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levels at 1272, 1809, and 2382 keV is not clear. Based on its decay pattern, this new 30-s activity appears to represent the $d_{5/2}$ proton orbital in ^{145}Tb.

*Operated by Union Carbide Corp. under Contract No. W-7405-eng-26 with the U.S. Department of Energy.
**Research supported in part by the U.S. Department of Energy and the Robert Welch Foundation.

JE 9 An Analytic Expression for the Pulse-height Spectrum of a Scintillation Detector. J.V.KANE JR J.V.Kane and Co. Wynnewood,PA.19096--A NaI(Tl) detector exposed to Cesium 137 gamma-rays has a pulse-height spectrum given by $P(M,N,X)={}_XN_{(1-X)}M-N$ times C(M,N) where M and X have the values 300,000 and 0.01 respectively. Using a level I 4k TRS-80 computer this expression has been calculated with the following program:

```
100 D=0:E=1:P=0.01/(1-0.01):G=300000:H=0
110 E=E*F*(G-D)/(D+1):D=D+1
112 IF E LT 16384 GOTO 120.
114 E=E/2:H=H+1:GOTO 112.
120 P.D,E,H:GOTO 110
```

This calculation gives a FWHM of 4.2%. Measurment gives 7%. This suggests that the early dynode stages of the photomultiplier are responsible for considerable loss of resolution and that substantial improovement can be obtained.

JE 10 2D Multiple Event Image Intensifier Scintillation Ion Detector.* H.E. Wegner, P. Thieberger, BNL - A 25 mm diameter three stage image intensifier[1] with a light gain of 85,000 has been adapted for the x-y position detection of low energy multiple heavy ion events. The input fibre optic plate of the image intensifier was coated with a close packed monolayer of 1 micron P11 phosphour grains.[2] The scintillation light is amplified and appears on the output fibre optic plate as small green intense light spots. A sensitive television camera system displays the events on a monitor for observation and photographic recording and a microcomputer system digitizes the coordinates of the events for analysis on a large computer. This detector is currently employed in the study of Coulomb explosion of accelerated complex molecules. Examples of typical data will be shown and the limitations of this system and future prospects will be discussed.

1. Varo Inc., Electron Devices Div., Garland, Tx 75040
2. Grant Scientific Corp., Bx 11729, Columbia SC 29211

*Research supported by the U.S. Department of Energy.

JE 11 Leak Checking a Superconducting Cyclotron Cryostat with Liquid Helium,* H.W. LAUMER, M.L. MALLORY, D.R. POE, Michigan State University.—Large superconducting magnets are now being developed in many areas of nuclear physics and will soon become an everyday tool of nuclear physicists. Successful operation of these magnets requires new technologies and engineering techniques. In the process of building a superconducting cyclotron, a method has been developed for finding ultra small helium leaks with liquid helium. The method consists of correlating helium leak rate in the cryostat vacuum jacket with the level of liquid helium. A change in the helium leak rate as the leak channel is covered or uncovered by liquid helium is clearly detected. Single phase liquid compared to gaseous helium at the same temperature would be expected to flow about twice as freely, but the opposite effect is observed. This decrease is attributed to two phase helium being transported through an ultra small channel. Using this method detection of greater than three magnitudes beyond the present state of the art at room temperature can be achieved.

*National Science Foundation Grant No. Phy 78-22696.

JE 12 THE EFFECTS ON IIao FILM OF PROTON BOMBARDMENT PRODUCED BY A CYCLOTRON.* E. C. Hammond, Morgan State University-- The impending space shuttle flight with its variety of Astronomical Research facilities requires a profile of the fogging effects on IIao film produced by proton bombardment caused by the solar wind and stellar wind. The experimental data is limited. The fogging of the film caused by such bombardment produces uncertainties in the results attained when the film is ultimately developed. To develop a quantitative as well as a qualitative understanding of the problem, our research group has used the University of Maryland Sectored Isochronous Cyclotron to measure quantitatively the effects of proton interaction with the IIao film. Several thicknesses of shielding were tested to simulate the orbital radiation effects on the IIao film. These studies will assist the Goddard engineer and scientist to determine the possible shielding requirements in the construction and engineering of the film carrier to be used in the space shuttle.

*Work supported by a grant from NASA, Goddard Space Flight Center, Greenbelt, Maryland.

