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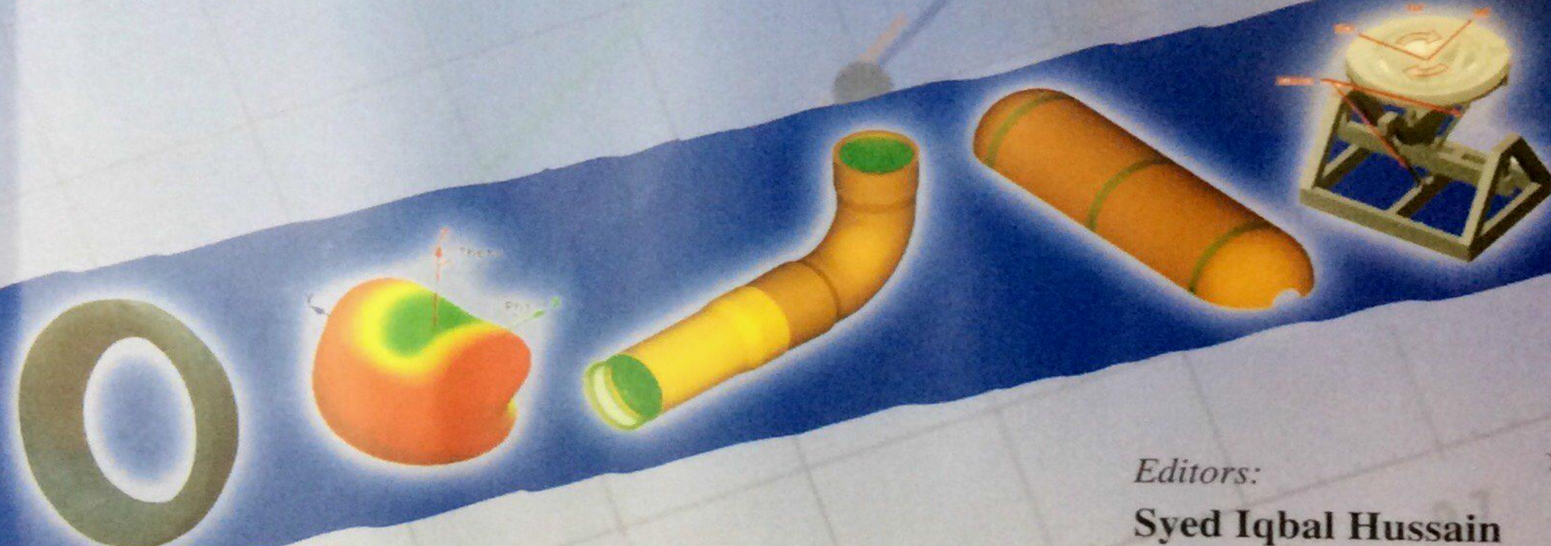

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Contents

The Multistage-Q System and the Inverse-Q System for Possible Application in Satellite-Launch Vehicle (SLV)

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Abstract
Steering a Satellite-Launch Vehicle (SLV) to strictly follow a predefined trajectory imposes unnecessary load on the control loop, and may, possibly, saturate servos. This If may introduce a permanent error in the vehicle-destination position and velocity vectors. Consequently, the payload (the satellite) would be deployed in a wrong orbit. The orbital-error correction utilizes onboard energy, which reduces the operational life of the satellite. Therefore, it is desirable that SLV is capable of altering its trajectory according to the new operating conditions, in order to achieve the required destination position and velocity vectors. In this paper, an innovative adaptive scheme is presented, which is based on "*the Multistage-Q System*". Using the control laws expressed in the elliptic-astrodynamical-coördinate mesh (*the normal-component-cross-product steering* and *the normal-component-dot-product steering*) this scheme proposes a design of autopilot, which achieves the pre-decided destination position and velocity vectors for a multistage rocket, when each stage is detached from the main vehicle after it burns out, completely. In "*the Inverse-Q System*", one applies *the extended-cross-product steering* to the vector sum of velocity vectors of spacecraft and interceptor.

Keywords: Q system, extended-cross-product steering, dot-product steering, elliptic-astrodynamical-coördinate mesh

Introduction

Flight dynamics of satellite-launch vehicle (SLV) is an example of unstable, non-autonomous, multivariable and nonlinear system. Several modern control techniques exist that can be effectively employed to design a robust controller for attitude control of aerospace vehicles.

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In the absence of an efficient guidance scheme, even a state-of-art control system cannot guarantee steering of vehicle to its required destination, under constrained conditions. Saturation and limited response rate of servomechanism are the universal nonlinear constraints of any control loop, which make impossible to implement all the control actions required to move a system from one state to any other state through any trajectory. Sometimes, it is almost impossible to reach the desired position in the state space. Saturation can give rise to inaccurate or even unstable response of a control loop.

