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روشنی کی رفتار سے حرکت کرنے والے ذرات (لکسان - Luxon) کی مساواتوں کو روشنی کی رفتار سے کم رفتار سے سفر کرنے والے ذرات (بریڈیان) کی مساواتوں میں تبدیل کرنے کا طریقہ معلوم کیا گیا ہے - اس طریقے سے حرکت سے متعلق مختلف مسائل کو آسانی کے ساتھ حل کیا جاسکے گا۔

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LUXON-BRADYON TRANSFORMATION*

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A method has been described to convert the equations valid for luxons (particles travelling with the speed of light) to be applicable for bradyons (particles travelling with speeds less than the speed of light). Problems related to the motion of particles can be solved easily by this method.

A particle which travels with the speed of light in vacuum and has zero proper mass is called 'luxon' e.g. photon, graviton, neutrino. A particle which has non-zero proper (rest) mass and travels with speed less than the speed of light in vacuum is called 'bradyon'[1]. The equations describing the motion of bradyons can generally be changed to luxon equations by substituting $v = c$ and $m_o = 0$ where v is the velocity of bradyon, c the velocity of light in vacuum and m_o the proper mass of bradyon. In this paper we describe a method to change luxon equations to

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bradyon equations.

Let us consider a particle having any arbitrary velocity between -c and +c. The equation

$$(1) \qquad E^2 = c^2p^2 + m_o^2c^4$$

for a free particle can be written as

$$(2) \qquad \pm c(m^2 - m_o^2)^{\frac{1}{2}} = p = mv$$

The particle having mass m moving with velocity v can be considered as a particle having mass $(m^2 - m_o^2)^{\frac{1}{2}}$ moving with velocity $\pm c$. Note that the particle is now expressed as equivalent luxon[1] of mass $(m^2 - m_o^2)^{\frac{1}{2}}$. The positive or negative sign is chosen according to the sign of v. $(m^2 - m_o^2)^{\frac{1}{2}}$ approaches zero as mass m tends to m_o. $m_f = (m^2 - m_o^2)^{\frac{1}{2}}$ is called 'relativistic effective mass'[2]. Therefore the equation of a particle can be written in a form similar to luxon equation

$$(3) \qquad E_f^2 = c^2p^2$$

$E_f = m_fc^2$ is called 'relativistic effective energy'. Using the concept of relativistic effective mass[2], we can state luxon-bradyon (LB) transformation[3]:

Any luxon equation can be changed to equivalent bradyon equation provided the mass of luxon in the equation is replaced by the relativistic effective mass of bradyon.

Consider the luxon equation $\lambda = h(mc)^{-1}$. Replacing m by the relativistic effective mass $(m^2 - m_o^2)^{\frac{1}{2}}$ and simplifying we get

(4) $\lambda = h(c^2/v^2 - 1)^{\frac{1}{2}}(m_o c)^{-1} = h(1 - v^2/c^2)^{\frac{1}{2}}(m_o v)^{-1}$

which is de Broglie relation for massive particles (bradyons).

Let $\overline{r}_o$ be the value of the four-vector in the laboratory frame and $\overline{r}_v$ be its value in the frame moving with velocity v.

(5a) $\overline{r}_o = (x_o, y_o, z_o, ict_o) = (x_{o1}, x_{o2}, x_{o3}, x_{o4})$

(5b) $\overline{r}_v = (x_v, y_v, z_v, ict_v) = (x_{v1}, x_{v2}, x_{v3}, x_{v4})$

We can write for any bradyon

(6a) $x_o^2 + y_o^2 + z_o^2 - c^2 t_o^2 = x_v^2 + y_v^2 + z_v^2 - c^2 t_v^2$

After rearranging we get

(6b) $(x_v^2 - x_o^2) + (y_v^2 - y_o^2) + (z_v^2 - z_o^2) = c^2(t_v^2 - t_o^2)$

Let $\overline{r}$ be the four-vector for a luxon i.e.

$$\overline{r} = (x, y, z, ict) = (x_1, x_2, x_3, x_4)$$

and so

(7) $$x^2 + y^2 + z^2 = c^2 t^2$$

We note that the luxon equation (7) can be changed to the equivalent bradyon equation (5b) if we replace x by $(x_v^2 - x_o^2)^{\frac{1}{2}}$, y by $(y_v^2 - y_o^2)^{\frac{1}{2}}$, z by $(z_v^2 - z_o^2)^{\frac{1}{2}}$ and t by $(t_v^2 - t_o^2)^{\frac{1}{2}}$. A similar result is obtained if we use current density four-vector.

Consider an arbitrary four-vector $\overline{q}$ representing luxon. We can write

(8) $$\bar{q} = (q_1, q_2, q_3, q_4)$$

The squared magnitude of $\bar{q}$ is given by

(9a) $$|\bar{q}|^2 = q_1^2 + q_2^2 + q_3^2 + q_4^2$$

or

(9b) $$|\bar{q}|^2 = \sum_{i=1}^{4} q_i^2 = 0$$

$|\bar{q}|^2$ can be called norm of $\bar{q}$. If $\bar{q}_v$ represents a bradyon moving with velocity v (considered to be at rest in a frame moving with velocity v) and $\bar{q}_o$ be its value when the bradyon is at rest, we have

(10a) $$\bar{q}_o = (q_{o1}, q_{o2}, q_{o3}, q_{o4})$$

(10b) $$\bar{q}_v = (q_{v1}, q_{v2}, q_{v3}, q_{v4})$$

Therefore

(11) $$|\bar{q}_o|^2 = |\bar{q}_v|^2$$

from the definition of four-vectors i.e.