SESSION JF: WEAK AND ELECTROMAGNETIC INTERACTIONS

Thursday morning, 29 January 1981
Nassau Suite Room B at 9:00 A.M.
R. C. Larsen, presiding

JF 1 Limits on CP Invariance Violation in $K^+_{\mu 3}$ Decays,* S.R. BLATT, M.K. CAMPBELL, J.K. BLACK, M.P. SCHMIDT, H. KASHA, R.K. ADAIR, Yale University; W.M. MORSE, L.B. LEIPUNER and R.C. LARSEN, Brookhaven National Laboratory--We have completed the first phase of the measurement of the polarization of μ^+ from the decay $K^+ \to \pi^0\mu^+\nu_\mu$. The polarization in the CP violating direction $P_\pi \times P_\mu$ is 0.00142 ± 0.00203. The ratio of the normal to the transverse polarization is 0.0198 ± 0.0025, giving a value of Imξ = 0.052 ± 0.066, based on 1.1×10^6 events, obtained in one week of running. We will report on our latest findings, including the second phase of the experiment. This work was done at the Brookhaven AGS.

*Work supported by the U.S. Department of Energy.

JF 2 New Limits on Neutrino Oscillation Parameters. J.L. VUILLEUMIER, F. BOEHM, J.F. CAVAIGNAC, F.v.FEILITZSCH, A.A. HAHN, H.E. HENRIKSON, D.H. KOANG, H.KWON, R.L. MOSSBAUER and B.VIGNON. Caltech[x], ILL and ISN[xx] Grenoble, T.U. Munich[xxx]. -- The electron-antineutrino spectrum has been measured at 8.7 m from the core of the ILL fission reactor using the reaction $\bar{\nu}_e p \to e^+ n$. The observed neutron correlated positron spectrum is compared with theoretical predictions based on the fission electron spectrum measured in ref. 1. If neutrino oscillations of the type $\bar{\nu}_e \to$ anything with large mixing occur the shape and yield of the measured spectrum are expected to be modified. No such modifications within the experimental uncertainty were observed and a limit of $\Delta^2 = (m_1^2 - m_2^2) < 0.14$ eV^2 (90% cl) can be set assuming full mixing. Limits for Δ^2 for a range of mixing angles will be presented.
Supported by [x]US DOE, [xx]French IN2 P3 [xxx]German Ministry of Res. and Techn.
1) K. Schreckenbach et al., Phys. Lett. to be published, and A.A. Hahn et al., Bull.Am. Phys.Soc. this issue.

JF 3 Absolute Measurement of the Beta Spectrum from ^{235}U Fission as a Basis for Reactor Neutrino Experiments A.A. HAHN, H.FAUST, F.v.FEILITZSCH, K.HAWERKAMP, K.SCHRECKENBACH, J.L.VUILLEUMIER. Caltech[x], ILL Grenoble, TU München[xx]. --The beta spectrum of fission products from the thermal neutron induced fission of ^{235}U has been measured on line at the ILL reactor with a magnetic spectrometer. In the energy range from E_e=2.0 to 9.0 MeV the spectrum was determined per fission with an accuracy of 5%. The maximum exposure time was three days. ...ng recent calculations the results of Davis et al.[1) ...e with the present work within the quoted errors. ...e experimental electron spectrum was converted to the corresponding $\bar{\nu}_e$ spectrum to form a basis for neutrino experiments at reactors[2)].

Supported by [x]US DOE, [xx]German Ministry of Res. and Techn.

1) B.R. Davis et al., Phys. Rev. C19, 2259 (1979)
2) F. Boehm et al., Phys. Lett. B (submitted) and J.L. Vuilleumier et al., Bull. Am. Phys. Soc. , this issue.

JF 4 Neutrino Emission and Recoil Energy in K-Capture Reactions - a Curious Coincidence. W.H. HONIG, Western Australian Institute of Technology, S. Bentley, 6102, Western Australia. --When Be^7 goes to Li^7 by K-capture a neutrino is emitted and the recoil energy, easily calculated, is 57.3 ev. Using fluid models* for the fundamental particles and for space, the electron in orbit before K-capture neutrino emission consists of a bubble of charge surrounding the nucleus inside of which both the nucleus and the field binding energy is located. The em field distribution inside the sphere is the neutrino itself. This electron neutrino mass is the ionisation potential of hydrogen with a mass equivalent of 13.595ev. The difference in ground state ionisation energy between Li^7 and Be^7 as given in common tables is 57.413 ev and is a prediction from these models for a more accurate determination of the energy of this decay and with the prediction that the electron neutrino mass equivalent is 13.595 ev. The Muon neutrino mass equivalent would be 206.8 times this, or 2811.45 ev. This neutrino model makes neutrinos electromagnetic waves and may resolve some of the cosmological problems on neutrino flux.

