TRENDS IN PHYSICS
PROC. SYMPOSIUM
KARACHI 1992

MODELING OF HEART FUNCTION

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Abstract

Acoustic properties of the heart are modeled by applying the concepts of wave theory. Heart is considered as a system of standing waves. The nature and origin of various frequencies are considered. Experimental suggestions are briefly mentioned.

Introduction

Heart is one of the most important organs of human body. Cardiac care occupies a very important role in the total well being of a person. However, the stressful life of the 20th century has also increased heart problems. Therefore, there is an increasing emphasis on the modeling of heart function.

Heart may be considered as an engine performing work (Cameron & Skofronick 1987). It may also be modeled as a vibrating system. One may be interested in the frequencies with which the heart may vibrate. The concept of standing waves may be used to calculate the possible frequencies of vibrations of the heart. The previous models of heart consider the heart as a sphere (Mazumdar & Woodaard-Knight 1984) or a bullet. However, the shape of the heart may be better approximated if we consider it a deformed ellipsoid of revolution.

It is not possible to describe the working of heart in detail in this paper. However, we have placed a block diagram of the cardiac cycle in Fig. 1. Details of working of heart may be found elsewhere (Bergel & Hunter 1979; Chiavelo & Jalife 1984; Glass, Hunter & McCulloh 1991; Guevara 1984).

Statement of the Problem

We are interested in studying the acoustic properties of heart. The frequencies obtained in the fourier transform of phonocardiogram may be calculated if we know the shape and properties of heart surface. Heart may be modeled as a system of standing waves. A discrete system is capable of sustaining standing waves with discrete frequencies (Halliday & Resnick 1988). We want to relate the origin of phonocardiogram frequencies to heart shape.

Acoustic Model of Heart

Heart is modeled as a deformed ellipsoid of revolution about the major axis. Using the concept of standing waves the frequencies obtained in the fourier

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transform of phonocardiogram may be calculated. In order to simplify the problem we have introduced cardiac coordinate system which is a modification of ellipsoidal coordinate system.

Heart is assumed to be an ellipsoid of revolution. Under these conditions there is rotational symmetry about the axis of revolution. Because of this symmetry we may drop one of the coordinates. Hence, the problem is reduced to two dimensions. We, now consider the projection of heart on the frontal plane. Let us take the frontal plane as xy plane, the anterioposterior axis, therefore, becomes the z axis. Cardiac coordinates, which are basically extension of elliptical coordinates, are defined.

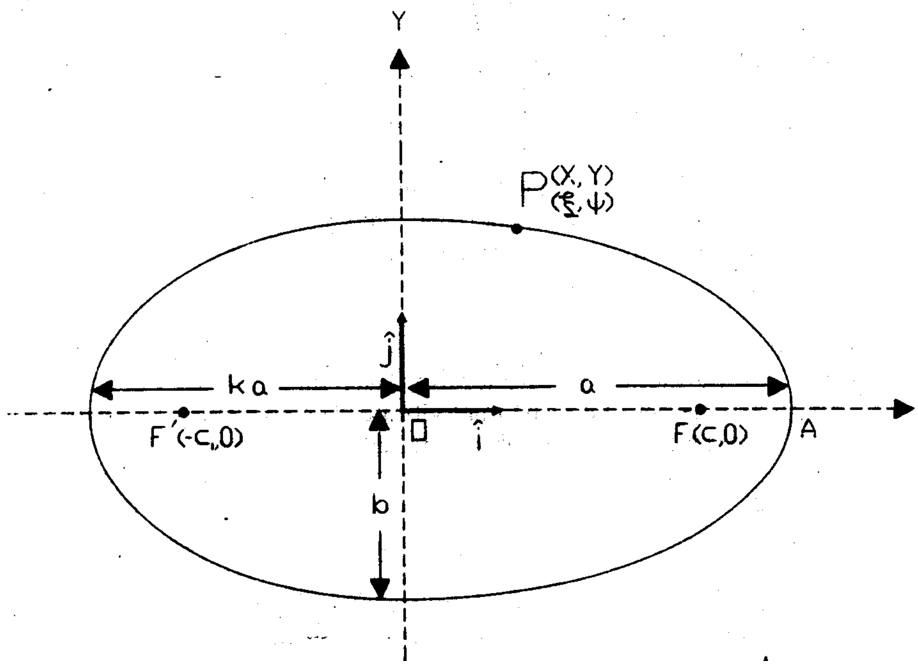

Fig. 1. Cardiac coördinates

Let us consider heart as a union of two semi ellipses, one of them having a and b as the semi major and the semi minor axes respectively, and the other having ka and b as the semi-major axes and the semi-minor axes respectively $(2 < k < 1)$. Let s(a, b, k) be the sum of the circumferences of the two semi ellipses. From the condition of standing waves

$$s(a,b,k) = \lambda_1, \qquad (n_1 = 1) \qquad (1)$$

Recall that standing waves may be set up in a closed string if the length of path is an integral multiple of wavelengths. The frequency, therefore, may be expressed as

$$\omega_1 = 2\pi\upsilon_1 = 2\,\pi v/\,\lambda_1 = 2\,\pi v\,/\,s(a,\,b,\,k) \qquad (2)$$

