the first entry as the only nonzero representing the electrical potential ϕ_i. The quantity t is time in the comoving frame of the signal. Let us transform eq. (1) in the laboratory frame under the Lorentz transformation λ_i. Under this similarity transformation the four vectors and matrices take the form

(2a) $\phi_i \rightarrow A_i = \lambda_i \phi_i$
(2b) $D_i(\tau) \rightarrow \Delta_i(\tau) = \lambda_i D_i(\tau)\tilde{\lambda}_i$
(2c) $N_i(t) \rightarrow n_i(t) = \lambda_i N_i(\tau)\tilde{\lambda}_i$
(2d) $\Re_i^{\,j}(t) \rightarrow k_i^{\,j}(\tau) = \lambda_i \Re_i^{\,j}(\tau)\tilde{\lambda}_i$

$\tilde{\lambda} = g\lambda g$ where $g_{\mu\gamma}$ is the metric tensor ($\mu, \gamma = 0.1, 2, 3$) with $g_{oo} = -1$, $g_{rr} = 1$; $r = 1, 2, 3$, $g_{\mu\gamma} = 0$ otherwise. Eq. (1), therefore becomes

$$(3) \qquad \ddot{A}_i + \Delta_i(\tau)\dot{A}_i + n_i^{\,2}(\tau)A_i = \sum_j k_i^{\,j}(\tau)A_j$$

The state transition matrix was constructed by defining new variables $\Omega_k = f(A_k, \dot{A}_k)$, $k = 1, \ldots, m$ where $m = 2n$ (n is the number of dendritic trees considered in the model, usually of the order of 10^{15}). Let us define a dimensionless parameter $t = \tau/\varepsilon$ (ε is a scaling parameter which can be taken as the average time of travel of a signal between two neurons). The coordinates are defined as

(4a) $\Omega_k = A_k$ if k is an odd number
(4b) $\Omega_k = dA_{k-1}/dt$ if k is an even number

In terms of Ω_k, eq. (3) can be written as

$$(5) \qquad dZ/dt = AZ$$

where $Z = [\Omega_k]$ is a column vector and A is the state transition matrix. The state transition matrix is a function of D's, N's and K's and ε. The scaling parameter ε is introduced to make all the elements of the state transition matrix dimensionaless. The state transition matrix is a linear transformation which connects the four potentials to their rate of change. Since we are transforming under a similarity transformation the eigenvalues remain invariant. Hence

(6a) $D_{lab} = D_{comoving}$
(6b) $N_{lab} = N_{comoving}$
(6c) $K_{lab} = K_{comoving}$

Generalization of the Covariant Model

In the covariant model described above we have written 4 x 4 matrices in place of simple parameters representing damping co-efficients, coupling constants and natural frequencies. However, in all the matrix representations in the comoving frame of the signal only one of the eigenvalues is non zero. In the spirit of complete symmetry among all the space-time coordinates we are introducing 4 eigenvalues for each parameter i.e. the diagonal entries of the matrices $D_i(\tau)$, $N_i(\tau)$ and $\mathfrak{R}_{ij}(\tau)$ are $D_i^{\mu}(\tau)$, $N_i^{\mu}(\tau)$ and $K_i^{ju}(\tau)$; $\mu = 0, 1, 2, 3$ respectively. All the nondiagonal entries are zero. The eigenvalues corresponding to $\mu = 0$ are the ones present even in the absence of magnetic fields. These will show up in the model of Wright and Kydd (1984). The other eigenvalues would not be observed because their projections on the four vector ϕ_i will vanish. These eigenvalues will show up when we transform eq. (1) to laboratory frame as (3). In the next section we shall consider the effects of weak magnetic fields on these eigenvalues.