*See other abstract, this meeting, for refs.on fluid model

JF 5 Massive Neutrino and the Anomalous Magnetic Moment of the Muon. A. ROSADO and A. ZEPEDA, Centro de Investigación del IPN . Recently the possibility of a non-zero mass of the neutrino has been considered by some authors. In such a case the neutrino might have an anomalous magnetic moment, K_ν. In this work we discuss the effect of K_ν on the anomalous magnetic moment of the muon, K_μ. The bound for K_ν obtained from our calculations is less restrictive than that obtained from astrophysical considerations.

JF 6 $\mu \rightarrow e\nu\nu\gamma$ and Magnetic Moment of the Neutrino. J.L. ARAUZ and A. ZEPEDA, Centro de Investigación del IPN . On the basis of recent indications about a nonvanishing neutrino mass we speculate about the possibility of an anomalous magnetic moment of the neutrino and compute its effect in the radiative muon decay. We analyze the photon and electron spectra as well as the angular distributions in order to isolate the characteristics due to the neutrino magnetic moment.

JF 7 Quantum Electrodynamics and My Space-Time Magic Numbers. ENOS E. WITMER, University of Pennsylvania.--The writer has pointed out that the masses of nuclei and elementary particles are dominated by pure number ratios involving the integers 3 and 4, which are respectively the number of dimensions of space and space-time, and by other integers derived from them, and especially by the integral powers of the integer 2. In particular if M' is the mass of a nucleus expressed in units of the electron rest mass, then $M'(Z,N) = 2^{14}A/3^2 + D(Z,N)$, where D(Z,N) is a relatively small quantity. This makes the M'(Z,N) remarkable numbers in the binary scale of notation[1]. Here I wish to point out that the very accurately known pure number g in quantum electrodynamics is given by $g/2 - 1 = 19/2^{14} = 0.0011596680$, which differs from the best experimental value by only about 10 ppm. Note the occurrence of the number 2^{14} in both of these expressions. More generally it seems to be true that from my point of view μ_e is the right unit for expressing nuclear magnetic moments.

[1]E.E. Witmer, Space-Time and Microphysics--A New Synthesis, University Press of America, Washington, DC 1979.

JF 8 The Possibility of Massive Particles Travelling with the Velocity of Light. S. ARIF KAMAL, Indiana U. Bloomington.-Special theory of relativity suggests that no massive particle can travel with the velocity of light This paper discusses the conditions in which a massive particle can travel with the velocity of light. Using free particle Dirac equation, uncertainities between velocity and Lorentz factor as well as between velocity and energy of a Dirac electron are calculated which are non-zero. Therefore an accurate determination of velocity would make Lorentz factor and energy indeterminate. For electron energies of 5 GeV and 200 GeV, it is shown that the uncertainity in velocity is greater than the difference between the velocity of light and the velocity expected from relativistic relation for that energy. By quantum mechanical treatment, it is shown that the probability of existance of particles having v = c is non-zero. The relativistic relation of mass is modified so that mass is non-infinite at v = c in the light of uncertainity relations.

JF 9 Experimental Test of Quantum Mechanics and Special Relativity in High Energy Physics, Marilyn E. Noz, New York Univ. and Y. S. Kim, Univ. of Maryland. Spacetime diagram is used to describe Heisenberg's position-momentum uncertainty and Dirac's C-number time-energy uncertainty relation. A Lorentz transformation is applied the spacetime diagram. The resulting Lorentz deformation of the probability distribution is shown to exhibit the Lorentz transformation property of relativistic extended hadrons. It is shown that this way of combining quantum mechanics and relativity leads to the resolution of the puzzles in Feynman's parton model.[1] It is pointed out that this mechanism can explain the the form factor behavior and the jet phenomenon. It is shown also that this particular approach corresponds to Dirac's "instant form" quantum mechanics[2] based on the representations of the Poincare group.