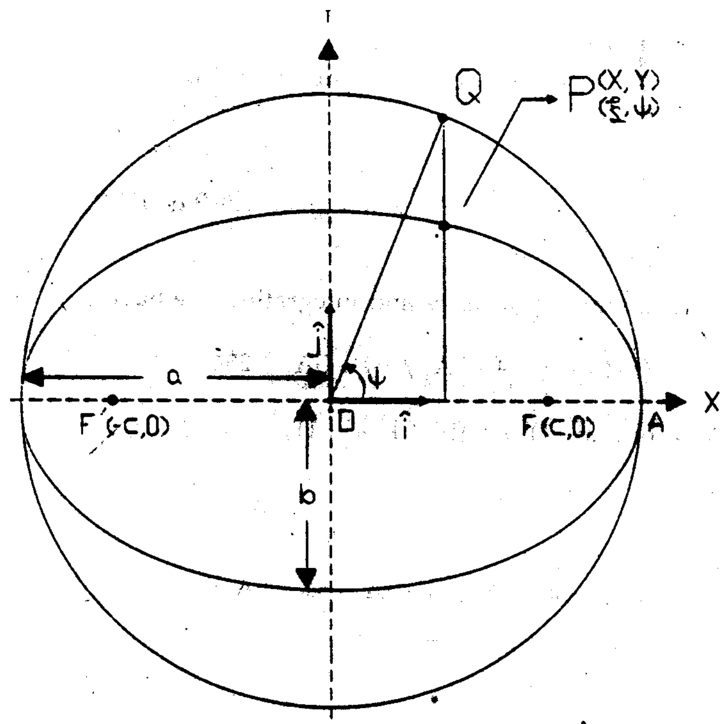

Fig. 2. Elliptical coördinates

where υ_1 is the frequency corresponding to the wavelength λ_1 and v is the velocity of the wave set up in the outer layer of heart membrane. Similarly

$$2\pi b = \lambda_2, \qquad (n_2 = 1) \tag{3}$$

$2\pi b$ is the circumference of the circle of radius b (Since we are considering the surface of heart as a surface of revolution, we are going to end up with a circle of radius b as a result of revolution). The frequency, therefore, becomes

$$\omega_2 = 2\pi\upsilon_2 = 2\pi v/\lambda_2 = v/b \tag{4}$$

From the above we note that

$$\omega_1 < \omega_2 \tag{5}$$

using $ds^2 = h_\xi{}^2 d\xi^2 + h_\psi{}^2 d\psi^2$ we may write down for the circumference of an ellipse

$$s = c \int_0^{2\pi} h\, d\psi \tag{6}$$

For cardiac coordinates

$$s(a, b, k) = a \int_{\pi/2}^{\pi/2} [1 + (b^2/a^2 - 1) \sin^2 \psi]^{1/2} \, d\psi$$

$$+ ka \int_{\pi/2}^{3\pi/2} [1 + (b^2 / k^2 a^2 - 1) \sin^2 \psi)^{1/2} \, d\psi \qquad (7)$$

Expanding the integrand in series and integrating, we have

$$s(a, b, k) = \pi a \, [(1+k) + A_1/4 - 3A_2 / 64 + 5A_3 / 256 - \ldots] \qquad (8)$$

where $A_i = (b^2 / a^2 - 1)^i + k\,(b^2 / k^2 a^2 - 1)^i; \quad i = 1, 2, 3, \ldots$

Therefore

$$\begin{aligned} \omega_2 / \omega_1 &= s(a, b, k) / 2\pi b \\ &= (a/2b)\,[(1+k) + A_1 / 4 - 3A_2 / 64 + 5A_3 / 256 - \ldots \end{aligned} \qquad (9)$$

