NTS Detailed Curriculum for Mathematics

Approved in the Academic Committee Meeting on December 30, 2010

Sr. No.	*Core Areas (in alphabetical order)*	*Weightage*
01	**Algebra**	**08%**
01.1	*Group Theory* Basic axioms of a group, abelian groups, center of a group, derived subgroup of a group, subgroups generated by subset of a group, system of generators, cyclic groups, cosets and quotient sets, Lagrange's theorem, permutations, even and odd permutations, cycles, length of a cycle, transpositions, symmetric and non-symmetric groups, alternating groups, normalizers and centralizers of a subset of a group, congruency classes of a group, normal subgroup, quotient groups, conjugacy, relation between elements and subgroups, homomorphism and isomorphism between groups, homomorphism and isomorphism theorems, group of automorphisms, finite p-groups, internal and external direct products, group action on sets, isotropy subgroups, orbits, 1^{st}, 2^{nd} and 3^{rd} Sylow theorems	03%
01.2	*Algebra of Matrices* Matrix and its types, determinants and its properties, inverse of a matrix, row and column operations, echelon and reduced echelon form, rank of a matrix, consistent and inconsistent systems (conditions for the existence of zero, one or infinite solutions), solution of non-homogenous equations (Gauss-elimination method, Gauss-Jordon method, inverse method, Cramer's rule), solution of homogenous equations and eigenvalue problems, groups and subgroups of matrices	01%
01.3	*Ring Theory* Finite and infinite fields, rings and its types (matrix rings, rings of endomorphisms, polynomial rings), integral domain, characteristic of a ring, ideals, types of ideals, quotient rings, homomorphism of rings, fundamental theorem of homomorphism of rings	01%
01.4	*Abstract Linear Algebra* Vector spaces, subspaces, linear span of subset of a vector space, bases and dimensions of a vector space, sums and direct sums of subspaces of a finite dimensional vector space, dimension theorem, linear transformation, null space, image space of a linear transformation, matrix of a linear transformation, rank and nullity of a linear transformation, relation between rank, nullity and dimension of domain of a linear transformation, orthogonal transformation, change of basis, inner-product spaces, projection of a vector on another vector, norm of a vector, Cauchy-Schwartz inequality, orthogonal and orthonormal bases, similar matrices and diagonalization of a matrix, Home(V, W), dimension and basis of Home(V, W), dual space and dual basis, annihilators	03%

Sr. No.	Core Areas (in alphabetical order)	Weightage
02	**Basics of Mathematics (Class V to VII)** Word problems related to real life	**10%**
02.1	*Class V & VI Mathematics* Sets, types of sets, whole numbers, integers, factors and multipliers, fractions, decimals, percentages, unitary method, BODMAS rule, ratio and proportion, financial arithmetic, algebraic operations, linear equations, distance, time and temperature, line segments, construction of angles, construction of triangles, construction of quadrilaterals, perimeter and area, average, graphs (block, column, bar and pie)	05%
02.2	*Class VII & VIII Mathematics* Basic operations on sets, Venn diagrams, verification of commutative, associative, distributive and De Morgan's laws through sets and Venn diagrams, rational numbers, real numbers, number systems with bases 2 and 10 and their conversions, exponents, square root of positive numbers, cubes and cube roots, HCF and LCM (using division and prime factorization) direct and inverse relations, taxes, profit, loss, discount and markup, compound proportion, income tax, Zakat and Ushr, operations with polynomials, algebraic identities involving $(x+a)(x-a)$, $(a+b)^2$, $(a-b)^2$ and a^2-b^2, factorization of algebraic expressions, simultaneous equations, solution by comparison, substitution, elimination, cross-multiplication and graphical methods, properties of angles, congruent and similar figures, congruent triangles, circumference and area of a circle, surface area and volume of sphere and cylinder, frequency distribution,	05%
03	**Calculus**	**08%**
03.1	*Differential Calculus* Functions, inverse functions, parametric equations and polar coordinates, limit and continuity; derivative of a function, maxima, minima and point of inflection; optimization problems; mean value theorem (Taylor theorem and the infinite Taylor series), curve sketching	03%
03.2	*Integral Calculus* Integral, definite and indefinite integral, the fundamental theorem of calculus, techniques of integration, area under the curve	02%
03.3	*Vector Calculus* Vectors and analytic geometry of 2 and 3 dimensional spaces, vector-valued functions and space curves, functions of several variables, limits and continuity, partial derivatives, the chain rule, double and triple integrals with applications, line integrals, the Green's theorem, surface area and surface integrals, the Green, the divergence and the Stokes theorems	03%
04	**Complex Analysis**	**07%**
04.1	*Complex Numbers* The algebra and the geometry of complex numbers	02%
04.2	*Theorems* Cauchy-Riemann equations, harmonic functions, elementary functions, complex exponents, contours and contour integrals, the Cauchy-Goursat Theorem, the Cauchy integral formulae, the Morera Theorem, maximum modulus principle, the Liouville theorem, fundamental theorem of algebra	02%
04.3	*Series and Integrals* Convergence of sequences and series, the Taylor series, power series representation of functions, the Laurent series, uniqueness of representation, branch point, zeros of analytic functions, residues and poles, the residue theorem, evaluation of improper integrals involving trigonometric functions, integrals around a branch point, the argument principle, the Roche theorem	03%