[1]Y. S. Kim and M. E. Noz, Phys. Rev. D 15, 335 (1977).
[2]P. A. M. Dirac, Rev. Mod. Phys. 21, 392 (1949).

SESSION KA: SYMPOSIUM OF THE DIVISION OF PARTICLES AND FIELDS: STRONG AND WEAK INTERACTION PHENOMENA OBSERVED IN e^+e^- COLLISIONS
Thursday afternoon, 29 January 1981; Sutton Ballroom North at 2:00 P.M.; N. P. Samios, presiding

KA 1 Meson Spectroscopy Using Hadronic Psi Decays. G. GIDAL, *Lawrence Berkeley Laboratory.* (40 min.)

KA 2 Decays of *B* Mesons Observed at CESR. F. SANNES, *Rutgers University.* (50 min.)

KA 3 What Do We Want to Learn from the Study of *B* Mesons? N. CABIBBO, *New York University and University of Rome.* (50 min.)

THE POSSIBILITY OF MASSIVE PARTICLES TRAVELLING WITH THE VELOCITY OF LIGHT

SYED ARIF KAMAL*

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Special theory of relativity suggests that no massive particle can travel with the velocity of light. The fundamental aim of this paper is to show that massive particles can travel with the velocity of light. An argument in favor of this concept can be given by consideration of Dirac equation. Using the uncertainity relation of Robertson[1],

$$\Delta A \, \Delta B \geqslant \tfrac{1}{2}\left|\langle [A,B] \rangle\right|$$

and noting that $\alpha_x\beta + \beta\alpha_x = 0$ for Dirac matrices, it can be shown that

(1) $$\Delta\alpha_x \, \Delta\beta \geqslant \left|\langle \alpha_x\beta \rangle\right|$$

Using free particle solution of Dirac equation[2] for positive energies

$$\psi_j = u_j \exp(i/\hbar)(xp_x + yp_y + zp_z - Et), \; j = 1,2,3,4$$

where $u_1 = 1$, $u_2 = 0$, $u_3 = cp_z(E_+ + m_oc^2)^{-1}$, $u_4 = c(p_x + ip_y)(E_+ + m_oc^2)^{-1}$, one obtains

(2) $$\Delta\alpha_x \, \Delta\beta \geqslant cp_y/E_+$$

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Bull. Amer. Phy. Soc. **26 (1)**, 47: abstract#JF8

Using $p_y = mv_y$, $E_+ = mc^2$ and taking the limit as v approaches c, we get (in this case m_o/m is negligible)

$$\Delta\alpha_x \Delta\beta \geqslant v_y/c \tag{3}$$

which is non-zero and so $\Delta\alpha_x \Delta\beta > 0$. $c\alpha_x$ is the velocity operator and $\langle\beta\rangle$ represents $(1 - v^2/c^2)^{\frac{1}{2}}$. As v approaches c, an accurate determination of α_x makes β uncertain and vice versa[3]. Therefore $m_o(1 - v^2/c^2)^{-\frac{1}{2}}$ becomes indeterminate as v approaches c. This result is not restricted to Dirac's case but it has general validity. Similarly uncertainity between α_x and Dirac Hamiltonian H_D can also be calculated using Robertson's relation[1]. Therefore

$$\Delta\alpha_x \Delta H_D \geqslant c(E_+ + m_o c^2)^{-2}(K_1^{\,2} + K_2^{\,2})^{\frac{1}{2}} \tag{4}$$

where

(5 $\quad$ $^2p_x{}^2p_y{}^2; \quad K_2 = p_y(E_+ + m_o c^2)^2 + p_y c^2(p_y^{\,2} + p_z^{\,2} - p_x^{\,2})$

Replacing α_x by v_x/c, H_D by E and taking the limit as v approaches c, we have

$$\Delta v_x \Delta E \geqslant 2c^2\left[(p_x^{\,2} + p_y^{\,2})(v_y/c)^4 + p_y^{\,2}(v_z/c)^4 + 2p_y^{\,2}(v_y v_z/c^2)^2\right]^{\frac{1}{2}}$$

where we have used

$$v_x^{\,2} + v_y^{\,2} + v_z^{\,2} = c^2 \text{ (approx.)}$$

Usin

$$p_x + p_y = (E/c) - m_o c - p_z$$

we get

$$(6)\ \Delta v_x \geqslant 2c(E/\Delta E)(v_y/c)^2 \left[1 + (v_z{}^2/c^2) + (v_z{}^4/c^2 v_y{}^2) - (m_o{}^2 c^4/E^2)\right]^{\frac{1}{2}}$$