Q system has been used for SLV by different authors [2, 12]. In this paper, we are presenting "*the Multistage-Q System*". In this formulation, sectionwise corrections are achieved, where destination point of first stage is the initiating point of the second stage and so on. In this way position and rate saturations (out of range deviations) are avoided. In "*the Inverse-Q System*", one applies *the extended-cross-product steering* [4] to the vector sum of velocity vectors of spacecraft and interceptor, expressed in the elliptic-astrodynamical coordinate mesh in order to derive them to zero. Use of *normal-component-cross-product steering* [4] and *normal-component-dot-product steering* [5] is recommended to achieve this objective. This paper includes a brief description of these control laws.

The attitude-control problem can be tackled by a number of ways. Real-time computer processing requires that the control law should be simple, and these techniques, generally, lead to higher-order controllers. On the other hand a nonlinear system, which can be linearized with continuous-time feedback, cannot necessarily be linearized with sampled feedback. A simpler attitude control loop can be designed for the attitude control of SLV based on effective-integral-control (EIC) [9, 10], or by Selective-State Feedback methodology [11].

Other schemes, which could be used to steer a satellite-launch vehicle include the Lambert scheme [1, 8].

The Q System

The Q system is described briefly in [1]. Let $\boldsymbol{r}$ be the radius vector representing the position of the spacecraft at an arbitrary time, t, after launch, represented by M in Fig. 1, the correlated velocity vector, $\boldsymbol{v}_C$, is defined as the velocity required by the spacecraft at the position $\boldsymbol{r}(t)$ in order that it might travel thereafter by free-fall in vacuum to the desired terminal condition. One defines a correlated spacecraft located at point M. The correlated spacecraft is assumed to experience only gravity acceleration g and moves with velocity $\boldsymbol{v}_C$. The actual spacecraft velocity is $\boldsymbol{v}_m$, and is affected by both gravity g and engine thrust acceleration $\boldsymbol{a}_T$.

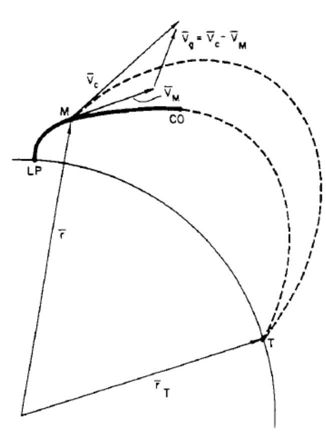

Fig. 1. Correlated trajectory and velocity-to-be-gained

The velocity to be gained, $\boldsymbol{v}_g$, may be expressed as:

$$\boldsymbol{v}_g = \boldsymbol{v}_C - \boldsymbol{v}_m$$

Computation of this velocity is only one element of the Q system. Of equal importance is a method to control the spacecraft in pitch and yaw, in order that the thrust acceleration causes all the three components of $\boldsymbol{v}_g$ to vanish, simultaneously. This may be accomplished by *extended-cross-product steering* [4] or *dot-product steering* [5]. Battin remarks in his book [1], pages 10-11: “If you want to drive a vector to zero, it is sufficient to align the time rate of change of the vector with the vector itself. Therefore, components of the vector-cross product

$$\boldsymbol{v}_g \times \frac{d\boldsymbol{v}_g}{dt}$$

could be used as the basic autopilot rate signals — a technique that became known as *cross-product steering*". However, this definition has a condition missing. The complete definition follows.

Extended-Cross-Product Steering

In order to drive a vector to zero, it is sufficient to align the time rate of change of the vector with the vector itself provided the time rate of change of the magnitude of this vector is a monotonically decreasing function [4]. This law may be termed as *extended-cross-product steering*. Let $\boldsymbol{A}$ be a vector, which needs to be driven to zero. Then, we must have

$$\boldsymbol{A} \times \frac{d\boldsymbol{A}}{dt} \to 0, \frac{d|\boldsymbol{A}|}{dt} < 0 \tag{1}$$

This is the basis of the following control law.