(12a) $$\sum_{i=1}^{4} q_{oi}^2 = \sum_{i=1}^{4} q_{vi}^2$$

This equation can be written in the form of (9b) i.e.

(12b) $$\sum_{i=1}^{4} (q_{vi}^2 - q_{oi}^2) = 0$$

We define a four-vector $\bar{Q}$

(13) $$\overline{Q} = (Q_1, Q_2, Q_3, Q_4)$$

where

(14) $$Q_i = (q_{vi}{}^2 - q_{oi}{}^2)^{\frac{1}{2}}; \qquad i = 1,2,3,4$$

Therefore

(15) $$|\overline{Q}|^2 = \sum_{i=1}^{4} Q_i{}^2 = 0$$

by virtue of eq. (12b).

Let the equation for a luxon be written in the form

(16) $$f(y_1, y_2, \ldots, y_n) = 0$$

where $y_1, y_2, \ldots, y_n$ are the observables. This equation can also be written as

(17) $$f(y_j) = 0; \quad j = 1,2,\ldots,n$$

The generalized luxon-bradyon (GLB) transformation can be stated as:

> Any luxon equation can be changed to equivalent bradyon equation provided we replace all the observables y_j in the luxon equation by $\left[y_{vj}{}^2 - y_{oj}{}^2\right]^{\frac{1}{2}}$ where y_{oj} is the value of the observable when the bradyon is at rest and y_{vj} is the value when it is moving with velocity v.

Therefore the bradyon equivalent to luxon equation (17) can be written as

(18) $$f(\left[y_{vj}{}^2 - y_{oj}{}^2\right]^{\frac{1}{2}}) = 0; \quad j = 1,2,\ldots,n$$

For the equations which involve vector notation e.g.

$$(19)\qquad \bar{a} = \hat{i}a_x + \hat{j}a_y + \hat{k}a_z$$

(where $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors along x, y, z axes) we have to replace this vector by a vector $\bar{a}_{vo}$ in order to change a luxon equation to a bradyon equation. The components of $\bar{a}_{vo}$ are given by

$$(20)\qquad \bar{a}_{vo} = \hat{i}(a_{vx}^2 - a_{ox}^2)^{\frac{1}{2}} + \hat{j}(a_{vy}^2 - a_{oy}^2)^{\frac{1}{2}} + \hat{k}(a_{vz}^2 - a_{oz}^2)^{\frac{1}{2}}.$$

where a_{ox}, a_{oy}, a_{oz} are the components of $\bar{a}$ when the bradyon is at rest and a_{vx}, a_{vy}, a_{vz} are the values when it is moving with velocity v.

The generalized luxon-bradyon transformation for inhomogeneous Lorentz transformations and the properties of GLB transformation matrices are described elsewhere[3]. We apply GLB transformation to prove that the velocity of de Broglie waves is c^2/v without actual wavepacket analysis.

The angular frequency ω and wavenumber k of a luxon are related by

$$(21)\qquad \omega = ck$$

Applying GLB transformation, we get

$$(22)\qquad (\omega_v^2 - \omega_o^2)^{\frac{1}{2}} = c(k_v^2 - k_o^2)^{\frac{1}{2}}$$

Also applying GLB transformation to the luxon equation

$$(23)\qquad p = \hbar k$$

we have

$$(24)\qquad (p_v^2 - p_o^2)^{\frac{1}{2}} = \hbar(k_v^2 - k_o^2)^{\frac{1}{2}} = p_v$$

Since $p_v = \hbar k_v$ for bradyons, eq. (24) gives $k_o = 0$. Therefore (22) becomes

(25) $$\omega_v^2 = c^2 k_v^2 - \omega_o^2$$

The phase velocity v_p can, therefore, be written as

(26) $$v_p^2 = \omega_v^2/k_v^2 = c^2(1 - \omega_o^2/c^2k_v^2)$$

Now

(27a) $$1 - \omega_o^2/c^2k_v^2 = 1 - \omega_o^2/(\omega_v^2 - \omega_o^2) = \omega_v^2/(\omega_v^2 - \omega_o^2)$$

Using $\hbar\omega = E = mc^2$, we get

(27b) $$1 - \omega_o^2/c^2k_v^2 = E_v^2/(E_v^2 - E_o^2) = m_v^2/(m_v^2 - m_o^2)$$

Noting that $m_v = m_o(1 - v^2/c^2)^{-\frac{1}{2}}$, we have

$$1 - \omega_o^2/c^2k_v^2 = c^2/v^2$$

and so

(28a) $$v_p^2 = c^2(c^2/v^2) = c^4/v^2$$

or

(28b) $$v_p = c^2/v$$

Therefore we have developed a method of 'equivalent luxon system' for the solution of dynamical problems. The following steps are suggested:

(i) Write down the bradyon equation and the corresponding luxon equation.

(ii) Replace all the parametrs in the luxon equation by the effective values. This is the bradyon equation and the bradyon is now equivalent to luxon with the effective parameters.

(iii) Treat the problem as the luxon problem and finally obtain the actual values from the effective values.

In the conventional method, we have to consider the variation of both the mass and the velocity in the relativistic range. Here the velocity is always equal to c and all the energy supplied changes its mass. Therefore we have to consider only one parameter. For example the force equation

(29a) $$F = dp/dt = m\,dv/dt + v\,dm/dt$$

can be written as

(29b) $$F = c\,dm_f/dt$$

This transformation has been obtained by inductive method. In future works a rigorous proof may be given for this transformation. The choice of the rest frame in GLB transformation is quite arbitrary. If the coordinates of the particle at any other velocity v_o are known, these can be used inplace of the rest frame coordinates. An equivalence of these equations is needed to be proved.

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