Consider 200 GeV electrons produced at Fermilab. Suppose that the focusing system is 3m away from the production area. $v_x = 3 \times 10^8$ m/s (approx.), $x = 3$m, $t = 10^{-8}$s. The beam is focused to a space of 3 microns. Therefore $y = z = 3 \times 10^{-6}$m, $v_y = v_z = y/t = z/t = 3 \times 10^2$ m/s. Substituting these values in (5) we get

$$(7) \qquad \Delta v_x \geqslant (0.12)/\Delta E$$

where ΔE is in GeV. To obtain minimum uncertainity in velocity we use maximum uncertainity in energy (5% of the actual value). Therefore $\Delta E = 10$ GeV and so

$$\Delta v_x \geqslant (11.9 \times 10^{-3})\ \text{m/s}$$

For E = 200 GeV, $c - v_x = (9.75 \times 10^{-3})$ m/s obtained from the relation $m = m_o(1 - v_x{}^2/c^2)^{-\frac{1}{2}}$. Therefore an accurate determination of H_D makes α_x indeterminate and vice versa.

Let us look into the matter from a different angle. Usually luminal frames are not given a physical meaning (even if mathematical use of 'infinite momentum frames' is possible) i.e. massive luxons are excluded[4]. From infinite momentum frames the space-time should appear as bidimensional space, projection of a suitable 3-dimensional hypersurface onto a plane normal to luminal ray direction. According to Recami and Mignani[4] on such a plane, both objects and photons appear immobile. This is in contradiction to the fact that photons cannot be brought to rest in any frame. However the problem can be solved by considering the f that time also transforms in such frames and the observed speed of photons is again equal to c.

Let us on the contrary try to give physical meaning even to luminal frames ($V = c$), trying to understand how they will see the world. Let us consider 'massive luxons' and in order to derive how their properties will appear, let us for instance derive their observable expressions by looking at usual massive particles from luxon frame. This idea is further supported when we calculate the probabilities of such massive luxons and these are finite.

Consider a luxon frame of reference. If a particle has velocity $\bar{u}$ in the laboratory frame of reference, we have from the velocity transformations

(8a) $$v_x = (u_x - V)(1 - u_x V/c^2)^{-1}$$

(8b) $$v_y = u_y(1 - V^2/c^2)^{\frac{1}{2}}(1 - u_x V/c^2)^{-1}$$

(8c) $$v_z = u_z(1 - V^2/c^2)^{\frac{1}{2}}(1 - u_x V/c^2)^{-1}$$

(V is the velocity of the frame moving in the positive x-direction), we get $v_x = -c$, $v_y = v_z = 0$ (because $V = c$). This holds only if $u_x \neq c$ (The case $u_x = c$ corresponds to luxons and is discussed before). Thus all particles whether massive or massless except those having $u_x = c$ are observed to be travelling with the velocity of light in the negative x-direction. A particle of non-zero rest mass observed to be travelling with the velocity of light can be given the name 'nooron' (or 'photo-particle')[5]. The velocity of a nooron can be +c or −c depending on the particular boost (luminal boost) chosen.

Note that in the one electron Dirac theory, the eigenvalues of the velocity operator $c\alpha_x$ come out to be $\pm c$, indicating that any velocity determination invariably leads to the conclusion that the particle, inspite of having a non-zero rest mass, moves with the speed of light[6]. Since a velocity measurement requires accurate successive position determinations and time measurements, with a consequent infinite uncertainity in the momentum and energy. This, however, gives an indication that in any interval massive particles can be observed to be travelling with the velocity of light. The Dirac's treatment gives $\pm c$ value for velocity only for such a short time which cannot be measured experimentally. In this paper an attempt is made to present a theory for particles travelling with the velocity of light having well defined mass (which is not infinite) and thus they represent a stable system. Classically $E = m_o c^2(1 - v^2/c^2)^{-\frac{1}{2}}$ shows that it would take infinite energy to get a particle of non-zero rest mass to speed c. But the conditions expressed by (3) and (4) show uncertainity in the value of $m_o(1 - v^2/c^2)^{-\frac{1}{2}}$ and hence in the value of E. Therefore energy becomes indeterminate as v approaches c. To incorporate the effect of this uncertainity the relation $m_o(1 - v^2/c^2)^{-\frac{1}{2}}$ is modified. This and the other modified relations satisfy the postulates of relativity and form a Lorentz group.

