



سائنٹفک سوسائٹی پاکستان

خلاصہ

شعبہ علوم طبیعی

انیسویں سالانہ کانفرنس

زیر اہتمام
جامعہ قائد اعظم اسلام آباد

۱۹۷۹ء

برینڈیان کے لیے REMO کا 2×2 بیان

شیخ انصار حسین - زیاد الکردی

عارف کمان اور لیسات علمی

شعبہ انجینئرنگ - جامعہ کراچی

روشنی کی رفتار سے کم رفتار سے حرکت کرنے والے ذرات (برینڈیان $Bradyon$)

کے لیے اضافی موثر کم کارگر (Relativistic Effective Mass Operator) کے

2×2 بیان کیا تشکیل سے جائزہ لیا گیا ہے اور یہ معلوم کیا گیا ہے کہ کون سی مسارات

بہترین نتائج دیتی ہیں -

2 x 2 REPRESENTATION OF REMO (RELATIVISTIC EFFECTIVE MASS OPERATOR) FOR
BRADYONS*

S. A. HUSAIN, ZEYAD D. M. AL-KURDI, SYED ARIF KAMAL[†] and LIAQUAT ALI

Department of Physics, University of Karachi
Karachi-32, Pakistan.

2 x 2 representation of relativistic effective mass operator
(S. A. Kamal, J. Nat. Sci. Math. 20(1980)15) for bradyons
has been studied in detail using different combinations of
Pauli spin matrices.

The concept of relativistic effective mass has been introduced by one
of the authors¹. Physical significance and properties of relativistic
effective mass operator (REMO) are described elsewhere^{1,2}. This concept
led to a useful transformation for changing any luxon equation to the
corresponding bradyon equation^{1,3}. Quantum mechanical treatment of the
relativistic effective mass operator (REMO) leads to some results which
are not explained by the special theory of relativity^{1,4}. In this paper
we examine in detail the 2 x 2 representation of REMO by using different
combinations of Pauli spin matrices. Usefulness of each of these

*

Revised and updated English translation of the paper (in Urdu) presented
during the Scientific Society of Pakistan's 19th Annual Science
Conference, Quaid-i-Azam University, Islamabad (Pakistan), September
1979.

†

Department of Physics, Indiana University
Bloomington, Indiana 47405, U.S.A.

representations in different velocity ranges is also discussed.

The equation relating the energy E and momentum p of a bradyon having proper mass m_0 can be written as

$$(1) \quad E^2 = p^2 c^2 + m_0^2 c^4$$

Recalling that $E = mc^2$, the above equation can be rearranged as

$$(2) \quad p = (m^2 - m_0^2)^{\frac{1}{2}} c$$

Therefore a bradyon of mass m moving with velocity v can be considered as a luxon of mass $(m^2 - m_0^2)^{\frac{1}{2}}$ moving with velocity c . $(m^2 - m_0^2)^{\frac{1}{2}}$ is called 'relativistic effective mass'. The following equation is satisfied by such particles

$$(3) \quad E_f^2 = p^2 c^2$$

where $E_f = (m^2 - m_0^2)^{\frac{1}{2}} c^2$ is the relativistic effective energy. Using $m = m_0 (1 - v^2/c^2)^{-\frac{1}{2}}$, we can write

$$(4) \quad m_f = (m^2 - m_0^2)^{\frac{1}{2}} = m_0 (c^2/v^2 - 1)^{-\frac{1}{2}}$$

Introducing spin we can write the relativistic effective mass operator (REMO) as

$$(5) \quad M = \delta_1 m + \delta_2 m_0$$

where δ_1 and δ_2 are the matrices satisfying the conditions

$$(6) \quad \delta_1^2 = 1 = -\delta_2^2, \quad \delta_1 \delta_2 + \delta_2 \delta_1 = 0$$

$M = \delta_1 m - \delta_2 m_0$ can also be taken as REMO for bradyons with the same conditions imposed on δ_1 and δ_2 . For spin $\frac{1}{2}$ particles σ_1 and $i\sigma_2$, σ_2 and $i\sigma_3$, σ_3 and $i\sigma_1$, σ_2 and $i\sigma_1$, σ_3 and $i\sigma_2$, σ_1 and $i\sigma_3$ can be taken as σ_1 and σ_2 respectively where σ 's are Pauli spin matrices.