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05	**Computational Mathematics**	**15%**
05.1	*Sets and Relations* Basic notions, set operations, extended-set operations, indexed family of sets, countable and uncountable sets, relations, cardinality, equivalence relations, congruence, partitions, partial order, representation of relations, mathematical induction	04%
05.2	*Elementary Logic* Logics of order zero and one, propositions and connectives, truth tables, conditionals and biconditionals, quantifiers, methods of proof, proofs involving quantifiers	04%
05.3	*Numerical Analysis* Computer arithmetic, approximations and errors; methods for the solution of nonlinear equations and their convergence: bisection method, regular falsie method, fixed point iteration method, Newton-Raphson method, secant method; error analysis for iterative methods. Interpolation and polynomial approximation: Lagrange interpolation, Newton's divided difference, forward-difference and backward-difference formulae, Hermite interpolation, numerical differentiation, integration and their error estimates, rectangular rule, trapezoidal rule, Simpson's one-three and three-eight rules, numerical solution of systems of algebraic linear equations: Gauss-elimination method, Gauss-Jordan method; matrix inversion; LU-factorization; Doolittle's, Crount's, Cholesky's methods; Gauss-Seidel and Jacobi methods	07%
06	**Differential Equations**	**\15%**
06.1	*Ordinary Differential Equations* Formation and solution of first-order-differential equations, formation and solution of higher-order-linear-differential equations; differential equations with variable coefficients, Sturm-Liouville (S-L) system and boundary-value problems, series solution and its limitations, the Frobenius method, solution of the Bessel, the hypergeometric, the Legendre and the Hermite equations, properties of the Bessel functions	08%
06.2	*Partial Differential Equations* First-order-partial-differential equations, classification of second-order partial-differential equations, canonical form for second-order equations; wave, heat and the Laplace equations in Cartesian, cylindrical and spherical-polar coördinates; solution of partial-differential equation by the methods of: separation of variables, the Fourier, the Laplace and the Hankel transforms, non-homogeneous-partial-differential equations	07%
07	**Functional Analysis**	**07%**
07.1	*Metric Spaces* Completeness and convergence	01%
07.2	*Normed Spaces* Linear spaces, normed spaces, Difference between a metric and a normed space, Banach spaces, Bounded and continuous linear operators and functionals, dual spaces, finite-dimensional spaces, F. Riesz lemma	02%
07.3	*The Hahn-Banach Theorem* The HB theorem for complex spaces, the HB theorem for normed spaces, the open mapping theorem, the closed graph theorem, uniform boundedness principle, the Banach-fixed-point theorem	02%
07.5	*Inner-Product Spaces* Inner-product space, Hilbert space, orthogonal and orthonormal sets, orthogonal complements, Gram-Schmidt orthogonalization process, representation of functionals, Reiz-representation theorem, weak and weak* convergence	02%