In order to appreciate why such a modification is needed let us consider a particle having any arbitrary velocity between -c and +c (This discussion is most general and not restricted in any way to noorons). The equation $E^2 = c^2p^2 + m_o^2c^4$ for a free particle can be written as

(9) $$\pm c\,(m^2 - m_o^2)^{\frac{1}{2}} = p = mv$$

Therefore the momentum mv can also be expressed as $\pm c(m^2 - m_o^2)^{\frac{1}{2}}$. The particle having mass m moving with velocity v can be considered as a particle having mass $(m^2 - m_o^2)^{\frac{1}{2}}$ moving with velocity $\pm c$. Note that the particle is now expressed as equivalent luxon of mass $(m^2 - m_o^2)^{\frac{1}{2}}$. Also $(m^2 - m_o^2)^{\frac{1}{2}}$ approaches zero as mass m tends to m_o. $(m^2 - m_o^2)^{\frac{1}{2}}$ is called 'relativistic effective mass'. In this case the equation of a particle can be written in a form similar to luxon equation

(10) $$E_{eff}^{\;2} = c^2 p^2$$

where $E_{eff} = c^2(m^2 - m_o^2)^{\frac{1}{2}}$ is 'relativistic effective energy'. All particles satisfy the above equation. For luxons $m_o = 0$ and so m becomes relativistic effective mass. Relativistic effective mass is denoted by m_{eff}. Using $m = m_o(1 - v^2/c^2)^{-\frac{1}{2}}$ we have

(11) $$m_{eff} = (m^2 - m_o^2)^{\frac{1}{2}} = m_o(c^2/v^2 - 1)^{-\frac{1}{2}}$$

Note that $m_{eff}/m = v/c$. We can state 'luxon-bradyon transformation' by use of relativistic effective mass[7,8] :

Any luxon equation can be changed to equivalent bradyon equation provided mass of luxon in the equation is replaced by relativistic effective mass of bradyon.

For example consider the luxon equation $\lambda = h(mc)^{-1}$. Replacing m by relativistic effective mass $m_o(c^2/v^2 - 1)^{-\frac{1}{2}}$ and simplifying we get

$$\lambda = h(c^2/v^2 - 1)^{\frac{1}{2}}(m_o c)^{-1} = h(1 - v^2/c^2)^{\frac{1}{2}}(m_o v)^{-1}$$

which is de Broglie relation for massive particles (bradyons).

Introducing the concept of spin the 'relativistic effective mass operator' (REMO) can be written as

$$M = \delta_1 m + \delta_2 m_o \tag{12}$$

where δ_1 and δ_2 are the matrices satisfying the conditions $\delta_1^2 = 1$, $\delta_2^2 = -1$, $\delta_1\delta_2 + \delta_2\delta_1 = 0$. If we write REMO as

$$M = (m^2 - m_o^2)^{\frac{1}{2}} = (H^2/c^4 - m_o^2)^{\frac{1}{2}}$$

(H is Hamiltonian operator), it is not of the form required by the general laws of the quantum theory[6] on account of its being quadratic in H. The wave equation must be linear in the operator $\partial/\partial t$ or H, otherwise a conserved probability function cannot be defined[9]. Therefore we have to accept (12) as the correct expression for REMO. For spin $\frac{1}{2}$ particles 2 x 2 matrices satisfying the above properties are σ_1 and $i\sigma_2$, σ_2 and $i\sigma_3$, σ_3 and $i\sigma_1$ and so on where σ's are Pauli spin matrices. $M = \delta_1 m - \delta_2 m_o$ can also be taken as REMO with the same conditions imposed on δ_1 and δ_2. Since $p = cm_{eff}$, the momentum operator $c(\delta_1 m + \delta_2 m_o)$ is defined in terms of quantities which are same in all coordinate systems (which may be oblique to one another) which are at rest with respect to each other. In another paper[10] the author has discussed the properties of REMO. Ali[11,12] obtained 4 x 4 representation of REMO.