The properties of REMO using σ_1 and $i\sigma_2$ as δ_1 and δ_2 have been discussed by one of the authors. We shall examine the properties of REMO using all possible combinations of Pauli matrices. The following simplifications are used to list the properties in tabular form.

$$(7a) \quad (1 - m_0^2/m^2)^{\frac{1}{2}} = K$$

$$(7b) \quad 1 + (1 - m_0^2/m^2)^{\frac{1}{2}} = 1 + K = L$$

The properties are:

(i) Eigenvalues: In all cases the eigenvalues come out to be $+(m^2 - m_0^2)$ and $-(m^2 - m_0^2)^{\frac{1}{2}}$.

(ii) Plane wave solutions: The plane wave solutions of

$$(8) \quad M\Psi = (\delta_1 m + \delta_2 m_0)\Psi$$

where

$$(9) \quad \Psi = [\Psi_j]; \quad j = 1, 2$$

and

$$(10) \quad \Psi_j = u_j \exp(i/\hbar)(\vec{p} \cdot \vec{r} - Et)$$

are obtained in different cases. Taking $u_{2\pm} = A_{\pm}$ (\pm is for positive and negative energies), the values of u_{\pm} are obtained as

Table 1

S.No.	δ_1	δ_2	$u_{1\pm}$
1	σ_1	$i\sigma_2$	$(m + m_0)c^2(E_{f\pm})^{-1}A_{\pm}$
2	σ_2	$i\sigma_3$	$mc^2(E_{f\pm} - im_0c^2)^{-1}A_{\pm}$
3	σ_3	$i\sigma_1$	$im_0c^2(E_{f\pm} - mc^2)^{-1}A_{\pm}$
4	σ_1	$i\sigma_3$	$mc^2(E_{f\pm} - im_0c^2)^{-1}A_{\pm}$
5	σ_3	$i\sigma_2$	$m_0c^2(E_{f\pm} - mc^2)A_{\pm}$
6	σ_2	$i\sigma_1$	$i(m - m_0)c^2(E_{f\pm})^{-1}A_{\pm}$

(iii) Probability density: The probability density is given by

$$(11) \quad |\psi|^2 = u_1^* u_1 + u_2^* u_2$$

Applying the condition $|\psi|^2 = 1$, we get the value of normalization constant $|A_{\pm}|$ for different combinations of δ_1 and δ_2 .

Table 2

S.No.	δ_1	δ_2	$ \psi ^2$	$ A_{\pm} $
1	σ_1	$i\sigma_2$	$2m(m - m_0)^{-1} A_{\pm} ^2$	$(m - m_0)^{\frac{1}{2}}(2m)^{-\frac{1}{2}}$
2	σ_2	$i\sigma_3$	$2 A_{\pm} ^2$	$(2)^{-\frac{1}{2}}$
3	σ_3	$i\sigma_1$	$2 A_{\pm} ^2$	$(2)^{-\frac{1}{2}}$
4	σ_1	$i\sigma_3$	$2 A_{\pm} ^2$	$(2)^{-\frac{1}{2}}$

Table 2 (contd.)

5	σ_3	$i\sigma_2$	$2L(m/m_0)^2 A_{\pm} ^2$	$(m_0/m)(2L)^{-\frac{1}{2}}$
6	σ_2	$i\sigma_1$	$2m(m+m_0)^{-1} A_{\pm} ^2$	$(m+m_0)^{\frac{1}{2}}(2m)^{-\frac{1}{2}}$

