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Section A: Present & Future Perspectives

THE DIRECTION QIBLA (MAKKA)

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A mathematical derivation of the direction of Qibla based on derivation of the geodesic is given for any point on the surface of earth. This should help to resolve the controversy regarding the Qibla direction in North America.

INTRODUCTION

The knowledge of the direction of *Qibla* (*Ka'aba*) is essential for the correctness of salat (prayer). It is the duty of every muslim to make an effort to find out the direction of Qibla and turn his face towards it before starting the prayer. Allah Subhanahu wa Ta'ala says in the Holy Quran:

وَمِنْ حَيْثُ خَرَجْتَ فَوَلِّ وَجْهَكَ شَطْرَ الْمَسْجِدِ الْحَرَامِ

"And whence-so-ever you come forth (for prayer) turn your face in the direction of the Sacred Mosque" (Sura 2:149).

This paper deals with the problem of calculating the direction of *Qibla*, and provides a mathematical procedure, taking into account the curvature of earth.

STATEMENT OF THE PROBLEM

In the United States of America, the direction of *Qibla* is, generally, taken as northeast, based on the definition that the direction of *Qibla* at a place is tangent to the geodesic (*i. e.*, the shortest distance along the surface of the globe) connecting that place and Makka. On the other hand, a group claims that because the United States (of

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America) is in the northwest of Saudi Arabia, so the direction of *Qibla* for the people in the United States is southeast. Its definition of the direction of *Qibla* is the following. Erect a normal to the surface of earth at Makka and at the place, where the direction of *Qibla* is to be determined. These two lines determine a plane. Since the earth is very nearly a perfect sphere (equatorial and polar radii do not differ by more than 0.35 %), the two normals pass through the center of earth and, hence, are coplanar. The direction of *Qibla* is the tangent to the curve made by the intersection of this plane and the surface of earth.

In order to resolve this discrepancy, the equation of geodesic is obtained using the calculus of variations in the next section.

CALCULATION OF GEOESIC

A geodesic is a curve, which represents the shortest path between any two points, when the path is constrained to lie on some surface. Let the origin of the coordinate system be the center of earth and the z axis coincide with the line joining the poles. Let, (r,θ,φ) be the spherical-polar-coördinate mesh.

The relationship between the cartesian and the spherical-polar coördinate meshes is given by:

$$x = r\sin\theta\sin\varphi\,;\;\; y = r\sin\theta\cos\varphi\,; z = r\cos\theta$$

The element of length on the surface of a sphere of radius R is given by

(1) $$ds = R\sqrt{d\theta^2 + d\varphi^2\sin^2\theta}$$

because $dr = 0$. The distance s between points P and Q on the sphere is

(2) $$s = R\int_P^Q d\varphi\sqrt{(\frac{d\theta}{d\varphi})^2 + \sin^2\theta}$$

Let

(3) $$f = \sqrt{\theta'^2 + \sin^2\theta}$$

where $\theta' = \frac{d\theta}{d\varphi}$. The distance *s* shall be maximum if *f* satisfies the Euler equation:

(4) $$\frac{\partial f}{\partial \varphi} - \frac{d}{d\varphi}(f - \theta' \frac{\partial f}{\partial \theta'}) = 0$$

Substituting *f* from eq. (3) and some algebraic manipulation yields:

(5) $$\frac{d\theta}{d\varphi} = \frac{a \csc^2 \theta}{\sqrt{1 - a^2 \csc^2 \theta}}$$

where *a* is a constant. Integrating and rearranging

(6) $$\cot \theta = b \sin(\varphi - \beta)$$

where $b^2 = \frac{1 - a^2}{a^2}$ and β is the constant of integration. Multiplying by $R\sin\theta$ and expanding $\sin(\varphi - \beta)$, one gets

(7) $$Ay - Bx = z$$

where

(8*a*, *b*) $$A = b\cos\beta, B = b\sin\beta$$

and

(9*a*-*c*) $$x = R\sin\theta\sin\varphi,\ \ y = R\sin\theta\cos\varphi,\ \ z = R\cos\theta$$

Eq. (7) is the equation of a plane passing through the center of sphere.