Normal-Component-Cross-Product Steering

In order to bring a vehicle to the desired trajectory one needs to align the normal component of velocity with its time rate of change and make its magnitude a monotonically decreasing function of time [4]. By normal component one means the component of velocity in the plane normal to reference trajectory. This plane passes through a point on the reference trajectory, which is closest to current location of center-of-mass of spacecraft. Mathematically,

$$\boldsymbol{v}_{\boldsymbol{perp}} \times \frac{d\boldsymbol{v}_{perp}}{dt} \to 0, \frac{d|\boldsymbol{v}_{\boldsymbol{perp}}|}{dt} < 0 \tag{2}$$

Therefore, components of the vector

$$\boldsymbol{v}_{\boldsymbol{perp}} \times \frac{d\boldsymbol{v}_{\boldsymbol{perp}}}{dt}$$

should be used as the basic autopilot rate signals. For elliptic-astrodynamical-coördinate formulation [3, 4, 6, 7] the perpendicular component of velocity may be expressed as:

$$\boldsymbol{v}_{\boldsymbol{perp}} = v_\xi \hat{\boldsymbol{e}}_\xi + v_z \hat{\boldsymbol{e}}_z \tag{3}$$

To correct for down-range error, one must have

$$\boldsymbol{v}_\xi \times \frac{d\boldsymbol{v}_\xi}{dt} \to 0, \frac{d|\boldsymbol{v}_\xi|}{dt} < 0 \tag{4a}$$

To correct for cross-range error, the following could be used as autopilot rate signals

$$\boldsymbol{v}_z \times \frac{d\boldsymbol{v}_z}{dt} \to 0, \frac{d|\boldsymbol{v}_z|}{dt} < 0 \qquad (4b)$$

Dot-Product Steering

Dot-product steering is a control law, which involves dot products of the vector, and its time rate of change [5]. In order to derive a vector $\boldsymbol{A}$ to zero, one needs to derive the factor $|\boldsymbol{A}|\left|\frac{d\boldsymbol{A}}{dt}\right|(1+\cos\theta)$ to zero, where θ is the angle between $\boldsymbol{A}$ and $\frac{d\boldsymbol{A}}{dt}$. In other words

$$\boldsymbol{A} \cdot \frac{d\boldsymbol{A}}{dt} + |\boldsymbol{A}|\left|\frac{d\boldsymbol{A}}{dt}\right| \to 0 \qquad (5)$$

In the trajectory problems, it is customary to require the normal component of velocity to vanish. Hence, one may develop a special case of *dot-product steering*.

Normal-Component-Dot-Product Steering

In order to bring a vehicle to the desired trajectory one needs to derive the factor $|\boldsymbol{v}_{\boldsymbol{perp}}|\left|\frac{d\boldsymbol{v}_{\boldsymbol{perp}}}{dt}\right|(1+\cos\phi)$ to zero, where ϕ is the angle between $\boldsymbol{v}_{\boldsymbol{perp}}$ and $\frac{d\boldsymbol{v}_{\boldsymbol{perp}}}{dt}$. The above condition may be expressed, mathematically, as

$$\boldsymbol{v}_{perp} \cdot \frac{d\boldsymbol{v}_{perp}}{dt} + |\boldsymbol{v}_{perp}|\left|\frac{d\boldsymbol{v}_{perp}}{dt}\right| \to 0 \qquad (6)$$

The steering law may be expressed in elliptic-astrodynamical coordinate mesh as:

$$\boldsymbol{v}_\xi \cdot \frac{d\boldsymbol{v}_\xi}{dt} + |\boldsymbol{v}_\xi|\left|\frac{d\boldsymbol{v}_\xi}{dt}\right| \to 0,\ \boldsymbol{v}_{\boldsymbol{z}} \cdot \frac{d\boldsymbol{v}_{\boldsymbol{z}}}{dt} + |\boldsymbol{v}_{\boldsymbol{z}}|\left|\frac{d\boldsymbol{v}_{\boldsymbol{z}}}{dt}\right| \to 0 \qquad (7)$$

The Multistage-Q System

Let us consider a 3-stage rocket. Destination point of the first (second) stage is the initiating point of the second (third) stage. Mathematically,

$$\boldsymbol{v}_{g1,final} \to \boldsymbol{v}_{g2,initial}; \boldsymbol{v}_{g2,final} \to \boldsymbol{v}_{g3,initial} \qquad (8)$$

By using these section-wise corrections, one can avoid position saturation and rate saturations. This can be effectively, established if the difference velocity vector is expressed in elliptic-astrodynamical-coördinate mesh and conditions expressed in terms of normal components of velocity, which have to be driven to zero.

The Inverse-Q System

Let us define a velocity vector, $\boldsymbol{v}$, representing the vector sum of velocity of the spacecraft and velocity of the interceptor. Mathematically,

$$\boldsymbol{v} = \boldsymbol{v}_{spacecraft} + \boldsymbol{v}_{interceptor} \qquad (9)$$

This velocity has to be derived to zero. *Extended-cross-product steering* [4] may be used to design basic autopilot, with *dot-product steering* [5] incorporated in the computation loop to check whether the control action has taken place or not, making it a closed-loop control system.