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08	**Geometry**	**04%**
08.1	*Analytical Geometry* Cartesian-coördinate mesh, slope of a straight line, equation of a straight line, parallel and perpendicular lines, various forms of equation of a line, intersection of two lines, angle between two lines, distance between two points, distance between a point and a line, equation of a circle, circles formed under various conditions, intersection of lines and circles	02%
08.2	*Conic Sections* Conic section (circle, parabola, ellipse and hyperbola), the general-second-degree equation	02%
09	**Mathematical Statistics and Probability**	**07%**
09.1	*Mathematical Statistics* Sampling theory, sampling distributions, sampling procedures, estimation of parameters, estimation of mean, variance, confidence intervals, decision theory, hypothesis testing and decision making, types of errors in tests, quality control, control charts for mean, standard deviation, variance, range, goodness of fit, chi-square test, regression analysis, method of least squares, correlation analysis	04%
09.2	*Probability* Introduction to probability theory, random variables, probability distributions, mean, standard deviation, variance and expectation, binomial, negative binomial, Poisson, geometric, hypergeometric and normal distributions, normal approximation to binomial distribution; distributions of 2 random variables	03%
10	**Mechanics**	**04%**
10.1	Particle kinematics, radial and transverse components of velocity and acceleration, circular motion, motion with a uniform acceleration, the Newton laws of motion (the inertial law, the force law and the reaction law), newtonian mechanics, the newtonian model of gravitation, simple-harmonic motion, damped oscillations, conservative and dissipative systems, driven oscillations, nonlinear oscillations, calculus of variations	02%
10.2	Hamilton's principle, lagrangian and hamiltonian dynamics, symmetry and conservation laws, Noether's theorem, central-force motion, two-body problem, orbit theory, Kepler's laws of motion (the law of ellipses, the law of equal areas, the harmonic law), satellite motion, geostationary and polar satellites, kinematics of two-particle collisions, motion in non-inertial reference frame, rigid-body dynamics (3-D-rigid bodies and mechanical equivalence, motion of a rigid body, inverted pendulum and stability, gyroscope)	02%
11	**Number Theory**	**04%**
11.1	Divisibility, euclidean algorithm, GCD and LCM of 2 integers, properties of prime numbers, fundamental theorem of arithmetic, congruence relation, residue system, Euler's phi-function, solution of system of linear congruences, congruences of higher degree	02%
11.2	Chinese remainder theorem, Fermat little theorem, Wilson theorem, primitive roots and indices; integers belonging to a given exponent (mod p), primitive roots of prime and composite moduli, indices, solutions of congruences using indices, quadratic residues, composite moduli, quadratic residues of primes, the Legendre symbol, the Quadratic reciprocity law, the Jacobi symbol, Diophantine equations	02%

Sr. No.	Core Areas (in alphabetical order)	Weightage
12	**Real Analysis**	**07%**
12.1	*Theoretical Basis* Ordered sets, supremum and infimum, completeness properties of the real numbers, limits of numerical sequences, limits and continuity, properties of continuous functions on closed bounded intervals, derivatives in one variable, the mean value theorem, sequences of functions, sequences and series, power series, point-wise and uniform convergence, functions of several variables, open and closed sets and convergence of sequences in R^n; limits and continuity in several variables, properties of continuous functions on compact sets, differentiation in n-space, the Taylor series in R^n, the inverse and implicit function theorems	04%
12.2	*Integration Theory* Series of numbers and their convergence, series of functions and their convergence, Dabroux upper and lower sums and integrals, Dabroux integrability, Riemann sums and Riemann integrals, Riemann integration in R^2, change of order of variables of integration, Riemann integration in R^3, and R^n, Riemann-Steiltjes integration, functions of bounded variation, the length of a curve in R^n	03%
13	**Vectors**	**104%**
13.1	*Vector Analysis* 3-D vectors, summation convention, Kronecker delta, Levi-Civita symbol, vectors as quantities transforming under rotations with $\in_{ijk}$ notation, scalar- and vector-triple products, scalar- and vector-point functions, differentiation and integration of vectors, line integrals, path independence, surface integrals, volume integrals, gradient, divergence and curl with physical significance, vector identities, Green's theorem in a plane, divergence theorem, Stokes' theorem, coördinate systems and their bases, the spherical-polar- and the cylindrical-coördinate meshes	02%
13.2	*Tensor Analysis* Tensors of first, second and higher orders, algebra of tensors, contraction of tensor, quotient theorem, symmetric and skew-symmetric tensors, invariance property, tensors in modeling anisotropic systems, physical tensors (moment of inertia, index of refraction), diagnolization of inertia tensor as aligning coördinate frame with natural symmetries of the system	02%
	Total	**100%**

NOTES

a) This structured testing is not designed to find out *aptitude* (WILLINGNESS to learn and create mathematics — term papers/projects may be used for this purpose) but identify *basic competencies* (ABILITY to do mathematics) in the form of "techniques" (more in NAT) and "concepts" (more in GAT)

b) The test is reliable only if conducted under standardized conditions.

c) For each item (which is scored) included in the test, piloting (*candidate piloting***:** include such questions in a standard test, but do not include their scores in the cumulative score, as is the practice in ETS examinations, GRE, SAT, etc.; *test-developer piloting***:** members of moderation committee solve test items, each item solved separately by 2 members and average time computed) needs to be done for the following purposes**:**

i) Determine the level of difficulty of a problem (difficult, moderate, easy) by noting if the score distribution is skewed (to left, none, to the right) — *candidate piloting*

ii) Determine the optimum time to be allocated for solution of a problem (for each item, the candidate should be given 3 × time taken by moderation committee) — *candidate piloting* and *test-developer piloting*

iii) Determine logical, grammatical and sequential anomalies in a test item — *test-developer piloting*

On Friday, October 12, 2013, Prof. Dr. Tahira Haroon, Chairwoman, Departement of Mathematics, COMSTATS Institute of Information Technology, Islamabad presented some modifications in the above curriculum, which included changes in percentages of the core areas in the curriculum. This curriculum was approved by NTS Academic Committee.

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