The above entries show that cases 2, 3 and 4 are of no interest because the probability is constant for all the velocities. In case 5, $|\psi|^2 = 2|A_{\pm}|^2$ at $v = 0$. As v approaches c , $|\psi|^2$ approaches infinity. The particle has now a greater tendency to travel with the velocity of light. One way to make $|\psi|^2$ finite at $v = c$ is to choose $|A_{\pm}| = 0$. Now $|\psi|^2 = 0$ for all $v \neq c$. Therefore this case actually represents luxons. In case 6, $|\psi|^2 = |A_{\pm}|^2$ at $v = 0$ and $|\psi|^2 = 2|A_{\pm}|^2$ at $v = c$. In this case the probability density remains finite for all values of v ($0 \leq v \leq c$). There is an increasing tendency of probability as v increases. Therefore case 6 is a representation of high speed particles. As discussed elsewhere^{1,2}, case 1 represents low speed particles. $|\psi|^2 = 2|A_{\pm}|^2$ at $v = 0$. As v approaches zero, $|\psi|^2$ approaches infinity. Therefore probability decreases as v increases. The infinity in case 1 is removed by introducing an uncertainty factor^{1,4}. Also note that there is a non-zero probability for the massive particles travelling with the velocity of light. This point is further discussed elsewhere^{1,4}.

(iv) Expectation values: Using the values of $|A_{\pm}|$ given in table 2, the expectation value in each case comes out to be

$$(12) \quad \langle M \rangle = \frac{1}{2} (m^2 - m_0^2)^{\frac{1}{2}}$$

(v) Calculation of uncertainties: Using the relation of Robertson⁶

$$(13) \quad \Delta A \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle|$$

we calculate the uncertainties of δ_1 and δ_2 , δ_1 and M , δ_2 and M .

Table 3

S.No.	δ_1	δ_2	$\Delta\delta_1$	$\Delta\delta_2$	$\Delta\delta_1 \Delta M$	$\Delta\delta_2 \Delta M$
1	σ_1	$i\sigma_2$	$\geq m_0/m$	$\geq m_0^2/m$	$\geq m_0$	
2	σ_2	$i\sigma_3$	$\geq K$	$\geq m_0 K$	$\geq mK$	
3	σ_3	$i\sigma_1$	≥ 0	≥ 0	≥ 0	
4	σ_1	$i\sigma_3$	$\geq m_0/m$	$\geq m_0^2/m$	$\geq m_0$	
5	σ_3	$i\sigma_2$	$\geq mL/m_0$	$\geq mL$	$\geq m^2 L/m_0$	
6	σ_2	$i\sigma_1$	$\geq m_0/m$	$\geq m_0^2/m$	$\geq m_0$	

Therefore we note that except for case 3, δ_1 and δ_2 , δ_1 and M , δ_2 and M cannot be determined simultaneously. In all these cases

$$(14) \quad \delta_1^\dagger \delta_1 = 1; \quad \delta_2^\dagger \delta_2 = 1$$

Note that

$$(15) \quad M^\dagger = (\delta_{1m} + \delta_{2m}^\dagger) = \delta_{1m} - \delta_{2m}$$

is another representation of REMO.

A 4×4 representation of REMO has also been obtained and its properties are discussed elsewhere^{7,8}. A definition of REMO can also be given for

1
tachyons based on Feinberg's theory . 9

1
S. A. Kamal, Luxon-Bradyon Transformation, thesis, Univ. Karachi, 1978 (unpublished).

2
S. A. Kamal, J. Nat. Sci. Math. (Lahore) 20(1980)15.

3
S. A. Kamal and S. A. Husain, Luxon-bradyon transformation, 19th Annual Science Conference, Quaid-i-Azam University, Islamabad, 1979.

4
S. A. Kamal, Bull. Amer. Phys. Soc. 26(1981)47.

5
Z. D. M. Al-Kurdi, 2 x 2 Representation of REMO (Relativistic Effective Mass Operator) for Bradyons, project, Univ. Karachi, 1978 (unpublished).

6
H. P. Robertson, Phys. Rev. 34(1929)164(L).

7
L. Ali, 4 x 4 Representation of Relativistic Effective Mass Operator for Bradyons, project, Univ. Karachi, 1978 (unpublished).

8
L. Ali, S. A. Husain, S. A. Kamal and Z. D. M. Al-Kurdi, 4 x 4 representation of REMO (relativistic effective mass operator) for bradyons, 19th Annual Science Conference, Quaid-i-Azam University, Islamabad, 1979.

9
G. Feinberg, Phys. Rev. 159(1967)1089.