Introducing the two component wavefunction $\psi(\bar{r},t) = [\psi_j(\bar{r},t)]$; $j = 1,2$ we can write (12) as

$$M\psi = (\delta_1 m + \delta_2 m_o)\psi \tag{13}$$

Plane wave solutions can be written as

$$\Psi_j(\bar{r},t) = u_j \exp i\hbar^{-1}(p_1x_1 + p_2x_2 + p_3x_3 - Et)$$

where u_j are complex constants. Substituting this in (13) and using $\delta_1 = \sigma_1$, $\delta_2 = i\sigma_2$ we obtain

(14a,b) $E_{eff}u_1 - (m + m_o)c^2u_2 = 0$; $(m - m_o)c^2u_1 - E_{eff}u_2 = 0$

The determinant of coefficients $(E_{eff}^2 - m^2c^4 + m_o^2c^4)$ vanishes by definition of E_{eff} and so non-trivial solution of (14) exists. Since $E_{eff} = \pm c^2(m^2 - m_o^2)^{\frac{1}{2}}$, there are both positive and negative energy states. Therefore

(15a)	$u_{1\pm} = (m - m_o)c^2(E_{eff\pm})^{-1} A_\pm$		(from 14a)
(15b)	$= E_{eff\pm}(m - m_o)^{-1}c^{-2} A_\pm$		(from 14b)
(15c)	$u_{2\pm} = A_\pm$		

Therefore the probability of existance of any massive particle at speed v is given by

(16) $$|\Psi|^2 = 2m\,(m - m_o)^{-1}\,|A_\pm|^2$$

Using $m = m_o(1 - v^2/c^2)^{-\frac{1}{2}}$ we get the following results

(a) as v approaches zero, $|\Psi|^2$ approaches infinity

(b) at $v = c$, $|\Psi|^2 = 2|A_\pm|^2$

Result (a) shows that there is infinite probability for a massive particle to be at rest which is physically inadmissible.

Result (b) shows that there is a finite non-zero probability of existance of massive particles travelling with the velocity of light.

This shows that particles of infinite mass can exist (from the concept of classical relativity).

Let us examine why $\delta_1 = \sigma_1$, $\delta_2 = i\sigma_2$ is used for REMO and is this the best choice. Al-Kurdi[13,14] discussed in detail the 2 x 2 representation of REMO and obtained probability densities for different choices of matrices. The results are

S. No.	choice of matrices δ_1	δ_2	probability density $\lvert\psi\rvert^2$
1	σ_1	$i\sigma_2$	$2m(m - m_o)^{-1}\lvert A\pm\rvert^2$
2	σ_2	$i\sigma_3$	$2\lvert A\pm\rvert^2$
3	σ_3	$i\sigma_1$	$2\lvert A\pm\rvert^2$
4	σ_1	$i\sigma_3$	$2\lvert A\pm\rvert^2$
5	σ_3	$i\sigma_2$	$2(m/m_o)^2[1 + (1 - m_o^2/m^2)^{\frac{1}{2}}]\lvert A\pm\rvert^2$
6	σ_2	$i\sigma_1$	$2m(m + m_o)^{-1}\lvert A\pm\rvert^2$

Case 1 has been treated by the author. Cases 2, 3 and 4 are of no interest because the probability is constant for all the velocities. In case 5, $|\psi|^2 = 2|A_\pm|^2$ at $v = 0$. As v approaches c, $|\psi|^2$ approaches infinity. Therefore in contrary to case 1, the particle has now a greater tendency to travel with the velocity of light. One way to make $|\psi|^2$ finite at $v = c$ is to choose $|A_\pm| = 0$. Therefore $|\psi|^2 = 0$ for all $v \neq c$. Therefore this case actually represents luxons. Another way to get rid of infinities in case 1 and case 5 is discussed in the next paragraph. In case 6, $|\psi|^2 = |A_\pm|^2$ at $v = 0$ and $|\psi|^2 = 2|A_\pm|^2$ at $v = c$. In this case the probability density remains finite for all values of v $(0 \leq v \leq c)$. There is an increasing tendency of probability as v increases. Therefore case 6 is a representation of high speed particles whereas case 1 represents low speed particles (because probability increases as v decreases).