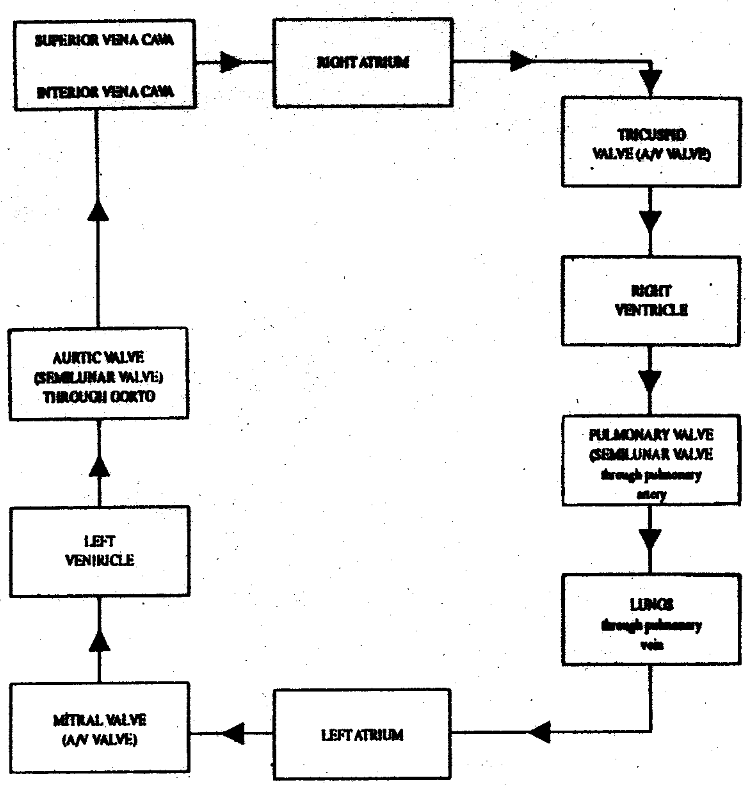

Fig. 3. Cardiac cycle

Therefore, we have evaluated the ratio of frequencies which are obtained in the fourier transform of phonocardiogram. This model, therefore, relates the ratio of frequencies to heart size. The best way to check this model would be to determine the ratio of frequencies in population having different heart sizes, which is obviously the pediatric population. In the next section we describe how to calculate the ratio using eq. (9). A comparison of $(\omega_1 / \omega_2)_{calculated}$ with $(\omega_1 / \omega_2)_{experimental}$ obtained from phonocardiogram would indicate how closely the model describes the dynamics of heart. A visual examination may be carried out by plotting a 2-D plot with $(\omega_1 / \omega_2)_{experimental}$ on x axis and $(\omega_1 / \omega_2)_{calculated}$ on y axis. A quantitative evaluation may be carried out by calculating the linear correlation coefficient of the two variables. The better the fit, the closer the linear correlation coefficient to unity.

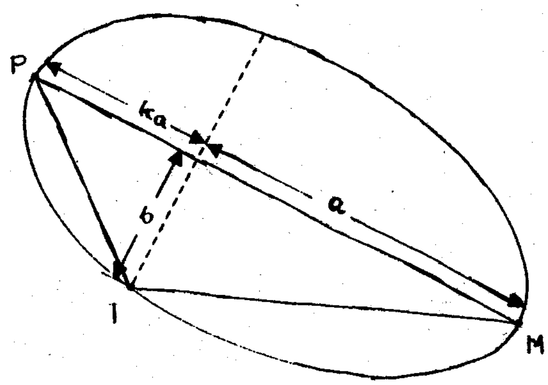

Fig. 4. Determination of heart parameters

Determination of Heart parameters

The ratio $(\omega_1 / \omega_2)_{calculated}$ may be determined from the location of heart sounds in children and adults. The values of a, b and k may be calculated by noting the positions where maximum intensity from mitral valve (M), tricuspid valve (T) and pulmonary valve (P) are obtained. Refer to Fig. 4. In the triangle PTM, let us call PM = β, PT = α, TM = γ, < TPM = θ. From the cosine law

$$\gamma^2 = \alpha^2 + \beta^2 - 2\,\alpha\beta \cos\theta \tag{10a}$$
$$\cos\theta = (\alpha^2 + \beta^2 - \gamma^2) / 2\,\alpha\beta \tag{10b}$$
$$b = \alpha \sin\theta \tag{10c}$$
$$a = \beta - \alpha \cos\theta \tag{10d}$$
$$k = (\alpha \cos\theta) / (\beta - \alpha\cos\theta) \tag{10e}$$

Preliminary data suggests that heart sound location may provide an estimate of the shape of heart, but the locations of maximum heart sounds may not be pinpointed. Therefore, for a check of this model it may be advisable to use alternate methods to estimate the shape of heart.

Discussions and Conclusions

The model of the heart presented here is based on a modified ellipsoid which is closer to reality as compared to spherical or bullet model. However, the model rests on some simplifications, e.g. it is assuming heart as ellipsoid of revolution, which may not be the case in reality. Further, there are deviations, from the ellipsoidal shape in the actual heart. The shape also varies from person-to-person and during various ages.

It is therefore recommended that after comparing this model with the actual phonocardiogram frequencies heart shapes in various age groups and various populations be studied to determine a better set of parameters to be used in the model. It may also be interesting to find out if the area of the heart sound location triangle PTM varies when the child is standing, mild stretching, lying or squatting positions. Further, velocity of sound in the membrane and the shift in frequencies because of movement of heart walls may be studied.

The model presented above may lead to a better understanding of human heart, may help in developing better cardiac care and may eventually lead to development of an artificial heart which performs almost all the functions of the real heart.

Acknowledgements

The authors would like to express their deepest appreciation to Dr. Akbar Bilal Siddiqui who spent many hours to explain the clinical side of the problem. Thanks are also due to Syed Atiq Alam for preparing illustrations for this work.

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