Conclusions

Control energy available to SLV is, mainly, used (or should, mainly, be used) for stabilization of vehicle and for disturbance-rejection action. Forcing the vehicle to follow an unnatural flight path can saturate the control loop, leaving be no room for disturbance-rejection action. This situation makes the whole guidance and control design inefficient. Sometimes, the saturation can make the loop unstable or, at least, a permanent error is introduced due to failure of implementation of control laws. Error in the position and the velocity vectors at the payload-ejection point can deploy the payload (satellite or an experimental payload) in a wrong orbit (or trajectory).

The multistage-Q system attempts to address this problem using corrections achieved when one stage is separated after burnout. The inverse-Q system may be used to remove an unwanted debris or out-of-service satellite from the orbit.

Acknowledgement

This work was made possible, in part, by *Dean's Research Grant* awarded by University of Karachi.

Appendix: SAK's Review of "An Introduction to the Mathematics and [the] Methods of Astrodynamics" by Richard H. Battin

This review appears in https://www.amazon.com as well as on first author's homepage https://www.ngds-ku.org/Papers/Battin.pdf — Book Rating: ★★★★★; full text of the book:
https://arc.aiaa.org/doi/pdf/10.2514/4.861543

The following comments refer to the 1987 edition. Some of these comments were communicated to Professor Battin, who, very kindly, acknowledged them.

The book by Richard Horace Battin, Senior Lecturer in Aeronautics and Astronautics, Massachusetts Institute of Technology, United States, covers essential mathematical background needed to work with astrodynamical problems. Topics covered include hypergeometric functions, elliptic integrals, continued fractions, coördinate transformations as well as essentials of two-body-central-force motion.

The author's way of discussing these topics with historical introduction and personal narrative makes the book interesting to read. There are minimal typographical errors, probably, because the author, personally, typeset this book. However, there are a few omissions and oversights. For example, on page 172 captions are given for Fig. 4.15 and Fig. 4.16, whereas the actual figures are missing (*The author has rectified this omission in the* 1999 *edition*). In addition:

a) On page 7 it is stated:

$$\Delta \boldsymbol{r} = \frac{s_g}{v_g} \boldsymbol{v}_g$$

where $s_g = \int \boldsymbol{v}_g \, dt$. In this equation, a scalar on the left-hand side is equated to a vector on the right-hand side. The equation should be modified as:

$$s_g = \left|\int \boldsymbol{v}_g dt\right|$$

b) On pages 10-11 it is stated: "If you want to drive a vector to zero, it is sufficient to align the time rate of change of the vector with the vector itself." This is *not* true, in general, but only if time rate of change is negative (cf. [4]).

c) On page 13 the author tries to show that curl of $\boldsymbol{v}_\text{c}$ in the equation:

$$\frac{\nabla \times \boldsymbol{v}_\text{c}}{\rho} = constant$$

vanishes by the following argument. "The demonstration concludes with an argument that the fluid is converging on the target point $\boldsymbol{r}_\text{T}$ so that the density in the vicinity of $\boldsymbol{r}_\text{T}$ is becoming infinite. *Hence, the constant is zero, implying that the curl is everywhere zero*." There are 2 problems in this line of argument: (*i*) The statement "hence, the constant (see note at the end of manuscript) is zero" is true, only if the numerator is finite. $B = \infty$ implies $A/B = 0$, only if $A \neq \infty$. Otherwise, one has to apply l'Hospital rule; (*ii*) Even if the constant is supposed to be zero, this does not imply that the curl is everywhere zero. $A/B = 0$, where $B = \infty$ does not imply that $A = 0$. In fact, *A* could have any finite value.

d) On page 109 equation of motion in a frame of reference moving with acceleration $-\boldsymbol{a}_1$ is written as:

$$m_2(\boldsymbol{a}_2 - \boldsymbol{a}_1) = \frac{m_1 + m_2}{m_1}\left(\frac{Gm_1m_2}{r^2}\right)\left[-\frac{\boldsymbol{r}}{r}\right]$$

Since the frame is non-inertial (accelerated) Newton's second law, $\boldsymbol{F} = m\boldsymbol{a}$, is not applicable in this frame.

e) On page 223 it is stated: "When we compare Eqns. (5.57) and (5.58), it is clear that we *must* have:

$$\sin E \cong \sqrt{\frac{6(E - \sin E)}{\sin E}}$$

………" This is *not the only choice* for sin*E*, which reduces (5.58) to (5.57) in the limit $E \to 0$. The word "must" is inappropriately used here.

I would recommend this book *very strongly* to any one involved in astrodynamical research.

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NOTE ADDED AFTER PUBLICATION OF THIS PAPER

The constant on page 32, line 2 should, actually, be a constant vector — Noor Fatima Siddiqui, Lecturer, Department of Mathematics, University of Karachi commented on March 25, 2016.

Web address of this document (*first author's homepage*): https://www.ngds-ku.org/Papers/C66.pdf

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