The above calculation clearly shows that the geodesic on a sphere is the path, which that plane forms at the intersection with the surface of a sphere, *i. e.*, a great circle.

CONCLUSION

The calculation of geodesic shows that there is, essentially, no difference in the two definitions regarding the direction of *Qibla*. The difference in direction, therefore, is not because of choosing one definition or another. Rather, it is a result of the *misconception* that the map of the world drawn on a two-dimensional paper (plane paper) gives the correct directions. This is *not* true. The correction “direction” can be obtained either from the globe or from the satellite photograph of the related portion of the earth (This is indicated in the portion of the relevant part of the globe in Fig. 1, where the geodesic starts off from USA in a N. E. direction relative to the lines of latitude and reaches Makka in S. E. direction — Editor).

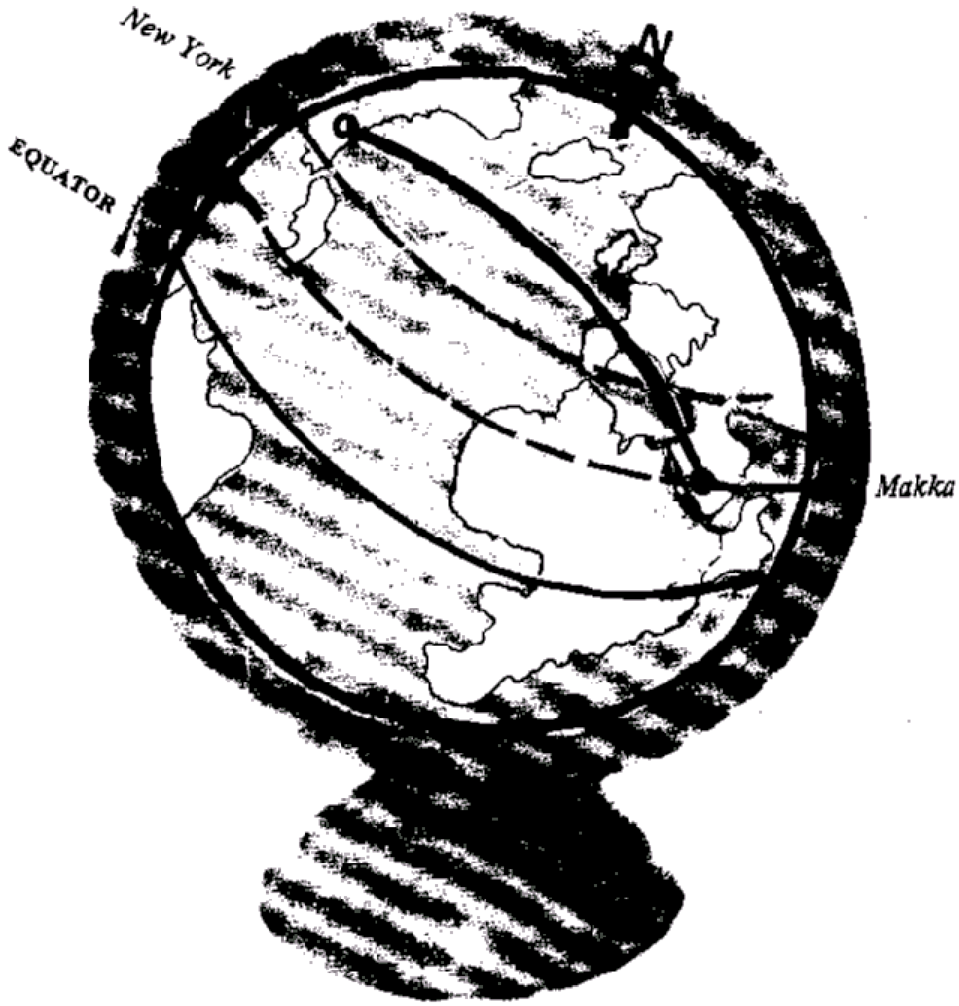

Fig. 1. Photograph of the shortest line on the globe between New York and Makka, showing the north-easterly direction from New York (left) relative to the 40^{th} parallel and the north-westerly direction from Makka (right) relative to the 20^{th} parallel.

Note: It is worth noting that on this basis (of great circles), the "direction of Qibla in Los Angles would be very nearly north".

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