It was pointed out earlier that an accurate determination of velocity makes $(1 - v^2/c^2)^{\frac{1}{2}}$ indeterminate. To incorporate this uncertainity we introduce a parameter $P(m_o, v)$ which is positive and gives uncertainity in velocity. The formula for mass can be written as

$$m = m_o \left[1 - v^2 \left\{P(m_o, v)\right\}^2 / c^2\right]^{-\frac{1}{2}} \tag{17}$$

For simplicity we write P instead of $P(m_o, v)$. Eq. (16) then becomes

$$|\Psi|^2 = 2 \left[1 - (1 - v^2P^2/c^2)^{\frac{1}{2}}\right]^{-1} |A_{\pm}|^2 \tag{18}$$

As m_o approaches zero, $P(m_o, v)$ tends to unity. This is the case of luxon. If $m_o \neq 0$, $P(m_o, v)$ can be taken as unity for intermediate values of velocity. As v approaches zero, $P(m_o, v)$ approaches infinity such that m is very nearly equal to m_o at v = 0. This is because of the fact that we cannot say that velocity is exactly zero (uncertainity principle). Therefore v $P(m_o, v)$ does not vanish and $|\Psi|^2$ remains finite as v tends to zero. As v approaches the velocity of light, $P(m_o, v)$ approaches a value very nearly equal to but slightly less than unity. Therefore m is very large but does not become infinite at v = c. $|\Psi|^2$ remains finite at v = c. Such a choice of $P(m_o, c)$ also removes infinity in case 5. $|\Psi|^2$ in case 5 becomes non-infinite at v = c. The most suitable case (case 6) is also very conveniently described after the introduction of $P(m_o, v)$. This is because infinity in the expression of mass $m = m_o(1 - v^2/c^2)^{-\frac{1}{2}}$ at v = c is removed.

Using the modified expression of momentum $p = mPv$, it can be shown that for 10^6 GeV protons $(1 - P) \leqslant 0.44 \times 10^{-12}$. It is suggested that the deviation of $P(m_o, v)$ from unity at both the limits $v = 0$ and $v = c$ should be small for smaller values of m_o and large for larger values of m_o. This effect was not observed in the case of fundamental particles because the contribution of $P(m_o, v)$ for low values of m_o is negligible[15].

Therefore we come to the conclusion that the energy

$$E = m_o c^2 (1 - P^2 v^2/c^2)^{-\frac{1}{2}}$$

does not become infinite as v approaches c. If we replace V (the velocity of the frame) by PV in Lorentz tranformation equations, the interval $(x_k^{(1)} - x_k^{(2)})^2$; $k = 1,2,3,4$ in Minkowski space remains invariant and hence these modified expressions form a Lorentz group[16].

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3 An informal discussion with Amer Mufti.

4 E. Recami and R. Mignani, Riv. Nuovo Cimento 4(1974)209;398.

5 The name 'photoparticle' first appeared in: S. A. Kamal, The photon frame of reference, Vision(Dept. Phys., Univ. Karachi), Nov. 1975, p. 4. The name 'nooron' is derived from 'noor' (Arabic) meaning light and is taken from the following verse of Al-Quran

اَللّٰهُ نُوْرُ السَّمٰوٰتِ وَالْاَرْضِ ط ... (سورة النور-آية ٣٥)

'Allah is the Light of the heavens and the earth. ....' (Sura 24:35).

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[15] It might appear that a massive particle travelling at the luminal velocity still maintaining a finite mass is against experimental evidence as shown by high energy particles in accelerators. Note that nobody has actually obtained data for $v = c$, and the relation which holds at lower speeds might not be correct for higher speeds. If v_1 is the velocity uptil which experimental results are available, there must exist a v_2 such that $v_1 < v_2 < c$ because v is a continuous variable, and the results that hold at $v = v_1$ might not hold at $v = v_2$. Therefore $m = m_0(1 - v^2/c^2)^{-\frac{1}{2}}$ shown to be correct at $v = v_1$ might be the special case of the more general relation $m = m_0(1 - P^2v^2/c^2)^{-\frac{1}{2}}$ which holds at $v = v_1$ and $v = v_2$ with proper values given to P.

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