Weak Magnetic Fields

Let us apply a weak magnetic field B_{ext} ($<< 10^{-14}$ tesla) which is assumed to be uniform throughout the region concerned (Since the ambient magnetic field noise $>> 10^{-14}$ tesla magnetically shielded room is needed for such studies). The magnetic field, not varying with time, is generated by a vector potential $\mathbf{A}_{ext}$ (x,y,z) such that $\mathbf{B}_{ext}$ ∇ x $\mathbf{A}_{ext}$ (For the purpose of testing our covariant model we choose the required form of $\mathbf{A}_{ext}$ and calculate the magnetic field by taking the curl. This magnetic field is externally applied to calculate the first-order shift in frequencies). Let us write a 4-potential as

$$(7) \qquad A_{ext} = \begin{bmatrix} 0 \\ \mathbf{A}_{ext} \end{bmatrix}$$

The components of this 4-potential are denoted by A_{ext}^{μ}; $\mu = 0,1,2,3$. The presence of weak magnetic field will not affect the Lampling coefficients and coupling of the individual A's. In the presence of this weak magnetic field the natural frequencies $N_i(\tau)$ will be modified to, say, $N_i'(\tau)$. In the lab frame eq. (3) now takes the form

$$(8) \qquad \ddot{A}_i' + \Delta_i(\tau)\dot{\mathbf{A}}_i' + n_i'^2(\tau)\, A_i' = \Sigma\, k_i^j(\tau)\, A_j'$$

where $A_i' = A_i + A_{ext}$. Since the signal velocities are very small as compared to the velocity of light, we take $t \approx \tau$ (t is time in the laboratory frame). The external field A_{ext} (x, y, z) does not depend on time. We, therefore, have

$$\ddot{A}_i' = \ddot{A}_i, \dot{A}_i' = \dot{A}_i$$

Subtracting (3) from (8) and introducing $n_i'^2 = n_i^2 + \delta n_i^2$, ($\delta n_i$'s are 4 x 4 matrices having eigenvalues δf^{μ}; $\mu = 0, 1, 2, 3$ representing the shifts in frequencies) we have

$$\delta n_i^2 A_i' = - (n_i^2 - \sum_j k_i^j) A_{ext}$$

we assume that δn_i, k_i^j and n_i can all be simultaneously diagonalized. In other words $[\delta n_i, k_i^j] = O$, $[\delta n_i, n_i] = 0$ etc. Applying the transformation $\delta n_i \rightarrow \lambda_i \delta n_i \lambda_i = \delta N_i$ etc. to write the above equation in the comoving frame of the signal, we have

$$\delta N_i^2 \tilde{A}_i' = - (N_i^2 - \sum_j \Re_i^j) (\tilde{A}_{ext})_i \tag{9}$$

where $\tilde{A}_i' = \phi_i + (\tilde{A}_{ext})_i$; $(\tilde{A}_{ext})_i = \lambda_i A_{ext}$. The matrices δN_i, N_i and $\Re_i^j$ are already diagonalized. Eq. (9) will yield four equations each one sufficient to determine an eigenvalue δf^{μ}. The results are

$$(\delta f^{\mu})^2 = [\sum_j K_i^{j\mu} - (N_i^{\mu})^2] \Theta_i^{\mu} \tag{10}$$

where $\Theta_{io} = - \gamma_i A_{ext} \cdot V_i / (\theta_i - \gamma_i A_{ext} \cdot V_i)$, $\Theta_{ir} = 1; r = 1, 2, 3.$

Although there are no well defined frequencies in the EEG pattern the frequencies have a finite variance about $<\omega>$. In such cases according to Crammer's central limit theorem the individual frequencies in our covariant model may be replaced by $<\omega>$ (Wright and Kydd, 1984). Therefore the shift in frequencies calculated here must be interpreted as shift in the average frequency for the purpose of experimental tests.

Conclusions

Upon examination of eq. (10) the following conclusions can be drawn: (a) Since the term $\gamma_i A_{ext} \cdot V_i$ is very small as compared to θ_i, we expect that

$$\delta_i^{o} \alpha [A_{ext}]^{1/2}$$

Taking the averages we expect that $< \delta^{o} > \alpha\ [A_{ext}]^{1/2}$. This conclusion may be checked experimentally. The other components δ_{ir}; r = 1, 2, 3 are independent of the applied field.

(b) If we apply magnetic field of same magnitude in the x-, y- and z- directions and observe δ_i^{o} in each case, we can measure the rates of velocities of signals in different directions

$$(\delta_i^{o})_x : (\delta_i^{o})_y : (\delta_i^{o})_z = \sqrt{(v_i)_x} : \sqrt{(v_i)_y} : \sqrt{(v_i)_z}$$

If we apply Central Limit Theorem of Cramer to replace each individual frequency and velocity by its average, the above equation may provide a way to evaluate the ratio of different components of signal velocity.

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